

Networked Volatility: Linking Oil, Petrobrás and VALE in Times of Crisis

Volatilidade em Rede: Conectando Petróleo, Petrobrás e VALE em Tempos de Crise

Volatilidad en Red: Conectando Petróleo, Petrobrás y VALE en Tiempos de Crisis

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ABSTRACT

Goal: This study analyzes the influence of international oil prices on Petrobrás (PETR4) and VALE (VALE3) stocks, also considering the impact of the S&P 500 index from January 2021 to January 2023, a period marked by exogenous events such as the COVID-19 pandemic and the Russia-Ukraine conflict. **Methodology/approach:** The research uses daily data extracted via the yfR package from Yahoo Finance and applies a Vector Autoregressive (VAR) model, using ADF, KPSS, BIC, AIC, and HQ tests to check stationarity and determine lag selection. Additional tests of normality, autocorrelation, and heteroskedasticity were performed, along with the Generalized Forecast Error Variance Decomposition (GFEVD) method proposed by Diebold and Yilmaz (2012) to measure spillovers, and Granger causality testing (1969). **Originality/relevance:** The study's originality lies in applying GFEVD to a recent and globally volatile sample, focusing on key Brazilian companies in the international commodities market. **Main results:** Oil (CL1) influenced 4.81% of PETR4 and 7.72% of VALE3 variance over a 10-day horizon. The spillover index dropped during COVID-19 peaks and the start of the war, indicating temporary disruptions. **Theoretical/methodological contributions:** The findings support the VAR-GFEVD model as a robust tool for analyzing asset interconnectivity. **Managerial contributions:** Results offer guidance for investors and policymakers in understanding how Brazilian stocks respond to external shocks, enhancing strategic decisions in volatile environments.

Keywords: spillover. vector autoregression. GFEVD. oil price.

RESUMO

Objetivo: Este estudo analisa a influência do preço internacional do petróleo sobre as ações da Petrobrás (PETR4) e da VALE (VALE3), considerando também o impacto do índice S&P 500, no período entre janeiro de 2021 e janeiro de 2023, marcado por eventos exógenos como a pandemia da COVID-19 e o conflito Rússia-Ucrânia. **Metodologia/abordagem:** A pesquisa utiliza dados diários obtidos via pacote yfR do Yahoo Finance e aplica a modelagem Vetor Autorregressivo (VAR), com testes ADF, KPSS, BIC, AIC e HQ para verificação da estacionariedade e seleção de defasagem. Adicionalmente, foram realizados testes de normalidade, autocorrelação e heterocedasticidade, bem como a técnica de Decomposição Generalizada da Variância do Erro de Previsão (GFEVD), conforme Diebold e Yilmaz (2012), para mensuração dos spillovers, além do teste de causalidade de Granger (1969). **Originalidade/relevância:** A originalidade está na aplicação do GFEVD a uma amostra recente e altamente volátil, com foco em empresas brasileiras estratégicas no mercado global de commodities. **Principais resultados:** O petróleo (CL1) influenciou 4,81% da variância de PETR4 e 7,72% da variância de VALE3 em um horizonte de 10 dias. O índice de spillover apresentou quedas durante os picos da COVID-19 e no início da guerra. **Contribuições teóricas/metodológicas:** O estudo evidencia a robustez do modelo VAR-GFEVD na análise de conectividade entre ativos. **Contribuições para a gestão:** Os resultados auxiliam investidores e gestores públicos a tomarem decisões em cenários de alta instabilidade externa.

Palavras-chave: spillover. vetor autorregressivo. GFEVD. preço do petróleo.

RESUMEN

Objetivo: Este estudio analiza la influencia del precio internacional del petróleo sobre las acciones de Petrobrás (PETR4) y VALE (VALE3), considerando también el impacto del índice S&P 500 entre enero de 2021 y enero de 2023, período marcado por eventos exógenos como la pandemia del COVID-19 y el conflicto entre Rusia y Ucrania. **Metodología/enfoque:** La investigación utiliza datos diarios obtenidos mediante el paquete yfR del sitio Yahoo Finance, aplicando el modelo de Vectores Autorregresivos (VAR) junto con las pruebas ADF, KPSS, BIC, AIC y HQ para verificar la estacionariedad y determinar el número óptimo de rezagos. También se realizaron pruebas de normalidad, autocorrelación y heterocedasticidad, además de aplicar la técnica de Descomposición Generalizada de la Varianza del Error de Predicción (GFEVD), según Diebold y Yilmaz (2012), y la prueba de causalidad de Granger (1969). **Originalidad/relevancia:** La originalidad radica en aplicar el modelo GFEVD a una muestra reciente y volátil, centrada en empresas brasileñas estratégicas en el mercado global de commodities. **Resultados principales:** El petróleo (CL1) explicó el 4,81% de la varianza de PETR4 y el 7,72% de VALE3 en un horizonte de 10 días. El índice de spillover disminuyó en los picos del COVID-19 y al inicio de la guerra. **Contribuciones teóricas/metodológicas:** El estudio refuerza la utilidad del modelo VAR-GFEVD para analizar la conectividad entre activos. **Contribuciones para la gestión:** Los hallazgos ayudan a inversores y gestores públicos a tomar decisiones ante choques externos en contextos inestables.

Palabras clave: spillover. vectores autorregresivos. GFEVD. precio del petróleo.

■ INTRODUCTION

The pricing of financial assets through arbitrage has been explored in various ways in the academic literature, especially in situations characterized by high volatility and uncertainty. As noted by Lopes, Viegas, Conte Filho, and Carvalho (2021), many companies listed on stock exchanges maintain deep ties with the global market, particularly those involved in international trade. These companies are not only affected by external events but also play a role in price formation and market efficiency within the segments in which they operate (Shen, 2025).

In this specific context of the contemporary financial market, a crucial element is the role played by investors. Ribeiro (2022) highlights how the individual mood of market agents can exert a significant influence on the returns of several financial instruments, creating gaps through which informational asymmetries emerge, along with potential opportunities for profitable arbitrage operations. The feasibility of arbitrage between domestic financial assets and their counterparts in foreign markets, as exemplified by the interconnection between Brazilian stocks and their ADRs, reveals complex pricing relationships, both symmetric and asymmetric, in the international context (Wu, Li, & Feng, 2024; Wang, 2019).

From a theoretical perspective, this study is grounded in Arbitrage Pricing Theory (APT), which suggests that asset returns are affected by several macroeconomic factors, among which commodity prices stand out (Mussa et al., 2011). For a more in-depth methodological analysis, this study adopts the theory of volatility transmission across markets proposed by Diebold and Yilmaz (2012). This theory makes it possible to assess the connectivity among assets and the extent to which external shocks influence them through the generalized forecast error variance decomposition (GFEVD). This methodology has been widely used in recent studies to understand interdependencies among markets during global crises (Le & Luong, 2022; Mensi, Vo, & Kang, 2023).

In addition, macroeconomic and microeconomic factors have been used to understand how the international price of oil affects exporting economies, as demonstrated by Castro et al. (2018). It is also possible to analyze how commodities influence broad economic indicators, as discussed by Bernanke, Gertler, and Watson (1997).

In recent years, oil prices have become one of the main drivers of fluctuations in financial markets around the world. In the European context, Theodoro (2021) investigated the impact of this commodity on renewable energy companies. In Brazil, Souza and Veríssimo (2013) examined the effects of changes in oil prices on the exchange rate and the trade balance. This scenario is particularly relevant for Brazilian companies with high exposure to the international market, such as Petrobras and VALE. Santos et al. (2015) show that changes in global oil prices have the potential to substantially affect the assets of these companies, supporting the hypothesis of causality.

