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Testing the Non-Parametric Conditional CAPM in the Brazilian Stock Market

Avaliação do CAPM Condicional Não Paramétrico no Mercado de Ações do Brasil

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Abstract

This paper seeks to analyze if the variations of returns and systematic risks from Brazilian portfolios could be explained by the nonparametric conditional Capital Asset Pricing Model (CAPM) by Wang (2002). There are four informational variables available to the investors: (i) the Brazilian industrial production level; (ii) the broad money supply M4; (iii) the inflation represented by the *Índice de Preços ao Consumidor Amplo* (*IPCA*); and (iv) the real-dollar exchange rate, obtained by PTAX dollar quotation.This study comprised the shares listed in the BOVESPA throughout January 2002 to December 2009. The test methodology developed by Wang (2002) and retorted to the Mexican context by Castillo-Spíndola (2006) was used. The observed results indicate that the nonparametric conditional model is relevant in explaining the portfolios' returns of the sample considered for two among the four tested variables, M4 and PTAX dollar at 5% level of significance.

Keywords: Capital Asset Pricing Model. Non-Parametric Conditional Model. Brazilian Stock Returns.

Resumo

Esse artigo analisa a evolução do retorno e do risco sistemático das carteiras de 11 setores da economia brasileira através do modelo do CAPM condicional não paramétrico, proposto por Wang (2002). São utilizadas quatro variáveis explicativas: (i) o nível da produção industrial brasileira; (ii) o agregado monetário M4; (iii) a inflação, representada pelo Índice de Preços ao Consumidor Amplo (IPCA); e (iv) a taxa de câmbio real-dólar, obtida pela cotação do dólar PTAX. A amostra compreendeu ações listadas na BOVESPA no período de janeiro de 2002 a dezembro de 2009. Os testes do modelo Wang seguiram também a metodologia de Castillo-Spíndola (2006) para economias emergentes. Os resultados encontrados indicam que o CAPM condicional não paramétrico de Wang é robusto para a explicação dos retornos das carteiras da amostra considerada, sendo que duas das quatro variáveis testadas, *i.e.*, M4 e dólar PTAX, foram significativas ao nível de 5%.

Palavras-chave: Modelo de Precificação de Ativos de Capital. Modelo Condicional Não Paramétrico. Retornos de Ações de Empresas Brasileiras.

1 Introduction

The Capital Asset Pricing Model (CAPM) determines the assets price given that optimal investment decisions are made when the market is in equilibrium. According to Sharpe (1970, p.77), some of the assumptions made for the development of the model are: (i) assets only remunerate non-diversifiable risk; (ii) investors have homogeneous expectations with respect to the expected returns and the variance of returns; (iii) it is possible to invest and raise funds at the risk-free rate in unlimited quantities; (iv) absence of market imperfections; (v) investors are rational and make decisions solely in terms of expected returns and assets risk; (vi) investors are risk-averse selecting between two portfolios with the same expected return the one with the lowest risk.

The conditional CAPM intends to relate the asset return with the risk-free rate, the market risk premium and the beta to the information available to investors at a time, making the parameters of the model change as a result of the change in the investors' expectations. According to Hansen and Richard (1987), the conditional CAPM is valid even when the traditional CAPM indicates deficiencies.

In the assessment of the conditional pricing models, Ghysels (1998) argues that the specification of the CAPM parameters is very important and should be carefully performed. The author also states that several models of conditional CAPM that use time series as a calculation method leads to erroneous specification of the model parameters, which could lead to pricing errors more serious than those found in models with constant beta. He *et al.* (1996), also confirm the above argument.

Wang (2002) mathematically derives a non-parametric test in order to incorporate the conditional information in estimating the CAPM parameters. Thus, is it possible to affirm that the non-parametric conditional CAPM is valid in the Brazilian stock market? Furthermore, what is the information or independent variables that influence and validate the non-parametric conditional CAPM in Brazil?

This paper was divided in five sections: the second sextion discusses the theoretical framework of the conditional CAPM. The third section dicusses the non-parametric model proposed by Wang (2002). The fourth section works the methodology used to obtain the results and the final considerations found in the following sections.

2 Conditional Capital Asset Pricing Model

The CAPM of Sharpe (1964) is based on the premise that all investors have the same expectations of mean and variance of the distribution of asset returns of an economy, and that the investment decision is exclusively based on these parameters in a single moment. However, some empirical studies, such as Engle (1982) and Bollerslev (1986) show that the distribution of asset returns vary over time. That is, as the distribution of the asset return changes, the investor's expectation also varies from one period to another. Thus, the expectations of assets return by investors are closer to being compared to random variables, instead of constant variables as assumed by the traditional CAPM.

In order to reflect the variable behavior of the expected return on the investment, the conditional CAPM allows that the measurement of risk of the asset, the beta and the market risk premium vary. According to Tambosi Filho *et al.* (2010, p. 61), the conditional CAPM models incorporate variances and covariances that change over time.

