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Discursive interactions of pre-service teachers on the algebraic structure of Groups: a look at the school mathematics

Interações discursivas de futuros professores sobre a estrutura algébrica de Grupos: um olhar para a matemática escolar

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Abstract: Several discursive practices are present in the classroom. In this article, we seek to identify potential relationships between discursive interactions, the role of the teacher educators, and the formative tasks and understand how such relationships promote learning about school mathematics teaching in an algebra subject in a mathematics teaching degree course. Two research cycles were conducted using a qualitative-interpretive approach and the design-based research method. These cycles analyzed planning and discursive interactions between participants in an algebra subject while solving formative tasks on the algebraic structure of groups and their assessments. The results show that discursive interactions were influenced by the choices of mathematical tasks and the educators' purposes during planning. Furthermore, the structure of these tasks provided tools for prospective teachers to discuss teaching cases involving the use of the symmetric element in different school contexts.

Keywords: Discursive Interactions. Algebraic Structures. Mathematical Language. Formative Tasks. Teaching Degree in Mathematics.

Resumo: Diversas práticas discursivas estão presentes em sala de aula. Neste artigo, busca-se identificar potenciais relações entre as interações discursivas, o papel do formador e as tarefas formativas, além de compreender de que modo tais relações promovem aprendizagens acerca do ensino da matemática escolar em uma disciplina de álgebra num curso de licenciatura em matemática. Utilizando uma abordagem qualitativa-interpretativa e o método design based-research, foram conduzidos dois ciclos de pesquisa. Esses ciclos analisaram o planejamento, as interações discursivas entre os participantes de uma disciplina de álgebra enquanto resolviam tarefas formativas sobre a estrutura algébrica de grupos, bem como suas avaliações. Os resultados mostram que as interações discursivas foram influenciadas pelas escolhas das tarefas matemáticas e pelos propósitos dos formadores durante o planejamento. Além disso, a estrutura dessas tarefas forneceu ferramentas para que os futuros professores discutissem sobre casos de ensino envolvendo o uso do elemento simétrico contextos escolares distintos.

Palavras-chave: Interações Discursivas. Estruturas Algébricas. Linguagem Matemática. Tarefas Formativas. Licenciatura em Matemática.

Introdução

When considering initial teacher education, we must focus on the basic education classroom and provide moments of reflection based on the experience of teaching practices to bring prospective teachers (PTs) closer to the situations they will face in their careers (Marcelo, 2009). Such practices can help teaching degree students to perceive and understand the mathematics present in learning processes, whether through examples from the classroom or through moments narrated by teachers (Fiorentini & Oliveira, 2013).

At these times, educators can use formative tasks and strategies that promote mathematical and didactic discussions to assist teachers' learning, such as the exploratory teaching approach (Cyrino & Oliveira, 2016; Ribeiro & Ponte; Ponte, 2020; Aguiar et al., 2021;). Although some studies indicate the potential of approaches that promote good discussions in teacher education, it is still necessary to explore how these approaches occur in different subjects of a mathematics teaching degree course (Ribeiro & Ponte, 2020; Marins et al., 2021), including Algebra, to be better explored in teacher education (Ribeiro, 2016).

Algebra teaching, in turn, can be supported by the study of algebraic structures as groups, which, despite being addressed in algebra subjects, present connections with other areas of mathematics, such as arithmetic and geometry (Zazkis & Marmur, 2018).

To prevent superficial discussions in the classroom, Sasseron (2013) highlights that a well-defined objective must be outlined. This objective should be related to the questions, proposed problems or tasks, and the issues, comments, and information to be addressed. Furthermore, it is essential to consider the participants' answers, which can be expressed through words or gestures (Kendon, 2004).