With a strong presence in the commodities market and a relevant impact on the Ibovespa index, Petrobras and VALE are emblematic cases for

examining the sensitivity of Brazilian assets to external shocks. According to Costa (2012), Petrobras plays a strategic role in the international arena. Huang, Masulis, and Stoll (1996), in turn, analyzed the relationship between the stock market and the oil market under the informational efficiency hypothesis, suggesting that, in efficient markets, there would be a contemporaneous correlation between these variables. However, studies such as Silva (2011) reveal more complex connections with time lags, indicating arbitrage opportunities and risk transmission mechanisms.

Although Petrobras has a direct economic link with the international oil market, the inclusion of VALE in the analyzed system stems from a different relationship. VALE operates predominantly in the iron ore market, but it has high exposure to the international commodity cycle, exchange rates, foreign demand, and global liquidity and risk-aversion conditions. Thus, the relationship between CL1 and VALE3 is not treated in this study as a direct operational relationship, but rather as a possible financial interdependence between strategic Brazilian assets that are sensitive to external shocks. Therefore, the analysis seeks to verify whether shocks in oil prices are indirectly transmitted to VALE3 through the systemic dynamics of the financial market and portfolio reallocation during crisis periods.

In addition to their significant weight in the Ibovespa index, Petrobras and VALE play a strategic role in Brazil's energy policy and macroeconomic performance. Petrobras is at the center of the national energy matrix, while VALE is one of the country's largest exporters. Analyzing the effects of oil prices on these companies contributes to investment decisions and public policy formulation, especially in contexts of global instability (Liu, Shao, & Zhang, 2020; Filho et al., 2021; Attarzadeh & Balcilar, 2022; Escribano et al., 2023).

Given this background, the objective of this study is to deepen the investigations initiated by Santos et al. (2015) by updating the empirical database through January 2023, a period marked by relevant events such as the COVID-19 pandemic and the beginning of the Russia-Ukraine conflict. Using VAR modeling and the methodology proposed by Diebold and Yilmaz (2012), this study seeks to assess the impact of oil prices (CL1), the S&P 500 index, and VALE3 stock returns on the variance of PETR4 returns, with the aim of identifying patterns of interconnection and potential structural breaks in the dynamics of volatility transmission among the assets (Da Silva et al., 2024).

The research gap explored in this study arises from the need to understand how shocks associated with the international oil market are connected to strategic Brazilian assets in a recent period of global instability. Although the literature has analyzed the relationship among oil, stock markets, and commodities in different contexts, there is still room to investigate the connectivity among CL1, PETR4, VALE3, and the S&P 500 in the Brazilian market between 2021 and 2023, with an emphasis on variance decomposition and the direction of volatility spillovers. This approach allows the study to move beyond analyses based solely on correlation or causality, as it identifies the relative contribution of each variable to the forecast error variance of the others.

The results obtained in this investigation are expected to offer valuable contributions to both the academic community and the financial and business environments. For investors, the findings may guide strategic decisions related to pricing and risk management in contexts of high uncertainty. In the realm of public policy formulation, the knowledge generated regarding

the response of Brazilian stocks to external shocks may support strategies related to the governance of the national energy infrastructure. Finally, the use of updated data and advanced econometric methods contributes to improving the understanding of volatility transmission processes in the Brazilian context (Salisu & Gupta, 2020; Ekanayake, 2024; Reis & Santana, 2024).

■ THEORETICAL FRAMEWORK

The International Oil Market and the Transmission of Shocks

The international oil market constitutes one of the main channels through which shocks are transmitted between the real economy and financial markets. Recent literature has reinforced that the relationship between oil and stock markets is neither static nor homogeneous. Escribano, Koczar, Jareño, and Esparcia (2023), by analyzing the connectivity between oil prices and stock markets from January 2000 to February 2023, show that periods of crises and conflicts tend to alter the correlation and transmission of shocks between oil and stock indexes. This finding is relevant to the present study because the 2021–2023 period combines the lingering effects of the COVID-19 pandemic, changes in global growth expectations, and the beginning of the Russia-Ukraine conflict, creating a suitable environment for investigating the connectivity between oil and strategic Brazilian assets.

As a global energy commodity, oil is sensitive to geopolitical events, supply changes, shifts in international demand, and expectations regarding economic growth. Therefore, its fluctuations may affect companies directly linked to the oil sector as well as financial assets exposed to global macroeconomic factors.

The international price of oil plays an extremely important role in the stock market, affecting not only companies directly involved in the oil industry but also several sectors of the global economy (Bamana, 2019). Consequently, the application of autoregressive vectors to the analysis of economic indexes has been widely used for time series assessment, as proposed by Sims (1980). In this regard, Santos (2015) sought to evaluate the impact of international oil prices on the Ibovespa index from 2008 to 2012. Given the representativeness of oil companies in the Ibovespa, the author identified significant shocks from oil prices to the index through a VAR(p) model.

In general, oil has consolidated its position as one of the main sources of energy for international industries. Rego and Marques (2006) show that its importance began in the 1950s, with the operations of companies such as Petrobras, Vale do Rio Doce, and Companhia Siderúrgica Nacional. Since then, oil has played a crucial role in the international economy. This variable has been used in several studies investigating its relationship with the volatility of financial assets, as observed by Osah (2023), who analyzes the influence of oil prices on 12 economies with different characteristics. Fluctuations in oil prices have the potential to affect the stocks of companies in the oil sector, including exploration, production, and refining, as well as to influence production costs, consumption patterns, and global demand for products and services.

Studies have also demonstrated relationships among different stock markets. Garbade and Silber (1983) investigate the asymmetric causal relationship between the commodity futures market and the spot market, analyzing the effect of arbitrage on price changes. Similarly, Sadorsky (2000) analyzes the integration among energy futures prices to test the arbitrage condition in crude oil, applying a vector autoregressive model to demonstrate cointegration among related variables.

In the field of futures markets, Bekiros and Diks (2008) investigate causal relationships between daily spot prices and futures prices with maturities ranging from one to four months for international oil prices. Using a vector error correction model (VECM), they identified that futures prices influence spot prices, indicating arbitrage opportunities. Hammoudeh and Choi (2006), also using a VECM, analyzed the effects of Brent oil and macro-economic indexes on the stock index of the Gulf Cooperation Council (GCC), finding that positive oil shocks benefit most markets.

When considering asymmetric and cointegration features, Cheikh et al. (2022) revisited the findings of Hammoudeh and Choi (2006) after the COVID-19 crisis, confirming evident shocks in some markets. Caporale et al. (2022) analyzed the effects of oil prices on stock returns in BRICS-T countries, identifying positive effects in the commodities sector, except in India, and negative effects in the Brazilian financial sector.

The importance of oil prices for volatility analysis is reinforced by Le and Luong (2022), who, using a TVP-VAR model, analyze dynamic spillovers among oil price shocks, stock returns, and investor sentiment in the United States and Vietnam. The authors identified oil as a net transmitter of shocks to U.S. stock returns between 2010 and 2020.

Along the same lines, Huang, Chen, Xu, and Xia (2023) show that volatility spillovers between energy commodities and financial markets exhibit dynamic behavior and are sensitive to crisis periods. Using a frequency-domain TVP-VAR connectivity approach during COVID-19, the authors identified that volatility transmission between energy and financial markets was marked by relevant time-varying patterns, with intensification during the pandemic period. This finding reinforces the relevance of analyzing shock transmission in recent windows of instability, such as the one adopted in this study.

Brazilian Listed Companies, Commodities, and International Market Exposure

In the Brazilian context, Petrobras and VALE represent strategic companies, with high liquidity in the stock market and strong exposure to external factors. Petrobras has a direct economic relationship with the international oil market, whereas VALE, although primarily linked to iron ore, is embedded in the global commodity cycle, exchange rate dynamics, and international investment flows. Thus, the joint analysis of these companies makes it possible to examine the financial interdependence among relevant Brazilian assets during periods of external instability.