As in the static non-conditional CAPM, the conditional CAPM model assumes that investors share identical subjective expectations, but these return expectations vary over time and they are conditioned to the information of moment t -1. The CAPM developed by Sharpe (1964), in his conditional version can be given by:

$$E\left(\tilde{R}_{i,t} \middle| \psi_{t-1}\right) = E\left(R_{f,t} \middle| \psi_{t-1}\right) + \beta_{i,m_t} \left[E\left(\tilde{R}_{m,t} \middle| \psi_{t-1}\right) - E\left(R_{f,t} \middle| \psi_{t-1}\right)\right] \quad (1)$$

where,

$E\left(\tilde{R}_{i,t}\right)$: expected return of the asset *i* in *t;*

$E\left(R_{f,t}\right)$: expected return of the risk-free asset in *t*;

Ψ_{t-1}: information available in *t* -1;

β_{i,m_t}: risk of asset *i* in relation to the market portfolio *m* in *t.*

The conditional beta of asset *i* is defined by:

$$\beta_{i,m_t} = \frac{\sigma\left(\tilde{R}_{i,t}, \tilde{R}_{m,t} \middle| \psi_{t-1}\right)}{\sigma^2\left(\tilde{R}_{m,t} \middle| \psi_{t-1}\right)} \qquad (2)$$

where,

$\sigma\left(\tilde{R}_{i,t}, \tilde{R}_{m,t} \middle| \psi_{t-1}\right)$: covariance between the return of asset *i* and the return of market portfolio *m* at *t* conditioned to the information of *t* -1;

$\sigma^2\left(\tilde{R}_{m,t} \middle| \psi_{t-1}\right)$: variance of the return of the market portfolio *m* in *t* conditioned to the information in *t* -1.

The conditional CAPM intends to relate the asset return with the risk-free rate, the market risk premium and the beta to the information available to investors at a time, making the parameters of the model change as a result of the change in the investors' expectations. Therefore, according to the studies presented above, the conditional CAPM represents in a more reliable way the valuation of the assets, since it enables both the risk premium and beta to vary from one period to another. According to Hansen and Richard (1987), the conditional CAPM is valid even when the static CAPM is not valid.

The main tests performed with the parametric conditional CAPM in Brazil and abroad are mentioned in the Table below.

Table 1: Summary of studies of the parametric conditional CAPM

Study	Objective	Conclusions
Bollerslev, Engle and Wooldridge (1988)	To use an M-GARCH model to estimate the returns of bills, bonds and stocks in the U.S. market in the period from 1959 to 1984.	The conditional covariances vary over time and are crucial in changing the market risk premium. The estimated betas were also variable from one period to another. The authors also state that other parameters, such as innovation in consumption, should be considered in the set of information available to the investor at the time of estimation of the conditional returns of the assets.
Harvey (1989)	To test the CAPM allowing conditional parameters and variables over time, namely: (i) covariance, (ii) expected return, and (iii) variance.	Even with the broad formulation, the conditional CAPM was statistically rejected. The results showed that the conditional covariances really changed from one period to another.
Bodurtha and Mark (1991)	To estimate and test the conditional CAPM of Sharpe-Lintner-Mossin, allowing returns and market risk premium variable over time.	The traditional conditional CAPM does not present as a statistically valid model. The use of GMM enabled the identification of other relevant variables in explaining the portfolio returns, including: (i) the Treasury bill return; (ii) the payment rate of dividends of shares; (iii) the rate of return of debentures with low credit ratings; (iv) the conditional variance of market return; and (v) the default premium for securities with high risk.