Considering that the discussion environment can be a classroom in which students are receiving education as prospective teachers and carrying out formative tasks, and considering that the educators' actions can influence the discussions that will take place (Trevisan et al., 2020; Trevisan et al., 2023)[1], in this article, we aim to identify potential relationships between discursive interactions, the role of the educator, and the formative tasks, besides understanding how such relationships promote learning about the school mathematics teaching in an algebra subject in a mathematics teaching degree course. To operationalize this objective, we intend to answer the question: How can educators anticipate discursive interactions among participants in an Algebra course and leverage them through formative tasks to promote learning about school mathematics teaching? What relationships are established between discursive interactions, the educator's role, and the formative tasks when addressing algebra in a mathematics teaching degree course?

Theoretical Framework

The theoretical framework seeks to establish solid and conceptual grounds based on approaches and models that highlight discursive interactions to broaden the understanding of the complexities underlying the initial education of mathematics teachers.

Initial teacher education and discursive practices

Beginning teachers face many challenges until they develop autonomy and establish their professional identity as mathematics teachers. In this sense, Barretto and Cyrino (2023) propose actions aimed at professional learning, such as reflections on teaching practice, to investigate how interaction between peers can contribute to the constitution of professional identity.

Mathematical discussions involve presenting ideas, arguing, justifying, and negotiating meanings in working with challenging mathematical tasks (Canavarro et al., 2012; Marcatto, 2022) and can provide productive learning moments for basic education students. Furthermore, such practices can help PTs make effective decisions in the classroom, which can be addressed with the support of exploratory teaching, where they can discuss and improve their mathematical and didactic knowledge while engaging in mathematical tasks and their possible applications in the classroom (Aguiar et al., 2021; Marins et al., 2021).

In this sense, resources such as formative tasks for teachers, which have as their starting point the exploration of mathematical tasks, can influence their' worldview and raise purposes for the constitution of identity while they develop self-

[1] This article is part of the multipaper doctoral thesis by the first author, under the guidance of the other authors in the Postgraduate Program in Teaching and History of Science and Mathematics at UFABC.

confidence during formative moments (Cyrino & Estevam, 2023). This process represents an even bigger challenge for prospective teachers.

In turn, formative tasks supported or constructed from videos that portray teaching cases can promote reflection on teaching practice or serve as a starting point for developing mathematical and didactic discussions (Rodrigues et al*.,* 2018; Jardim et al., 2023a). Such collective discussions and reflections motivated by formative tasks can create conditions for teachers to construct and reformulate knowledge autonomously, enabling reflection and modification of their conceptions (Sousa & Paiva, 2023).

The professional learning opportunities for teacher model (PLOT) and discursive interactions

Ribeiro and Ponte (2019) consider "professional learning opportunities (PLO) collective moments in which practicing teachers work and discuss mathematical and didactical situations in order to amplify their professional knowledge for teaching" (p. 50).

To support teacher education focusing on professional learning opportunities, Ribeiro and Ponte (2020) present the PLOT model to subsidize the design of professional development processes. This model is based on an interactive and interconnected perspective of three domains: the role and actions of the teacher educator (RATE), the professional learning tasks for teachers (PTLT), which seek to elucidate the use of mathematical tasks in teaching, and the discursive interactions among participants (DIAP). By interconnecting such domains, one expects PLOTs to be implemented through a unifying process.

These domains are related in three operationalization phases. The first phase involves teacher educators' organization, the second phase initiates the interactions among participants (teachers and educators), and the third phase aims to promote teachers' professional learning through the agglutination of the three domains. Each domain has four components: two in the conceptual dimension, characterizing the structure and theoretical basis, and two in the operational dimension, guiding the use of the model.