The joint inclusion of PETR4 and VALE3 is also justified from the perspective of connectivity among assets exposed to global factors, even if through different channels. Attarzadeh and Balcilar (2022) show that spillovers among oil, the stock market, clean energy, and technology may vary according to market conditions, becoming more intense during periods of

extreme shocks. Thus, the analysis of VALE3 within the same system as PETR4 and CL1 does not assume a direct operational relationship with oil, but rather a possible indirect exposure to systemic shocks, global liquidity conditions, risk aversion, and portfolio reallocation movements in commodity and equity markets.

With the objective of verifying the influence of economic variables on Petrobras common shares, Lopes et al. (2021) analyzed stock data from 2009 to 2020, applying the Ordinary Least Squares model to identify positive or negative correlations. The authors found an influence of Brent oil, the Ibovespa itself, and the company's production level. Previously, Leite et al. (2016) applied the same model to the 2005–2015 period, with the aim of analyzing the effect of international oil prices on Petrobras shares, reaching similar conclusions.

Aroui et al. (2011) find that oil volatility influences stocks in seven industrial sectors in Europe and the United States, with a unidirectional relationship in Europe and a bidirectional relationship in the United States.

Regarding the timing of influence among variables, Brooks et al. (2001) used high-frequency data to examine the lead-lag relationship between the FTSE 100 index and its futures contracts, finding evidence that the futures market anticipates the spot market.

In Brazil, Silveira (2016) analyzed data from 2006 to 2014 and identified a significant influence of Brent oil prices on stock returns. Specifically regarding PETR4, Silva (2011) found a contemporaneous relationship between oil prices and Petrobras shares, as well as a lead-lag effect using high-frequency data.

To reinforce the possibility of arbitrage, Santos et al. (2015) used high-frequency data to investigate the intraday influence of oil futures contracts, the S&P 500 index, and Petrobras preferred shares. The results indicated causality from oil and the index to PETR4, with a 15-minute lag.

Regina et al. (2012) analyzed the cross-impacts among PETR4, VALE5, West Texas oil prices, and iron ore prices, identifying a correlation among these variables. Along similar lines, Wolff, Santos, and Souza (2011) found direct transmission from commodity indexes to Vale stock, but not to Petrobras stock in the long run.

Volatility Transmission, Arbitrage, and Investor Behavior

In addition to identifying relationships between prices and returns, recent studies have advanced the measurement of connectivity among financial assets through variance decomposition models. This approach makes it possible to assess not only whether there is an association between variables, but also the direction and intensity of volatility spillovers, which is a central aspect for understanding asset behavior during crisis periods.

The Generalized Forecast Error Variance Decomposition (GFEVD), proposed by Diebold and Yilmaz (2012), is a central methodology for measuring spillovers. Applied to the U.S. market, it made it possible to identify volatility transmission among stock, bond, foreign exchange, and commodity markets.

Recent studies have also advanced the analysis of dynamic connectivity between oil and stock markets in emerging economies. Qin, Cong, Ma, and Rong (2024) show that the connectivity between oil and emerging stock markets is heterogeneous, dynamic, and sensitive to market conditions,

being higher in extreme scenarios than during normal periods. This evidence reinforces the importance of investigating the Brazilian case using a recent sample, since PETR4 and VALE3 are liquid assets that are relevant to the domestic stock market and exposed to different channels of international shock transmission.

Nazareno (2023) applies the GFEVD methodology to analyze the contribution of oil prices to inflation, demonstrating their importance as a transmitter of volatility. Gurrola-Ríos et al. (2021) analyzed spillovers between the S&P 500 and Latin American markets before and after the pandemic, confirming positive transmission from the S&P 500.

Diebold, Liu, and Yilmaz (2017) applied GFEVD to the commodity market and identified the energy sector as the main transmitter of volatility. Balcilar, Gabauer, and Umar (2021) observed a strong connection between 11 agricultural commodities and oil futures prices.

Xiaoe and Huang (2018) analyzed the dynamic connectivity of oil prices and identified Brent and WTI as net transmitters of volatility. Zhang and Wang (2014) found strong dependence of the Chinese market on the global oil market. Hung (2022) showed that the S&P 500 transmits volatility to oil and gold, and that oil, in turn, transmits volatility to gold.

On the African continent, Mensi, Vinh, and Kang (2023) evaluated spillovers across different market regimes and confirmed oil as a net transmitter. Roudari et al. (2023) observed that positive shocks in oil production positively affect the stock markets of Norway and Japan. Trinh and My (2023) identified impacts of oil prices on Vietnam's industrial production, inflation, and domestic prices.

The recent literature indicates that shock transmission between oil and stock markets does not occur homogeneously across countries, sectors, and crisis periods. This heterogeneity makes it relevant to analyze the Brazilian case, especially because PETR4 has a direct link with the oil sector, whereas VALE3 represents an exporting company associated with the global commodity cycle, but without a direct operational relationship with oil. Thus, the contribution of this study lies in examining whether the observed connectivity stems from direct sectoral exposure, as in the case of PETR4, or from indirect mechanisms of financial transmission, such as risk aversion, international liquidity, exchange rates, and portfolio reallocation, as in the case of VALE3.

Based on Arbitrage Pricing Theory and the literature on volatility transmission, shocks to international oil prices are expected to help explain part of the variance of PETR4 returns, given its direct sectoral relationship with the commodity. For VALE3, a less direct and potentially more limited relationship is expected, associated with the financial interdependence among Brazilian assets linked to the international commodity cycle. The S&P 500 is also expected to act as a relevant international control variable, as it reflects global market conditions and shifts in risk aversion or risk appetite.

Moreover, the literature highlights that extreme events can alter the direction and intensity of connectivity between oil and stock markets. Cui, Maghyreh, and Liao (2024), by analyzing risk connectivity between oil and stock markets during the COVID-19 pandemic and the Russia-Ukraine conflict, show that spillovers are sensitive to major crisis events and geopolitical factors. This evidence supports the choice of the time frame adopted in this study and reinforces the need to assess whether international oil

shocks were transmitted, even partially, to strategic Brazilian assets during the 2021–2023 period.

Given the extensive literature linking oil prices to stock market dynamics, especially in times of crisis and geopolitical instability, studies that analyze this phenomenon in the recent Brazilian context using modern volatility decomposition techniques remain scarce. This study seeks to address this gap by integrating VAR-GFEVD models into the analysis of Petrobras and VALE stocks from 2021 to 2023, a period marked by simultaneous shocks and market disruptions.

METHODOLOGY

Data

To analyze the relationships among the international price of oil, Petrobras stock prices (PETR4), VALE stock prices (VALE3), and the S&P 500 index, the variables described below were used, considering a time frame from January 1, 2021, to January 1, 2023. This period was selected because it follows the onset of the COVID-19 pandemic, with the aim of avoiding the capture of more pronounced extreme effects or exogenous distortions in the estimated model. The data were collected at a daily frequency.

The information was obtained through the yfR package (version 1.1.0), which interfaces with the public Yahoo Finance database, as described by Perlin (2021).

The inclusion of VALE3 in the model does not assume that oil is a direct operational determinant of its core activity. The justification for its presence in the system stems from its relevance in the Brazilian stock market, its high liquidity, its international exposure, and its inclusion among Brazilian companies associated with the global commodity cycle. Thus, the model seeks to measure the connectivity among strategic assets traded on B3, assessing the possibility of indirect transmission of oil shocks to VALE3 in a context of global instability.

The variables and their specifications are presented below:

Variables:

- **CL1:** International oil price futures contracts with a one-month maturity.
- **PETR4:** Preferred stock of Petrobras.
- **VALE3:** Common stock of VALE.
- **S&P 500:** Benchmark index of the New York stock market.
- **Dollar (BRL/USD):** Daily quotation of the U.S. dollar against the Brazilian real, used for data standardization.
- **Measurement:** Data from the last daily quotation from January 1, 2021, to January 1, 2023.
- **Package tickers:** “BRL=X” (Dollar), “CL=F” (CL1), “PETR4.SA” (PETR4), “VALE3.SA” (VALE3), “^GSPC” (S&P 500).