Study	Objective	Conclusions
Engel, Frankel, Froot and Rodrigues (1995)	To test the conditional CAPM using a method that would allow the expected conditional returns to vary due to two reasons: (i) change of the ratio of capital invested in each asset of the investor's portfolio; and (ii) modification of the covariance matrix of each asset.	The version of the conditional CAPM which assumed the constant variance was not statistically significant. On the other hand, the conditional CAPM, with GARCH effect demonstrated explanatory power of portfolio returns, with proven statistical validity.
Jagannatan and Wang (1996)	To develop a broader conditional CAPM, including the parameter of return on human capital in the calculation of market portfolio return.	The variable of size included in the model did not explain the stock returns, and therefore, the conditional CAPM that considers the return on human capital has proved to be valid.
He, Kan, Ng and Zhang (1996)	To use the conditional CAPM to allow the covariances between asset returns and market factors to vary over time. The model was also applied to analyze whether the parameters of market value and the book-to-market rate were consistent in explaining stock returns in a conditional multifactor pricing model. Still, an informational vector was created from five variables to incorporate the influence of information available to investors in the asset returns.	As in the study of Fama and French (1992), the model of a factor slightly explained the expected returns of stocks, even when variables covariances were applied to the CAPM. In turn, the three-factor model indicated better performance than the one-factor model, but it was still not statistically significant. Finally, even the five-factor model showed limited predictive power.
Ferson and Harvey (1999)	To test the conditional version of the three-factor model of Fama and French (1993) and the four-factor model of Elton *et al*. (1995), using information available to investors in past periods.	The conditional variables were significant in explaining asset returns. The three- and four-factor models in both the static and conditional version were statistically rejected in explaining the returns of the common shares.
Bonomo and Garcia (2004)	Test the static CAPM and the conditional CAPM proposed by Bodurtha and Mark (1991), in which both components of the conditional beta follow an ARCH process.	The traditional CAPM was not rejected by the authors. However, the conditional CAPM based on the ARCH effects showed greater adherence to the data, better capturing the dynamics of risk measures and expected returns.
Ribenboim (2004)	To test the conditional CAPM in accordance with the methodology proposed by Schwert and Seguin (1990). The test sample were the stocks listed on Bovespa in the period from June 1989 to March 1998.	The conditional CAPM was accepted for group 1, with portfolios of the most traded stocks on the market, and rejected for group 2, consisting of low liquidity stocks. The constant beta model was not rejected at 5% level of significance.
Tambosi Filho, Costa Jr. and Rossetto (2006)	To test the model of Jagannathan and Wang (1996) in the Brazilian stock market, from January 1994 to December 2002.	The variables market size, market risk premium, human capital return and market portfolio return were significant in explaining the historical returns of the portfolios. The explanatory power of the asset pricing model significantly increased when the conditional CAPM was considered.

Source: Prepared by the authors of this article

3 Non-Parametric Conditional Capital Asset Pricing Model

Wang (2002) presents a solution to the problem of erroneous specification of the parametric conditional CAPM parameters. The author proposes a new model that uses a non-parametric method to incorporate the information available to investors into the parameters and the test of the conditional CAPM. This model assumes that the covariances are stationary and that the considerations of protection to risk by the investor are not important enough.

Wang (2002) assumes that the excess return of the market portfolio *m* in $t+1$, $r_{m,t+1}$, and the excess return of asset *i* on the same date, $r_{i,t+1}$, are stationary variables. The conditional CAPM considered by the author is given by:

$$E\left[r_{i,t+1}\middle|I_t\right]=E\left[r_{m,t+1}\middle|I_t\right]\beta_{i,t} \qquad (3)$$

where, $\beta_{i,t} = \dfrac{\operatorname{cov}\left(r_{i,t+1}, r_{m,t+1} \middle| I_t\right)}{\operatorname{var}\left(r_{m,t+1} \middle| I_t\right)}$.

Whereas the equation (3) is equivalent to:

$$E\left[r_{i,t+1} \middle| I_t\right] = E\left[r_{m,t+1} \middle| I_t\right] \frac{E\left[r_{i\ t,+1} r_{m,\ t+1} \middle| I_t\right]}{E\left[r^2_{m,t+1} \middle| I_t\right]} \quad (4)$$

for $i = 1, \ldots, n$.

Where,

$r_{i,t+1}$: excess return of asset i, which is obtained by the difference of return of asset i in $t+1$ and the rate of risk--free asset also in $t+1$;

$r_{m,t+1}$: market premium, which is calculated as the difference between the return on market portfolio m in $t+1$ and the risk free rate also in $t+1$.

Whereas $E\left[m_{t+1} r_{i,t+1} \middle| I_t\right]$ equal to the residue of the expression 4.41, we can rewrite it:

$$E\left[r_{i,t+1} \middle| I_t\right] = E\left[r_{m,t+1} \middle| I_t\right] \frac{E\left[r_{i,t+1} r_{m,\ t+1} \middle| I_t\right]}{E\left[r^2_{m,t+1} \middle| I_t\right]} + E\left[m_{t+1} r_{i,t+1} \middle| I_t\right] \text{ or}$$

$$E\left[r_{i,t+1} \middle| I_t\right] - E\left[r_{m,t+1} \middle| I_t\right] \frac{E\left[r_{i,t+1} r_{m,t+1} \middle| I_t\right]}{E\left[r^2_{m,t+1} \middle| I_t\right]} = E\left[m_{t+1} r_{i,t+1} \middle| I_t\right] \quad (5)$$

where,

m_{t+1}: stochastic discount factor m in $t+1$.

If $E\left[m_{t+1} r_{i,t+1} \middle| I_t\right] = 0$, the residue of the expression 5 is zero and the conditional CAPM is statistically valid in explaining the asset returns.