Concerning the DIAP domain, the four components that comprise it are constituted from the meanings linked to participants' involvement in discussions and are distributed and characterized according to Table 1:

Table 1

Characteristics and meanings of the components of the DIAP domain

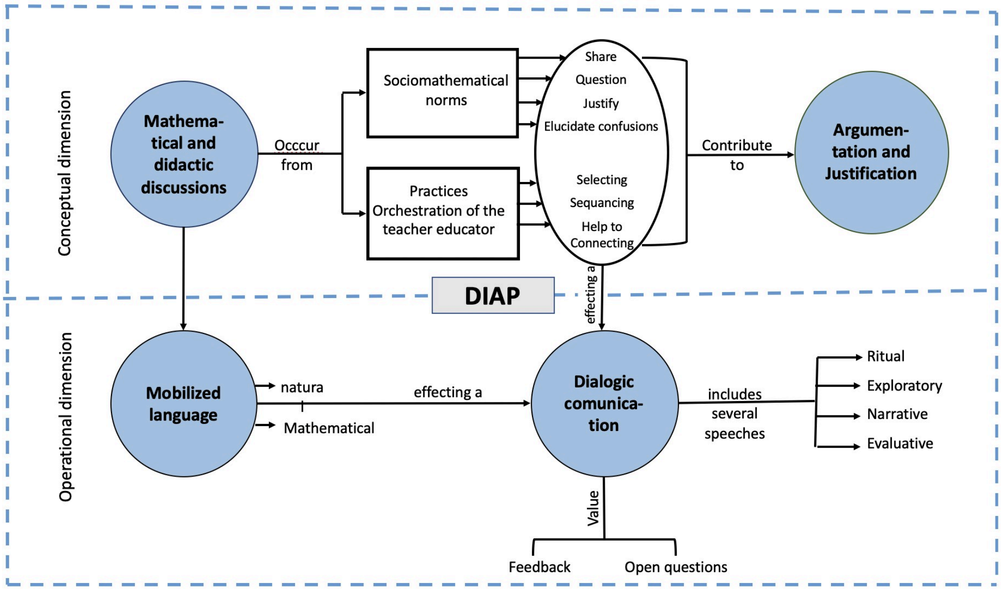
	Component	***Component characteristic***	***Constitution of the component to***
Conceptual dimension	*Mathematical and didactic discussions*	Articulate the mathematical and didactic discussions related to mathematical tasks.	Promote mathematical and didactic discussions to favor professional learning for teachers.
	Argumentation and Justification	Involve valid mathematical and didactic arguments and justifications.	Engage teachers in an environment that promotes argumentation and justification when discussing mathematical tasks for students.
Operational dimension	*Mobilized language*	Consider the use of mathematical and didactic language appropriate and relevant to the teaching level of mathematical tasks.	Encourage the use of correct mathematical language appropriate to students' level of education.
	Dialogic communication	Promote dialogic and interactive communication among all participants.	Lead teachers to recognize the importance of dialogical communication between them and their students.

Source: Constructed from Ribeiro and Ponte (2020)

To deepen the understanding of the DIAP domain, as Sasseron (2020) does when addressing the classroom, we consider discursive interactions as ways in which the teacher educator and the PTs relate to materials and professional knowledge constructed during a formative process, which occurs through debates involving the exchange of ideas and justification. Based on this and the transposition of the ideas presented by Sasseron (2013) on promoting discursive interactions, Trevisan et al. (2023) propose a model (Figure 1) that considers aspects related to the different components of the DIAP.

Figure 1

Model for the analysis of the DIAPs in a formative process.



Source: Translate from Trevisan et al. (2023, p. 696)

Thus, based on what was proposed by Trevisan et al. (2023), who present the connections between the components of the DIAP domain and incorporating elements considered relevant for discussions in an initial education context, we will detail each component of the DIAP domain to analyze the interactions during an Algebra subject.

Discussions and arguments in the DIAP domain

Rodrigues et al. (2018) addressed the communicative aspects of teachers conducting mathematical and didactic discussions about mathematics classes for basic education in the context of mathematics teaching degrees. During these discussions, teachers expressed different ways of reasoning and reconstructing meanings related to providing feedback to the students.

When observing such discussions in a continuing teacher formative environment, Trevisan et al. (2023) (Figure 1) use sociomathematical norms promoted through actions such as sharing, justifying, questioning, and exposing confusion (Elliott et al., 2009) and practices aimed at orchestrating discussions to describe how mathematical and didactic discussions contribute to argumentation and justification.