VAR(p) Econometric Procedure

This section presents the econometric procedure used to estimate the dynamic relationships among the variables in the study. The VAR(p) model, proposed by Sims (1980), makes it possible to analyze multivariate systems in which each variable is explained by its own lags and by the lags of the other variables in the system. In general terms, it is a linear model with n variables defined by n equations, in which the variables are functions not only of their own lags but also of the lags of the other variables. In this sense, the general form of the VAR(p) model is given by Equation 1:

$$Y_t = \mu + \pi_1 Y_{t-1} + \dots + \pi_p Y_{t-p} + \varepsilon_t \quad (1)$$

In this case, Y_t is an “ $n \times 1$ ” vector of stationary variables, that is, variables integrated of order zero, $I(0)$, with covariance remaining constant throughout the period. Thus, each regressed time series incorporates the autoregressive coefficients and the lagged values of the other series. Moreover, the errors must follow a multivariate normal distribution with zero mean and variance Ω , that is, $\varepsilon_t \sim NM(0, \Omega)$. In addition, the expected value between ε_t and ε_j for all $t \neq j$, is equal to zero, indicating the absence of autocorrelation, $E(\varepsilon_t, \varepsilon_j) = 0$, $\forall t \neq j$, as well as $\sum \varepsilon_t < \infty$.

Tests and VECM

When the series in levels are not stationary, it becomes necessary to transform the variables, usually through first differencing, to avoid spurious estimations and meet the conditions required for VAR estimation. If there is evidence of a long-run relationship among the variables, cointegration tests are performed on the series. Once such a relationship is identified, the application of the VECM becomes necessary. The VECM, or Vector Error Correction Model, measures short- and long-run effects through linear combinations (Bueno, 2020).

In this context, Granger causality is evaluated, as it verifies whether the lagged values of one variable contribute to improving the forecast of another variable in the system. Thus, one variable is said to Granger-cause another when its lags reduce the forecast error of the dependent variable.

As in Santos et al. (2015), to assess the integration of the series, that is, whether they are nonstationary in covariance, tests are applied to identify the presence of a unit root in the series, such as the Augmented Dickey-Fuller (ADF) test, in addition to graphical inspection. The definition of the number of lags follows the BIC (Schwarz), AIC (Akaike), and HQ (Hannan-Quinn) information criteria, for which lower values are preferable. Along these lines, the number of lags is defined by the majority of the tests. In cases of divergent results, the specification indicated by the BIC (Schwarz) is selected, as recommended by Lutkepohl and Kratzig (2004).

According to Johansen and Juselius (1990), after defining the number of lags, the Johansen cointegration test is performed, which is based on the

trace test and maximum eigenvalue test statistics. Through this procedure, three possible results are obtained for $\text{Rank}(\pi)$:

1. **$\text{Rank}(\pi) = 0$.** This indicates that a VAR model should be estimated, since there are no cointegrating vectors.
2. **$\text{Rank}(\pi) = n$.** In this case, the number of cointegrating vectors is equal to the number of variables contained in the vector Y'_t , indicating that a VAR model should be estimated using the variables in levels.
3. **$0 < \text{Rank}(\pi) < n$.** In this case, $Y'_t \sim I(1)$ indicates that there are cointegrating vectors among the variables; therefore, the VEC model should be estimated.

To estimate the impulse response function, Cholesky decomposition is required, indicating the paths of random shocks on the trajectory of the variables. In this case, the equation is expressed as follows:

$$\Delta y_t = c + \pi y_{t-1} + \phi_1 y_{t-1} + \phi_1 \Delta y_{t-1} + \dots + \phi_p \Delta y_{p-1} + \varepsilon_t \quad (2)$$

In the case, πy_{t-1} is interpreted as the error correction term. In addition, to test for autocorrelation in the residuals, the Breusch-Godfrey test is performed, while the Jarque-Bera test is used to assess the normality of the residuals. The ARCH-LM test is also applied to validate the homoscedasticity of the residuals. Finally, it is also examined whether the roots lie outside the unit circle to corroborate the hypothesis of model stability.

Network Connectivity Dynamics

Diebold and Yilmaz (2012) consolidated an approach for measuring volatility spillovers across financial markets based on the forecast error variance decomposition associated with VAR models. This methodology makes it possible to assess the intensity, direction, and connectivity of shocks among the variables in the system.

In this case, for the forecast error variance decomposition, Equation 1 is rewritten as follows $Y_t = \mu + \sum_{i=1}^p \pi_i Y_{t-i} + \varepsilon_t$, and, if the model is covariance-stationary, the moving-average representation of the VAR exists. Therefore, it is possible to compute the h -step-ahead forecast error variance decomposition, as shown in Equation 3:

$$Y_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i} \quad (3)$$

So, $A_i = \pi_1 A_{i-1} + \pi_2 A_{i-2} + \dots + \pi_n A_{i-n}$, where A_0 is the identity matrix. Unlike the decomposition based on Cholesky orthogonalization, the GFEVD, as shown in Equation 4, makes it possible to measure the contribution of shocks without imposing a specific ordering of the variables in the system. This feature is relevant for connectivity studies because it reduces the sensitivity of the results to the order in which the variables are included in the VAR.

$$\theta_{ij}^g(\mathbf{H}) = \frac{\sigma_{jj} \sum_{h=0}^{H-1} (e_i' A_h \sum e_j)^2}{\sum_{h=0}^{H-1} (e_i' A_h \sum A_h' e_i)} \quad (4)$$

Where σ_{jj} is the standard deviation of the error term for the j -th equation, and e_i is the selection vector, with one as the i -th element and zero otherwise. Since the shocks in the framework are not orthogonal, the equation must be normalized in the form presented in Equation 5.

$$\tilde{\theta}_{ij}^g(\mathbf{H}) = \frac{\theta_{ij}^g(\mathbf{H})}{\sum_{j=1}^N \theta_{ij}^g(\mathbf{H})} \quad (5)$$

Thus, based on Diebold and Yilmaz (2012), the values of $\tilde{\theta}_{ij}^g(\mathbf{H})$ $i \neq j$, are defined as the pairwise directional connectedness measures, which can also be used to calculate total connectedness. Along these lines, these values generate spillover indices that make it possible to identify the receivers and transmitters of volatility.

For purposes of interpreting the connectedness matrix, a value equal to 25 in a cell outside the main diagonal indicates that shocks originating from one variable explain 25% of the forecast error variance of another variable over an h -step-ahead horizon. Thus, the matrix makes it possible to identify both the transmitters and receivers of volatility within the system. Once the table is constructed, the total, directional, and net directional connectedness indices can be developed.