Rewriting equation 5 for the subset of information, x_t, available to investors, we have:

$$E\left[r_{i,t+1} \middle| x_t\right] - E\left[r_{m,t+1} \middle| x_t\right] \frac{E\left[r_{i,t+1} r_{m,\ t+1} \middle| x_t\right]}{E\left[r^2_{m,t+1} \middle| x_t\right]} = E\left[m_{t+1} r_{i,t+1} \middle| x_t\right] \quad (6)$$

The stochastic discount factor is given by:

$$m_{t+1} = 1 - \frac{\hat{g}_m\left(x_t\right)}{\hat{g}_{mm}\left(x_t\right)} r_{m,t+1} \quad (7)$$

where,

$\hat{g}_m\left(x_t\right)$: market risk premium in $t+1$, or conditional mean, estimated in a non-parametric way;

$\hat{g}_m\left(x_t\right)$: variance of the market portfolio in $t+1$ also estimated in a non-parametric way.

The estimation of the non-parametric conditional mean is obtained by:

$$\hat{g}_m\left(x_t\right) = E\left[r_{m,t+1} \middle| x_t\right] = n^{-1} h^{-k} \hat{f}\left(x\right)^{-1} \sum_{t=1}^{n} K\left(\frac{x - x_t}{h}\right) r_{m,t+1} \quad (8)$$

where,

$\hat{f}(x) = n^{-1}h^{-k}\sum_{t=1}^{n} K\left(\frac{x-x_t}{h}\right)$: non-parametric probability distribution;

$h = 1,06 * n^{-1/5} * \sigma(x_t)$: estimation window according to Silverman (1986) method;

n : number of observations;

$K(x_{i,t}) = \frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}}$: probability density function estimated using the Standard Gaussian Function in the study;

x : set of variables of information;

In turn, the estimation of the conditional variance in a non-parametric way is obtained by:

$$\hat{g}_{mm}(x_t) = E\left[r^2_{m,t+1} \middle| x_t\right] = n^{-1}h^{-k}\hat{f}(x)^{-1}\sum_{t=1}^{n} K\left(\frac{x-x_t}{h}\right) r^2_{m,t+1} \quad (9)$$

Knowing $\hat{m}_{t+1}r_{i,t+1} = \hat{e}_{i,t+1}$ and x_t, which is a vector of observable variables of information, one way to test the condition $E\left[m_{t+1}r_{i,t+1} \middle| x_t\right] = 0$, which is equivalent to $E\left[e_{i,t+1} \middle| x_t\right] = 0$, is through the regression of $e_{i,t+1}$ with x_t, by checking whether the coefficients of the regression are equal to zero. That is, to test the validity of the conditional CAPM, according to Wang (2002), it is necessary to perform the non-parametric regression given by the following expression:

$$\hat{e}_{i,t+1} = x_t'\delta_i + u_{i,t+1} \quad (10)$$

where,

i : portfolio;

$u_{i,t+1}$: residue of the model.

Where $E\left[u_{i,t+1} \middle| x_t = 0\right]$ and $E\left[e_{i,t+1} \middle| x_t\right] = x_t'\delta_i$.

To estimate the parameter $\hat{\delta}_i$ in a non-parametric way, Wang (2002) used the following equation:

$$\hat{\delta}_i = \left(\frac{1}{n}\sum_{t=1}^{n}\hat{w}_t z_t z_t'\right)^{-1}\left(\frac{1}{n}\sum_{t=1}^{n}\hat{w}_t z_t \hat{e}_{i,t+1}\right) \quad (11)$$

where,

$\hat{w}_t = \hat{f}(x_t)\,\hat{g}_{mm}(x_t)$: weighing of the regression given by Wang (2002);

$z_t = (1 \quad x_t')'$.

For each portfolio of shares, a given delta was estimated, obtaining the set:

$$\hat{\delta}_N = \left(\hat{\delta}_1' \quad \hat{\delta}_2' \quad \ldots \quad \hat{\delta}_j'\right)' \quad (12)$$

where,

N: number of variables of information used in the non-parametric regression;

J: number of portfolios.

That is, to validate the non-parametric conditional CAPM and the conditionality of the variables of information, it is necessary to statistically demonstrate that the coefficient $\hat{\delta}_N$ is equal to zero. This test statistic follows a chi-square distribution, as follows:

$$\hat{T}_\delta = N\hat{\delta}_N'\hat{\Lambda}_N^{-1}\hat{\delta}_N : \chi^2(N) \quad (13)$$

where,

$$\hat{\Lambda}_N = \frac{1}{n}\sum_{t=1}^{n}\hat{\gamma}_N(y_{t+1})\hat{\gamma}_N(y_{t+1})' - 4\hat{\delta}_N\hat{\delta}_N'.$$