About argumentation, Sasseron (2020) emphasizes its importance as a basis for knowledge (which we specify here as professional knowledge, including mathematical or didactic knowledge). The author defines it as a process that establishes an affirmation that relates, through justification or refutation, a proposition and a conclusion. According to Sasseron (2020), teachers' actions to promote argumentation in the classroom are based on pedagogical and epistemological purposes. The first is related to the development of actions that address space and time management in the classroom. In contrast, the second is related to work and the construction of scientific arguments. In a new transposition, considering the context of the mathematics teaching degree, teacher educators' actions must be based on didactic and mathematical purposes linked to professional knowledge, which aligns with Ribeiro and Ponte (2020) and Aguiar et al. (2021).

Regarding argumentation and justification in mathematics, Aguilar and Nasser (2012) argue that teachers must understand and accept students' different levels of argumentation. Furthermore, these authors say that many mathematics educators have emphasized the conception of proof as a convincing argument, which may be related to teaching proof to validate a statement, which requires the development of students' deductive reasoning. Therefore, it is important to consider the age range of students and their underlying knowledge.

Aguilar and Nasser (2012) considered that the types of proof presented by Sowder and Harel (1998) aim to elucidate how argumentation and justification can be discussed and addressed in mathematics classes in basic education. According to Sowder and Harel (1998), the types of evidence can be categorized as follows: i) *proof scheme based on external elements,* which occurs through persuasion with the use of symbols in a ritual and authoritarian way; ii) *empirical proof scheme*, in which justifications are carried out exclusively through examples and; iii) *analytical proof scheme,* considered the most rigorous type of proof, with justifications that approach the formal logical-deductive model widely discussed in academia.

In turn, Elliott et al. (2009) use the term "justification" to include the "how" and "why" a mathematical solution method for a problem or task is valid. They claim that justifications consist of a mathematical argument that allows for a deeper understanding of the ideas involved. This is important, considering that PTs must improve their understanding and interpretation of their mathematical knowledge and what their students may present.

Language and communication in the DIAP domain

From an operational point of view, the language used and the dialogic communication implemented demonstrate how discussion and argumentation practices favor teachers' professional learning (Ribeiro & Ponte, 2020). These aspects mediate interactions between individuals, contributing to the development of knowledge (David, 2004).

Language use in mathematics education has expanded the understanding of the use of words and mathematical symbols to consider the complexity present in the variety of communicative means, such as speech, writing, sight, and gestures, among others, understood as part of communication in the classroom, as pointed out by Morgan et al. (2014). Among these communicative means, gestures are visible actions in the interaction between subjects. The movements are part of a person's communication; they direct attention and may involve object manipulation (Kendon, 2004). According to Morgan et al. (2014), doing mathematics involves speaking, writing, or using communicative means, as its entities are not directly accessible; it is a discursive practice linked to language.

In this direction, Lorensatti (2009) considers mathematical language a system with symbols that relate to specific rules, understood by the community that uses it, inseparable from mathematical knowledge development. Through mathematical language, we can decipher mathematical codes and interpret mathematical problems and/or tasks (Lorensatti, 2009).

Finally, considering the classroom context, teachers must use natural and mathematical languages to interpret what students present and connect and systematize ideas and concepts to teach according to the school context, as Morgan et al. (2014) pointed out. "Didactic language" is what we call this approach.

Bringing together language and communication, Heid-Metzuyamin et al. (2015), based on the ideas of Sfard (2008), described some learning opportunities offered to PTs by analyzing the type of discourse used. Sfard proposes that learning occurs through participation in a discourse, and mathematics is considered a discourse with specific characteristics. According to Sfard (2008), discourse is composed of specific keywords, narratives, and routines, while participation in it can be *ritual* when the focus is on connecting with or pleasing other participants or *exploratory* when the goal is to produce mathematical narratives in and of themselves.