The total connectedness index is given by Equation 6.

$$S^g(\mathbf{H}) = \frac{\sum_{i,j=1, i \neq j}^N \tilde{\theta}_{ij}^g(\mathbf{H})}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(\mathbf{H})} \times 100 = \frac{\sum_{i,j=1, i \neq j}^N \tilde{\theta}_{ij}^g(\mathbf{H})}{N} \times 100 \quad (6)$$

In addition, the directional connectedness from all other states (j) to state (i) is given by:

$$S_{\cdot,i}^g(\mathbf{H}) = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ij}^g(\mathbf{H})}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(\mathbf{H})} \times 100 = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ij}^g(\mathbf{H})}{N} \times 100 \quad (7)$$

The directional connectedness from state (i) to all other states (j) is given by:

$$S_{i,\cdot}^g(\mathbf{H}) = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ji}^g(\mathbf{H})}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(\mathbf{H})} \times 100 = \frac{\sum_{j=1, j \neq i}^N \tilde{\theta}_{ji}^g(\mathbf{H})}{N} \times 100 \quad (8)$$

The net directional connectedness from state (i) to all other states is given by:

$$S_i^g(\mathbf{H}) = S_{i,\cdot}^g(\mathbf{H}) - S_{\cdot,i}^g(\mathbf{H}) \quad (9)$$

Finally, the pairwise net connectedness from state (i) to state (j) can be described as follows:

$$S_{ij}^g(H) = \left(\frac{\tilde{\theta}_{ij}^g(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)}, - \frac{\tilde{\theta}_{ji}^g(H)}{\sum_{i,j=1}^N \tilde{\theta}_{ij}^g(H)} \right) \times 100 = \left(\frac{\tilde{\theta}_{ij}^g(H) - \tilde{\theta}_{ji}^g(H)}{N} \right) \times 100 \quad (10)$$

RESULTS AND DISCUSSION

This section presents the empirical results of the study and discusses their econometric and economic implications. Initially, a descriptive analysis of the series used in the study was conducted. As shown in Table 1, the series were treated and converted into U.S. dollars in order to reduce the influence of exchange rate fluctuations on the prices analyzed. During the sample period, the international price of oil with a one-month maturity, represented by CL1, had an average value of USD 81.19. VALE3 had an average value of USD 16.71, PETR4 had an average value of USD 5.48, and the S&P 500 index had an average value of 4,186 points. The remaining values for minimum, maximum, median, first quartile, and third quartile are presented in Table 1.

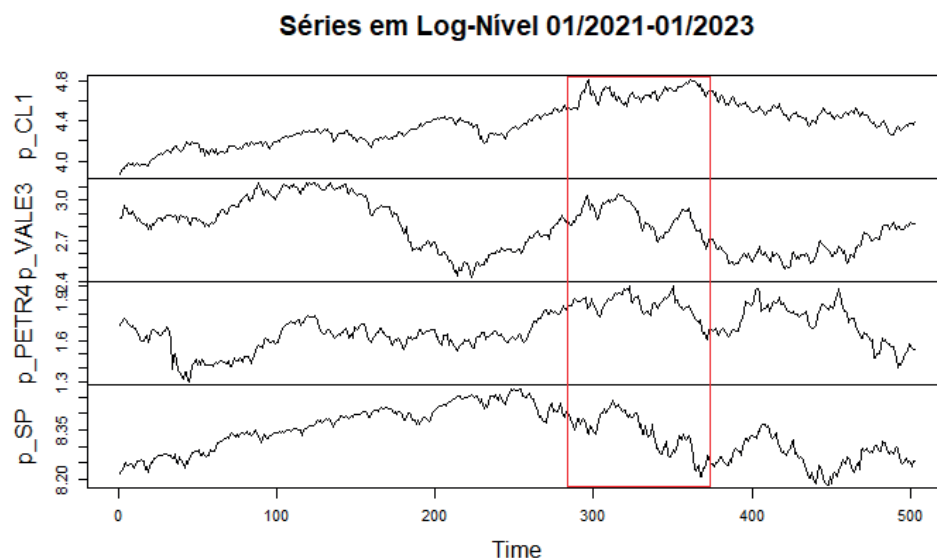
Table 1

Descriptive Statistics of the Series

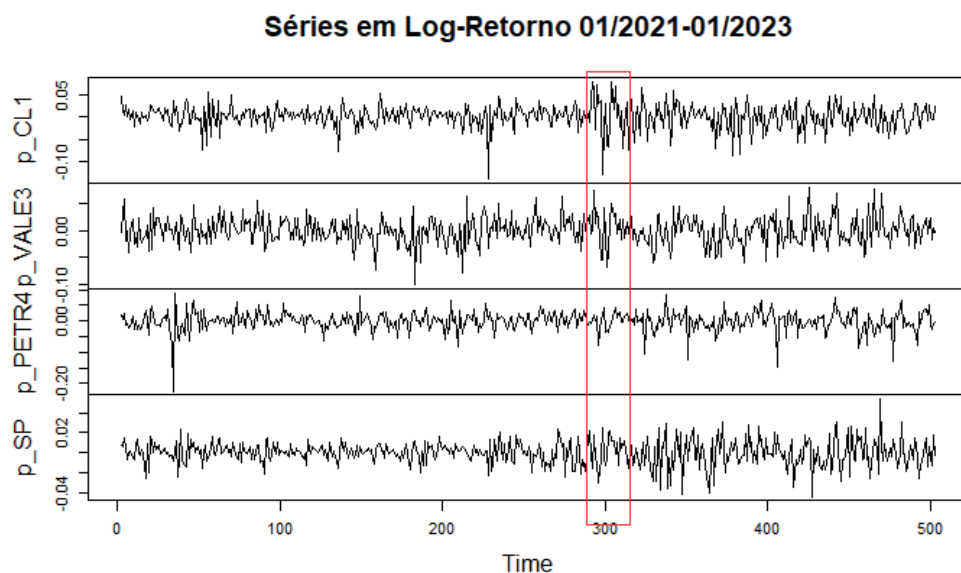
	p_CL1	p_VALE3	p_PETR4	p_SP
Min	47.62	11.27	3.675	3577
1st Qu.	68.40	13.92	4.967	3919
Median	78.96	16.66	5.338	4181
Mean	81.19	16.71	5.479	4186
3rd Qu.	91.86	18.88	6.120	4437
Max.	123.70	22.88	7.418	4797

Source: Prepared by the authors

Figure 1 presents the series in log-levels over the analyzed period. Sharp fluctuations are observed in the series, especially in oil prices and PETR4, suggesting partially synchronized movements between these assets. This initial behavior indicates the possibility of an association among the variables, which is subsequently assessed through econometric tests and connectivity analysis. In the period highlighted in red, it becomes clear that the sharp appreciation in oil prices, driven by the Russia-Ukraine conflict, boosted the other stocks.

Figure 1*Series in Log-Levels over the Analyzed Period***Source:** Prepared by the authors

Regarding the log-returns of the time series, the same behavior is observed, as shown in Figure 2.

Figure 2*Log-Returns of the Time Series***Source:** Prepared by the authors

Next, the Pearson correlation matrix of the log-returns was estimated in order to assess preliminary linear associations among the variables analyzed. The results indicate positive correlations between CL1 and the other assets in the system, suggesting an initial association between the international price of

oil and the returns of PETR4 and VALE3. This evidence, however, should be interpreted only as a preliminary indication, since the dynamic connectedness among the assets is subsequently analyzed through the GFEVD.

Table 2

Correlation Matrix of the Log>Returns

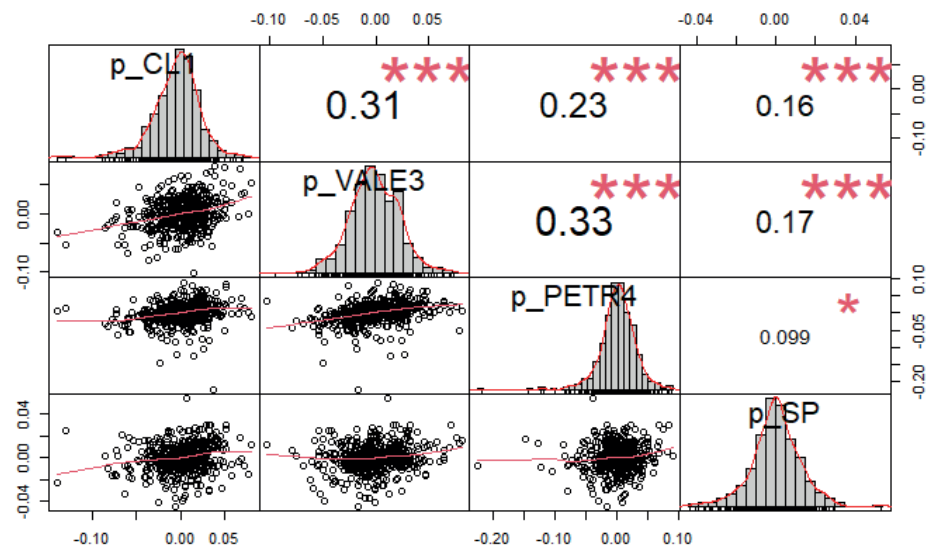
	p_CL1	p_VALE3	p_PETR4	p_SP
p_CL1	1.00			
p_VALE3	0.3135	1.00		
p_PETR4	0.2323	0.3312	1.00	
p_SP	0.1599	0.1661	0.099	1.00

Source: Prepared by the authors

It is also observed that the correlations are statistically significant at the 1% level (Figure 3), which reinforces the existence of linear associations among the variables. Even so, because correlation does not allow the direction or dynamic intensity of shocks to be identified, it becomes necessary to analyze volatility spillovers through variance decomposition.