The parameter $\hat{\gamma}_N(y_{t+1})$ is given by:

$$\hat{\gamma}_N(y_{t+1}) = \hat{f}(x_t)\left[\hat{g}_{mm}(x_t)r_{i,t+1} - \hat{g}_m(x_t)r_{m,t+1}r_{i,t+1} + \hat{g}_i(x_t)r_{m,t+1}^2 - \hat{g}_{mi}(x_t)r_{m,t+1}\right] \quad (14)$$

where:

$$\hat{g}_i(x_t) = E\left[r_{i,t+1} \middle| x_t\right] = n^{-1}h^{-1}\hat{f}(x)^{-1}\sum_{t=1}^{n}K\left(\frac{x-x_t}{h}\right)r_{i,t+1} \quad (15)$$

$$\hat{g}_{mi}(x_t) = E\left[r_{m,t+1}r_{i,t+1} \middle| x_t\right] = n^{-1}h^{-1}\hat{f}(x)^{-1}\sum_{t=1}^{n}K\left(\frac{x-x_t}{h}\right)r_{m,t+1}r_{i,t+1} \quad (16)$$

That is, for each variable of information, after verifying that coefficient $\hat{\delta}_N$ is equal to zero, it can be stated that the residue of the non-parametric conditional CAPM in cross section is statistically null, indicating the validity of the selected variable and the model in explaining the asset returns.

According to the results presented by the author, the estimated betas proved to be nonlinear, thus varying over time according to the influence of variables of information DPR, EWR, DEF and RTB. However, even showing better performance than the traditional CAPM, the non-parametric conditional CAPM was not statistically valid.

Following the methodology proposed by Wang (2002), Castillo-Spíndola (2006) tested the non-parametric conditional CAPM for the Mexican economy, with monthly data from March 1987 to January 2002. For the variables of information, data from February 1987 to December 2001 were used.

To conduct the tests, the author divided the sample of shares in five industries, namely: (i) processing industry; (ii) construction; (iii) trade; (iv) transport; and (v) services.

In turn, the variables of information used by Castillo-Spíndola (2006, p. 290) were:

a) percentage variation of the Brazilian industrial production level;
b) percentage variation of inflation, represented by the *national consumer price index;*
c) percentage variation of broad money supply M4;
d) percentage variation in the real-dollar exchange rate;
e) spread between the monthly rate of return on commercial papers issued in Mexico and the risk-free rate;
f) percentage variation in the Dow Jones index.

The non-parametric conditional CAPM was valid and statistically significant for all the economic information used. Therefore, the variables of information selected by the author influence the risk measure of the stocks, the beta, and also the market risk premium.

4 Methodology

The selected sample of stocks represents the companies of the theoretical portfolio of Ibovespa for the first quarter of 2012. The prices selected are from January 2002 to December 2012. This selection criterion was chosen because the companies of Ibovespa are those that present greater liquidity in the domestic market. Still, the liquidity criterion for the selection of stocks is arbitrary. However, according to Fama and French (1988), this practice is common in the literature of empirical tests.

We also chose to include in this study the stocks of financial companies, as tests of the conditional CAPM versions in Brazil, such as the study of Ribenboim (2004) and Tambosi Filho *et al.*(2006), already use the stocks of these companies.

To maintain the liquidity criterion, we decided to exclude the stocks that were not traded for more than five working days.

The information data used to test the non-parametric conditional CAPM date from December 2001 to November 2012. In turn, both the exchange rate data of Ibovespa and the effective monthly SELIC were obtained for the period from January 2002 to December 2012.

5 Results Obtained

The following table shows the results of the deltas obtained by applying the test of the non-parametric conditional CAPM of Wang (2002) regarding the four variables of information selected, which are: (i) industrial production; (ii) M4; (iii) IPCA; and (iv) PTAX dollar.

Table 1: Deltas estimated for the four instrumental variables considering the 11 sectors studied $\left(\hat{\delta}_1' \quad \hat{\delta}_2' \quad \ldots \quad \hat{\delta}_j'\right)$

Sector	Industrial production	M4	IPCA	PTAX dollar
Banks	0.205	0.988	-0.830	0.469
Beverage	0.225	0.246	-0.533	0.128
Energy, gas and water	0.387	0.147	-20.845	0.335
Holdings	0.113	0.904	0.130	0.227
Aircraft Industry	-0.009	-0.925	-23.104	0.406
Iron and steel	-0.110	18.443	21.623	0.283
Mining	0.319	0.518	17.260	-0.004
Paper and Pulp	0.135	0.648	-10.446	0.157
Petrochemical	0.413	0.679	-14.301	0.186
Telecommunications	0.235	-0.050	-3.079	0.358
Retail	0.319	0.571	0.738	0.173

Source: Prepared by the authors of this article

It is possible to observe that both the Industrial Production and the PTAX Dollar produced deltas closer to zero, and therefore, it is expected that such instrumental variables are relevant to statistically explain the evolution of the conditional betas of the portfolios of the 11 sectors considered over time and also the market risk premium. See below the appropriate statistical test to assess whether these variables may actually be used in the conditional CAPM model.

Table 2: Values of the chi-square test regarding the four instrumental variables $\hat{\delta}_N = (\hat{\delta}_1' \quad \hat{\delta}_2' \quad \ldots \quad \hat{\delta}_j')$

Variable of information	Value of the test $[\hat{\delta}_N]$	P-value
Industrial production	-0.117	0.000***
M4	0.849	0.357
IPCA	70.459	0.008***
PTAX dollar	0.355	0.551

*** Significant at 1% level.

Source: Prepared by the authors of this article

It can be argued that the non-parametric conditional CAPM can be used in the Brazilian stock market, as both the M4 as the PTAX dollar are variables that satisfactorily explain the evolution of the conditional betas of the portfolios of the 11 sectors considered over time. That is, as the two variables of information presented *p-value* greater than 1%, the second and fourth null hypotheses tested could not be rejected, which validated the model of Wang (2002) to the national context and for the two variables.

Regarding the information of industrial production and IPCA, one can say that both do not validate the application of the non-parametric conditional CAPM because the statistical tests of the model with these variables produced *p-value* lower than 5%.

Therefore, we found that the betas and the market premiums conditional to the M4 and the real-dollar exchange rate, represented by the PTAX dollar price, can be used in the domestic market practice, for example, in a model of weighted average cost of dynamic capital.

As for the same variables tested in the Brazilian context and considering their similar information regarding Mexico, Castillo-Spíndola (2006) concluded that the non-parametric conditional CAPM is valid at 10% level of significance for the Mexican economy. According to the author, this model is statistically significant, even when the test is performed with all variables of information together.

As the non-parametric conditional CAPM was valid for the variables of information M4 and PTAX dollar, it was possible to calculate the conditional beta according to the available data of the aforementioned variables.

Table 3 presents data from the beta conditioned to variable M4. Note that the sector that presented the highest average risk for the market, from January 2002 to November 2012, was the iron and steel sector, with average beta equal to 1.06. In turn, the one that showed the greatest variation of this parameter was the mining sector, with a coefficient of variation equal to 0.52. Finally, for the mining sector there is the largest beta conditioned to the M4, which was equal to 2.98 in December 2008.

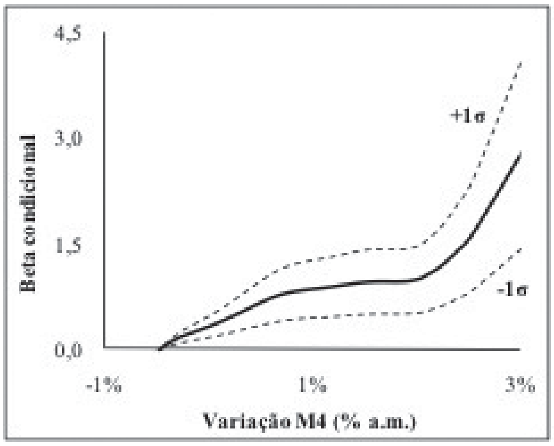
Table 3: Descriptive statistics of the betas conditioned to the variable of information M4

Sector	Average beta	Median beta	Standard Deviation (%)	Coefficient of variation	Lowest beta	Highest beta
Banks	1.06	1.08	12.2%	0.11	0.34	1.31
Beverage	0.51	0.56	11.0%	0.22	0.10	0.59
Energy, gas and water	0.84	0.81	14.3%	0.17	0.71	1.64
Holdings	0.85	0.83	15.6%	0.18	0.53	1.72
Aircraft Industry	0.70	0.76	28.7%	0.41	0.05	1.70
Iron and steel	1.27	1.24	15.4%	0.12	0.85	1.78
Mining	0.83	0.88	42.9%	0.52	-0.68	2.98

Sector	Average beta	Median beta	Standard Deviation (%)	Coefficient of variation	Lowest beta	Highest beta
Paper and Pulp	0.70	0.68	15.1%	0.21	0.34	1.08
Petrochemical	0.98	0.96	12.4%	0.13	0.76	1.55
Telecommunications	0.76	0.76	15.2%	0.20	0.03	1.12
Retail	0.78	0.78	14.3%	0.18	0.03	1.09

Source: Prepared by the authors of this article

The variable behavior of the conditional beta can be viewed at the following chart. The solid line represents the conditional beta and the dashed lines show this metric by adding and subtracting one standard deviation. It is important to note that Chart 1 shows the univariate relation between the conditional beta and the variation of a single variable of information, the M4.



[Conditional beta/ M4 variation (% a.m.)]

Figure 1: Beta conditioned to variable M4 – sector: mining
Source: Prepared by the authors of this article

Table 4 shows the data of the beta conditioned to variable PTAX dollar. The sector with the highest average risk for the market, from January 2002 to November 2012, was the iron and steel sector, with average beta equal to 1.24. On the other hand, the mining sector showed the greatest variation of this parameter, with a coefficient of variation equal to 0.37. In turn, the beverage sector was the one that had the largest beta conditioned to the PTAX dollar in the period under analysis, which was equal to 1.56 in September 2002.