Figure 3

Correlation of the Log>Returns



Source: Prepared by the authors

Subsequently, the Augmented Dickey-Fuller (ADF) test was used to verify the presence of a unit root in the series. As a complementary procedure, the KPSS test was applied, whose null hypothesis assumes stationarity of the series. The joint use of both tests allows stationarity to be assessed more robustly, since their null hypotheses are distinct. In general, the results for the series in log-levels indicate the presence of a unit root at the 1% significance level.

After taking the first difference of the series, they are found to be stationary at the 1% significance level, as shown in Table 3, consistent with Silva (2011) and Santos et al. (2015).

Table 3

Results of the Augmented Dickey-Fuller Test

Serie	trend			drift		none
	tau3	phi2	phi3	tau2	phi1	tau1
Critical Value (1%)	-3.96	6.09	8.27	-3.43	6.43	-2.58
Log-level CL1	-1.81	1.84	2.49	-2.20	2.69	0.79
Log-return CL1	-11.00	40.36	60.54	-10.86	58.95	-16.71
Log-level VALE3	-1.77	1.07	1.61	-1.64	1.36	-0.19
Log-return VALE3	-8.02	21.44	32.15	-8.02	32.18	-8.03
Log-level PETR4	-2.50	2.17	3.24	-2.40	2.89	-0.36
Log-return PETR4	-15.81	83.34	125.01	-15.81	125.04	-15.83
Log-level S&P	-2.29	2.14	3.20	-1.89	1.78	-0.03
Log-return S&P	-16.41	89.73	134.6	-16.36	133.84	-16.38

After applying the Augmented Dickey-Fuller test, the KPSS test was used as a complementary procedure to verify the stationarity of the series. Considering the 5% significance level, the series in log-levels present p-values equal to 0.01, which leads to the rejection of the null hypothesis of trend stationarity. In contrast, the series in log-returns present p-values equal to 0.10, meaning that the null hypothesis of the KPSS test is not rejected, as shown in Table 4. These results indicate that the series in log-returns are stationary, supporting the decision to use the variables in first logarithmic differences in the estimation of the VAR model.

Table 4

Results of the KPSS Test

Serie	KPSS Trend	p-valor
Log-level CL1	1.18	0.01
Log-return CL1	0.04	0.10
Log-level VALE3	0.34	0.01
Log-return VALE3	0.08	0.10
Log-level PETR4	0.59	0.01
Log-return PETR4	0.06	0.10
Log-level S&P	0.59	0.01
Log-return S&P	0.04	0.10

Source: Prepared by the authors

In order to determine the number of lags (p) to be used in the model, the Schwarz information criterion (BIC), the Akaike information criterion (AIC), and the Hannan-Quinn criterion (HQ) were considered, with (p) ranging from 1 to 15. In cases of divergent results, the specification indicated by the BIC was selected, that is, ($p = 1$), as shown in Table 5.

Table 5

Selection of the Number of Lags

p	AIC.n.	HQ.n.	SC.n.
1	-30.29	-29.47	-28.19
2	-30.27	-29.39	-28.03
3	-30.23	-29.29	-27.85
4	-30.21	-29.22	-27.7
5	-30.2	-29.16	-27.55
6	-30.18	-29.09	-27.39
7	-30.16	-29.01	-27.23
8	-30.12	-28.92	-27.06
9	-30.1	-28.85	-26.9
10	-30.1	-28.79	-26.76
11	-30.1	-28.73	-26.62
12	-30.06	-28.64	-26.45
13	-30.04	-28.57	-26.29
14	-30.01	-28.49	-26.13
15	-30.01	-28.43	-25.98

Source: Prepared by the authors

Subsequently, the Johansen cointegration test (1990) was applied, with the objective of identifying at least one cointegrating vector. However, the results of the trace test and the maximum eigenvalue test do not indicate the existence of at least one cointegrating vector at the 1% significance level, as shown in Table 6. This differs from the approach proposed by Santos et al. (2015), who worked with high-frequency data and were therefore more susceptible to cointegration.

Table 6*Cointegration Tests*

Rank	Critical Value (1%)	Trace	Critical Value (1%)	Max Eigenvalue
$r \leq 3$	12.97	1.12	12.97	1.12
$r \leq 2$	24.60	4.87	20.20	3.75
$r \leq 1$	41.07	14.29	26.81	9.43
$r = 0$	60.16	34.82	33.24	20.53

Source: Prepared by the authors

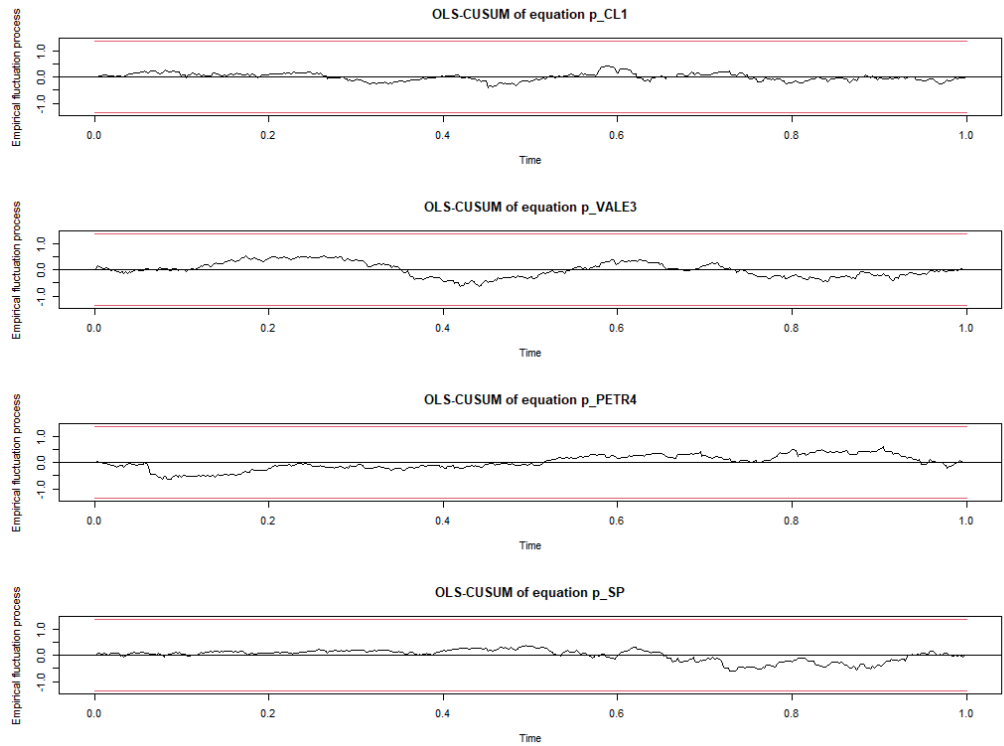
After estimating the VAR(1), residual diagnostic tests were performed. The Jarque-Bera test indicated rejection of the null hypothesis of residual normality. Although this evidence should be considered when interpreting the results, the literature suggests that, in sufficiently large samples, violations of normality tend to be less problematic for the consistency of VAR estimates. Next, the Portmanteau test was applied to verify the presence of residual autocorrelation, and the null hypothesis of no autocorrelation was not rejected. The ARCH-LM test was also applied, in which the null hypothesis of no ARCH effects in the residuals was not rejected. The results are presented in Table 7.