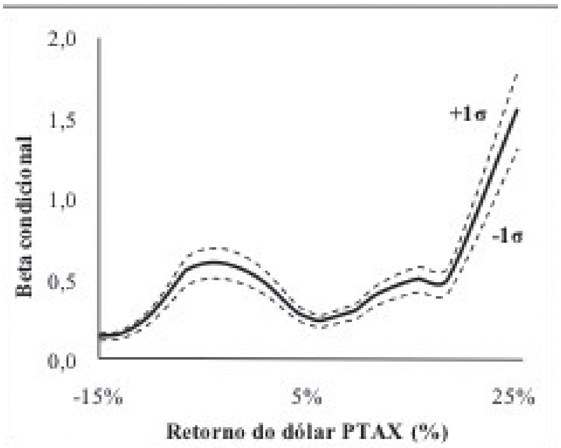
Table 4: Descriptive statistics of the betas conditioned to the variable of information PTAX dollar

Sector	Average beta	Median beta	Standard Deviation (%)	Coefficient of variation	Lowest beta	Highest beta
Banks	1.02	0.98	9.8%	0.10	0.93	1.45
Beverage	0.51	0.55	15.8%	0.31	0.14	1.56
Energy, gas and water	0.94	0.97	13.5%	0.14	0.35	1.39
Holdings	0.80	0.80	9.0%	0.11	0.59	0.97
Aircraft Industry	0.71	0.75	11.3%	0.16	0.16	0.78
Iron and steel	1.24	1.21	11.2%	0.09	0.93	1.48

Sector	Average beta	Median beta	Standard Deviation (%)	Coefficient of variation	Lowest beta	Highest beta
Mining	0.82	0.93	30.3%	0.37	-0.60	1.07
Paper and Pulp	0.74	0.74	10.1%	0.14	0.20	0.98
Petrochemical	0.98	0.97	8.0%	0.08	0.60	1.25
Telecommunications	0.82	0.82	11.1%	0.14	0.53	1.14
Retail	0.83	0.90	18.7%	0.22	0.05	1.32

Source: Prepared by the authors of this article

The beta conditioned to the variable PTAX dollar and its variable behavior is shown in Chart 2.



[Conditional beta/ Return of PTAX dollar (%)]

Figure 2: Beta conditioned to variable PTAX dollar – sector: beverage
Source: Prepared by the authors of this article

Despite not validating the non-parametric conditional CAPM, Wang (2002) shows that the relation of risk of the portfolios with the market, their betas, varies from one period to another, reflecting the influence of the changes in the information available to investors, that is, indicating the conditionality of the model parameters.

6 Final Considerations

The rate of return that should be required of the equity capital is a central issue in the consideration of investment options. The asset pricing model proposed by Sharpe (1964), the static CAPM, has been the most widely used to estimate the rate of return.

According to the traditional CAPM, the investor behavior is analyzed and evaluated in a hypothetical economy in a single period. Several authors, including Fama and French (2002) and Jagannathan and Wang (1996), proved that the market conditions change from one period to another, causing the static parameters of the model to not be realistic. Therefore, in the practical context, it should be considered that the risk premium and also the relationship of the risk of the asset with the market, the beta, change from one moment to another.

The conditional CAPM is aimed precisely to consider the dynamic and the change in the market conditions in the estimation and explanation of the assets return. Jagannathan and Wang (1996) showed that the conditional CAPM is statistically significant in explaining stock returns for the U.S. market. Bonomo and Garcia (2004, p. 43) state that the conditional CAPM indicates a higher adherence to data from the Brazilian stock market, which reflects more realistically the dynamic of the risk measures and the expected assets return. However, Ghysels (1998) demonstrates that the conditional CAPM models that use time series such as the calculation method lead to the erroneous specification of the model parameters, which can lead to pricing errors more serious than those found in models with constant beta. To correct this parameter specification problem, Wang (2002) proposed a conditional CAPM that considers non-parametric methods to estimate the market premiums, betas and asset returns conditioned to the information available to investors.

The purpose of this paper was to test the validity of this model in the Brazilian stock market in the period from 2002 to 2012, applying a test methodology similar to that of Wang (2002) and Castillo-Spíndola (2006). The results indicate that:

- the non-parametric conditional CAPM can be used in the Brazilian stock market, as both the M4 and the real-dollar exchange rate explain the historical evolution of the asset returns;
- the beta and the market risk premium are conditioned to the information available to investors and vary over time;
- the variables of industrial production level and IPCA did not demonstrate statistical significance in explaining the parameters of the non-parametric conditional CAPM and, therefore, did not validate the model.

Knowing fully the variables of information that influence the decision making of the investors and the expected assets return is a daunting task. Several researchers studied the relationship between the macroeconomic variables and the asset prices. Dornbusch and Fischer (1980) focused on the relationship between the stock market and the exchange rate. The authors found that the increase of foreign prices compared to the domestic prices increases the competitiveness of domestic companies in the foreign market, which increases the prices of their stocks. Moreover, assuming that investors invest part of their wealth in the domestic market, they concluded that an increase in wealth increases the demand for stocks, which drives up the prices of these assets. In turn, Blanchard (1990) analyzed the relationship between the asset prices and the level of economic activity, in order to characterize the relationship between the variables. The author noted that the stock market does not generate increases in the level of economic activity of a country and that variations in the asset prices arise from changes of the economic policies in force.

The results of this study should be considered with caution. Choosing the variables of information available to the investors was based on the study of Castillo-Spíndola (2006). Even having a theoretical framework, this choice does not cease to be arbitrary and limited, which indicates a limitation of this study.

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Appendix A – Calculation Program of the Non-Parametric CAPM – Mathematica 7.0 Software

```
<<Graphics`Graphics`
<<NumericalMath`SplineFit`
<<Statistics`ContinuousDistributions`
div=Flatten[Import["C:\Simulação\dpetwang.txt","Table"]];
```

$$n=Length[div];nt=n-1;ng=11;restr=1;N=nt;$$

$$r=Import[\text{"C:\\Simulação\\petwang.txt","Table"}];$$

$$rm=Flatten[Import[\text{C:\\Simulação\\ibov.txt","Table"}]];$$

$$coefbeta=\{\ \};delta=\{\ \};cart=Table[i,\{i,1,ng\}];\ k[x\_]:=\frac{e^{\frac{-x^2}{2}}}{\sqrt{2\pi}};$$

$$h=1.06*n^{-1/5}*StandardDeviation[div];(*\text{Método de Silverman }(1986)-\text{Escolha do }h*)$$

$$f[y\_]:=\frac{1}{n*h}\sum_{j=1}^{n}k\left[\frac{y-div[[j]]}{h}\right];$$

$$gp[x\_]:=\frac{1}{n*h}*f[x]^{-1}\sum_{j=1}^{nt}k\left[\frac{x-div[[j]]}{h}\right]*rm[[j+1]];$$

$$gr[x\_,m\_]:=\frac{1}{n*h}*f[x]^{-1}\sum_{j=1}^{nt}k\left[\frac{x-div[[j]]}{h}\right]*r[[All,m]][[j+1]];$$

$$grx=Transpose[Table[gr[div,m],\{m,1,ng\}]];$$

$$gpp[x\_]:=\frac{1}{n*h}*f[x]^{-1}\sum_{j=1}^{nt}k\left[\frac{x-div[[j]]}{h}\right]*rm[[j+1]]^{2};$$

$$gpx[x\_,m\_]:=\frac{1}{n*h}*f[x]^{-1}\sum_{j=1}^{nt}k\left[\frac{x-div[[j]]}{h}\right]*r[[All,m]][[j+1]]*rm[[j+1]];$$

$$gpx=Transpose[Table[gpx[div,m],\{m,1,ng\}]];$$

$Do[$

$ri=Flatten\left[Take\left[Transpose[r],\{j\},n\right]\right];$

$$gpr[x\_]:=\frac{1}{n*h}*f[x]^{-1}\sum_{j=1}^{nt}k\left[\frac{x\text{-}div\left[[j]\right]}{h}\right]*ri\left[[j+1]\right]*rm\left[[j+1]\right];$$

$b=gp[div]/gpp[div];$

$AppendTo\left[coefbeta,gpr[div]/gpp[div]\right];$

$m=Table\left[1-b\left[[x+1]\right]*rm\left[[x+1]\right],\{x,1,nt\}\right];$

$e=Table\left[m\left[[x]\right]*ri\left[[x+1]\right],\{x,1,nt\}\right];$

$w[j\_]:=f\left[div\left[[j]\right]\right]*gpp\left[div\left[[j]\right]\right];$

$$AppendTo\left[delta,\left((1/nt)*\sum_{j=1}^{nt}w[j]*div\left[[j]\right]^2\right)^{-1}\left((1/nt)*\sum_{j=1}^{nt}w[j]*div\left[[j]\right]*e\left[[j]\right]\right)\right];$$

$Clear[e],\{j,1,ng\}];$

$gpp=gpp[div];gp=gp[div];f=f[div];$

$$var=Table[f\left[[i]\right]*\left(gpp\left[[i]\right]*r\left[[All,m]\right]\left[[i+1]\right]\right)-gp\left[[i]\right]*rm\left[[i+1]\right]*r\left[[All,m]\right]\left[[i+1]\right]$$

$$+grx\left[[i,m]\right]*rm\left[[i+1]\right]^2-gpx\left[[i,m]\right]*rm\left[[i+1]\right],\{m,1,ng\},\{i,1,ng\}];$$

$dif=4*Table\left[delta*i,\{i,1,ng\}\right];deltat=Table\left[delta,\{1\}\right];$

$capmvar=PseudoInverse\left[var.Transpose[var]-dif\right];$

$teste=ng*Mean\left[deltat.capmvar.delta\right]$

$pvalor=CDF\left[ChiSquareDistribution[1],teste\right]$