Table 7*Model Tests*

Test	Chi ²	df	p-value
Jarque-Bera (Normality)	3847.70	8	0.0000
Portmanteau (Autocorrelation)	238.81	240	0.5096
ARCH (Heteroskedasticity)	514.65	500	0.3157

Source: Prepared by the authors

Based on Figure 4, it can be observed that all variables pass the stability test for the return residuals, without exceeding any threshold. Thus, it is possible to proceed with the estimation of the spillovers proposed by Diebold and Yilmaz.

Figure 4*OLS-CUSUM Test for Residual Stability***Source:** Prepared by the authors

In order to assess the impact of one variable on the forecast error of another, the forecast error variance decomposition (FEVD) was performed. As shown in Table 8, over a four-day horizon, shocks to CL1, VALE3, and the S&P 500 explain, respectively, 5.62%, 7.28%, and 0.00% of the forecast error variance of PETR4. These results indicate that, over this horizon, most of the forecast variance of PETR4 remains associated with shocks to the stock itself, although CL1 and VALE3 make a non-negligible contribution to its dynamics.

Table 8*Results of the Variance Decomposition: PETR4*

Forecast Error Variance Decomposition: Shock to PETR4				
Period	p_{PETR4}	p_{CL1}	p_{VALE3}	p_{SP}
1	0.8749	0.0569	0.0681	0.0000
2	0.8707	0.0562	0.0727	0.0002
3	0.8705	0.0562	0.0728	0.0003
4	0.8705	0.0562	0.0728	0.0003

Source: Prepared by the authors

Table 9 presents the forecast error variance decomposition for VALE3. Compared to PETR4, a higher relative contribution of CL1 to the forecast variance of VALE3 is observed over the analyzed horizon. This result should be interpreted as evidence of indirect financial connectedness between oil and VALE3, rather than as a direct operational relationship between oil and iron ore.

Table 9

Results of the Variance Decomposition: VALE3

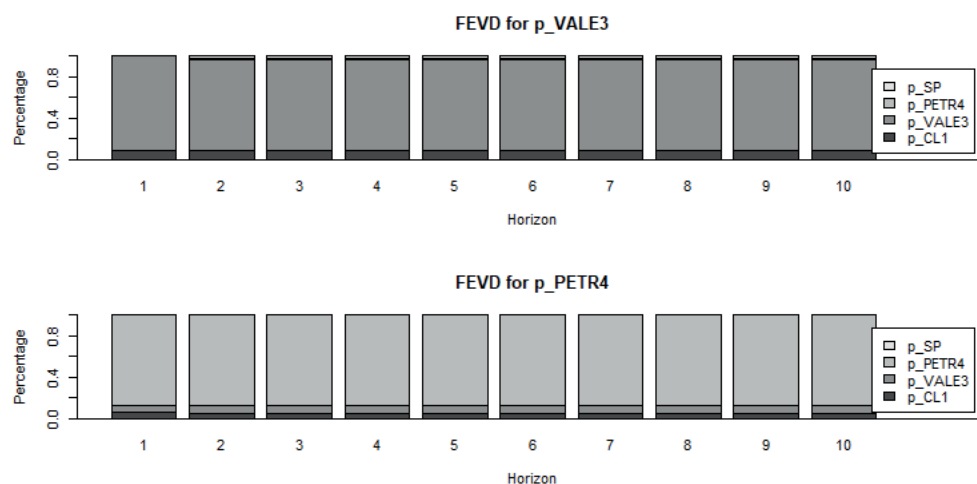
Forecast Error Variance Decomposition: Shock to VALE3				
Period	p_PETR4	p_CL1	p_VALE3	p_SP
1	0.0000	0.0908	0.9091	0.0000
2	0.0055	0.0952	0.8772	0.0220
3	0.0056	0.0952	0.8771	0.0220
4	0.0056	0.0952	0.8771	0.0220

Source: Prepared by the authors

Figure 5 presents the variance decomposition over a 10-day horizon. It is observed that, after the initial periods, the relative contributions of the variables tend to stabilize, indicating that the main transmission effects are concentrated in the first days of the forecast horizon.

Figure 5

Variance Decomposition over a 10-Day Horizon



Source: Prepared by the authors

The connectedness measures were estimated for a 10-step-ahead horizon, following the approach of Diebold and Yilmaz (2012). When examining the results for the international oil price, it is possible to infer that CL1 accounts for 4.81% of the 10-step-ahead forecast error variance when predicting PETR4, while from the perspective of VALE3, it represents 7.72% of its forecast error variance, according to the spillover dynamics presented in Table 10. In addi-

tion, the S&P 500 index explains 0.76% of the forecast error variance of PETR4, while it explains 5.09% of that of VALE3.

Table 10

Spillover Dynamics: Diebold and Yilmaz (2012)

10-Day Horizon					
	p_CL1	p_VALE3	p_PETR4	p_SP	C. from others
p_CL1	83.03	8.09	5.15	3.72	16.96
p_VALE3	7.72	78.38	8.80	5.09	21.62
p_PETR4	4.81	9.18	85.25	0.76	14.75
p_SP	4.09	3.39	0.75	91.77	8.23
C. to others (spillover)	16.62	20.66	14.70	9.58	15.39
C. to others including own	99.65	99.05	99.94	101.35	400.000

Source: Prepared by the authors

The results in Table 10 indicate that the estimated spillovers are economically relevant, although limited in magnitude. Over a 10-day horizon, CL1 accounts for 4.81% of the forecast error variance of PETR4 and for 7.72% of the forecast error variance of VALE3. These percentages suggest that oil exerts influence on the analyzed assets, but it is not the main determinant of their return dynamics during the period. Most of the variance remains explained by own shocks, indicating the predominance of asset-specific factors, as well as domestic, sectoral, and macro-financial factors not directly captured by the estimated system.

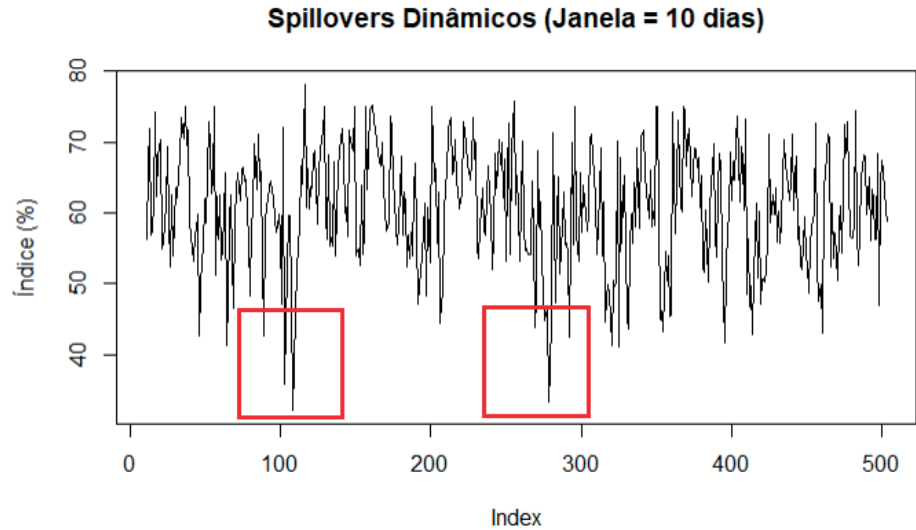
In the case of PETR4, the influence of oil is consistent with its direct sectoral exposure to the energy market. Even so, the percentage of 4.81% shows that the relationship between oil and Petrobras shares is neither mechanical nor exclusive, since the company is also affected by institutional factors, pricing policy, governance, exchange rates, domestic conditions, and investor expectations. In the case of VALE3, the 7.72% contribution of CL1 should not be interpreted as a direct operational relationship between oil and iron ore, but rather as evidence of financial connectedness between Brazilian assets exposed to the global commodity cycle. This result reinforces the interpretation that oil shocks may be indirectly transmitted to liquid exporting companies through systemic channels, such as risk aversion, international liquidity, exchange rates, and portfolio reallocation (Escribano et al., 2023; Qin et al., 2024).

Figure 6 shows variations in the dynamic spillover index over the analyzed period. The highlighted areas coincide with moments of heightened international instability, including peaks associated with the COVID-19 pandemic and the beginning of the Russia-Ukraine conflict. This evidence suggests temporary changes in connectedness among the assets, although the interpretation of these movements should be made with caution, since the model does not formally estimate structural breaks associated with these events. Even so, the temporal association between movements in the index and crisis periods is consistent with recent evidence that pandemic and

geopolitical shocks may alter the connectedness among oil, commodities, and stock markets (Huang et al., 2023; Cui et al., 2024).

Figure 6

Dynamic Spillovers

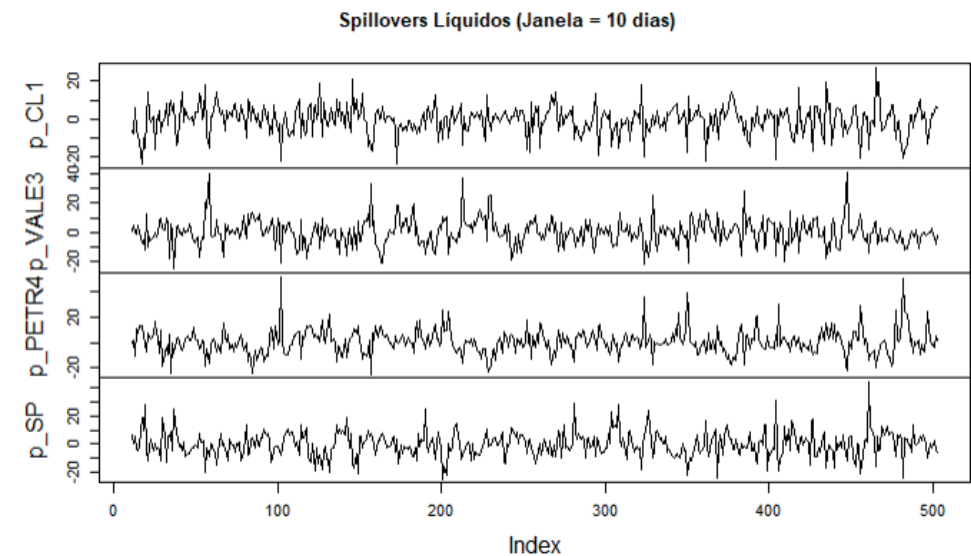


Source: Prepared by the authors

Regarding the net spillovers of the system, a variable can be classified as a transmitter when the difference between what it receives and what it transmits is positive.

Figure 7

Net Spillovers



Source: Prepared by the authors

As shown in Table 11, only the S&P 500 index is a net transmitter of spillovers, where “To” represents what is received and “From” represents what is transmitted to the other variables. If the variable is a transmitter, it is classified as “TRUE.”

Table 11*Net Transmitters of Volatility*

	To	From	Net	Transmitter
p_CL1	16.62	16.96	-0.34	FALSE
p_VALE3	20.66	21.62	-0.94	FALSE
p_PETR4	14.70	14.75	-0.05	FALSE
p_SP	9.58	8.23	1.34	TRUE

Source: Prepared by the authors

The identification of the S&P 500 as a net transmitter of volatility reinforces the importance of international financial conditions for the dynamics of the Brazilian assets analyzed. This result suggests that, during the period from 2021 to 2023, global shocks captured by the U.S. stock market played a relevant role in the propagation of volatility within the system (Attarzadeh & Balcilar, 2022; Escribano et al., 2023). Thus, even though the central objective of the study is associated with oil, the inclusion of the S&P 500 makes it possible to show that the connectedness among CL1, PETR4, and VALE3 occurs within a broader international financial environment, in which external shocks may simultaneously affect different assets traded in the Brazilian market.

Finally, the first-order Granger causality test (Granger, 1969) was applied, in accordance with the lag structure used in the VAR(p) model, to identify causal relationships within the system. However, no statistically significant directional transmission was observed. This result contrasts with the conclusions of Santos et al. (2015) and Silva (2011), which may be explained by the fact that those studies analyzed different periods and used high-frequency data.

CONCLUSION

The study under analysis investigated how the prices of four variables were interconnected in order to examine the influence of international oil prices on Petrobras shares. These variables included PETR4, which is the preferred share of Petrobras S.A. traded on the Brazilian stock exchange, B3, as well as VALE3, which is the share of Vale S.A., a company in the mining sector, also traded on B3. In addition, the study considered CL1, which refers to the most liquid and shortest-maturity oil futures contract available in the international market, and the S&P, which represents the S&P 500 index in the United States. The methodology used for the quantitative approach employed the vector autoregressive model developed by Sims (1980). Prior to this, Pearson correlation (Pearson, 1896) was examined. The objective was to analyze how the forecast

error variance, as defined by Diebold and Yilmaz (2012), was decomposed, as well as the Granger causality relationship proposed by Granger (1969).

Based on a sample from January 2021 to January 2023, the results indicated that Granger causality (Granger, 1969) does not identify statistically significant directional transmission, in contrast to the results of Santos et al. (2015), which indicated an influence of CL1 and SP1 on PETR4. However, the opposite results reflect different time periods, since the period analyzed in this study includes exogenous events such as the Russia-Ukraine conflict, which strongly affected the commodity, as well as the COVID-19 pandemic. Moreover, the data frequency was relatively different, with one study using intraday data and the other using daily data.

The research question can be answered through the results obtained from the system connectedness indices based on the methodology of Diebold and Yilmaz (2012), which indicate a value of 15.39%. In general, the CL1 variable explains the variance of PETR4 and VALE3, although with low representativeness, indicating that other external factors influence their fluctuations. Through dynamic spillovers, it is observed that during the peaks of COVID-19 cases in the pandemic, as well as at the beginning of the Russia-Ukraine conflict, the system experienced a decline in connectedness. During the analyzed period, only the S&P 500 was a net transmitter of volatility in the system, indicating that the other components, on average during the period analyzed, were influenced by this component.

Indeed, understanding the influence of commodities, such as oil prices, on Brazilian companies is extremely important for their management, especially when the commodity itself is a raw material or indirectly influences the company's production, as is the case with Petrobras and VALE. For the government, this analysis becomes crucial for decisions regarding the management of national oil, its price, and its influence on the shares of state-controlled companies. From the perspective of investors, this analysis contributes by highlighting the need for diversification or portfolio adjustments during periods such as the one analyzed, which includes exogenous events, since, despite being limited, there is some influence of international oil prices on the fluctuations and volatility of the assets analyzed.

For future research related to the analysis carried out, particularly in the academic field, if the limitations faced in obtaining the data are overcome, the same analyses may be conducted using high-frequency information. Such a study, for example, would contribute even more effectively to traders of PETR4 and VALE3 shares in arbitrage positions. Other information assessment criteria may also be applied to refine the results found.

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