Ritual participation involves manipulating mathematical symbols without reference to meaningful objects and using human actions to manipulate these symbols (e.g., multiply, reduce, invert). Exploratory participation presents a mathematical discourse developed through a process of objectification, valuing experimentation with errors, and the exploration of unproductive paths to reach a conclusion in a non-direct manner.

Regarding pedagogical discourse, Shabtay and Heyd-Metzuyanim (2017) point out that it shapes and guides teachers on *what* to teach, *how* to teach, *why* some teaching actions are more effective than others, and *who* can or cannot learn.

On the other hand, Nemirovsky et al. (2015) identified two types of pedagogical discourses teachers use when discussing teaching cases presented in videos. The first type, called *grounded narrative*, links descriptions of events in the classroom and considers the evidence made available to teachers to establish a connection between reality and fiction to link a set of evidence presented in videos and other records related to professional practice. The second type of discourse identified by Nemirovsky et al. (2015) is the *evaluative* discourse, where the values, virtues, and commitments involved in the case in question are considered, while participants try to evaluate the use of good or bad practices based on their own evaluation criteria. This discourse may involve hypothetical situations about what should have been done by the observed teachers and students.

Arithmetic and algebra in the teaching degree

A teaching degree course in mathematics is expected to address arithmetic and algebra to deepen and solidify mathematical knowledge, expanding discussions regarding their teaching in basic education (Brasil, 2001). This action requires that subjects such as Algebra provide a "foundation that allows the desired teaching practice with an understanding of concepts and not just domains of algorithmic procedures" (SBEM, 2013, p. 23).

Ribeiro (2016) points out that the possible connections between algebra and school mathematics are not always carried out straightforwardly. The author indicates that a possible way to address this issue would be to think of a subject that discusses school mathematical concepts, such as functions and numbers, in light of teaching practice, which would include considering algebraic structures to support the constitution of mathematical knowledge specific to the mathematics teacher.

Thus, the approach to algebraic structures, such as groups, should not be abstract. Instead, their principles should be emphasized as a resource from which PTs can draw examples and counterexamples and support the discussion of mathematical tasks using these properties, whether implicit or explicit.

When considering the algebraic structure of groups –consisting of a set associated with an operation that satisfies the properties of associativity, the existence of the neutral element, and the symmetric element for every element– some other properties are consequences of its definition. As Domingues and Iezzi

(2018, p. 145) point out, if *(G, *)* is a group, then we can ensure the following properties: i) uniqueness of the neutral element of (G, *), ii) uniqueness of the symmetric element of each element of G; iii) if a $\in$ G then *(a')'=a*; iv); if *a, b* $\in$ G then *(a * b)'=b' * a'*, among others.

Based on such properties and well-defined didactic purposes, the teacher educator can seek approaches that provide PTs with learning opportunities that reveal procedures and unveil concepts addressed in basic education (Jardim et al., 2023b). In this sense, it is possible to discuss the meaning of the symmetrical element in arithmetic, algebraic contexts (Wasserman, 2014; Zazkis & Marmur, 2018) and even in geometric contexts, in which it is possible to explore the concept of symmetry associated with algebra (Gonçalves et al., 2022).

Study Context

This study was carried out with a mathematics teaching degree from the Federal Institute of São Paulo, São Paulo campus, emphasizing the moment the algebraic structure of groups was addressed in the Algebra subject. For this, a set of classes was planned, developed, and reflected upon (PDR cycle) (Trevisan et al., 2020), using two PLTTs refined from one year to the next.

Planning took place through online meetings where the teacher educator and the researcher (first author of this article) discussed the purposes and elaboration of the two PLTTs and how they would be developed. Nine planning meetings were held in total: seven in the first cycle (P0, $P_1$1, $P_1$2,..., $P_1$6) for the preparation of the PLTTs and their respective lesson plans, and two meetings for the refinement of these materials in the second cycle ($P_2$1, $P_2$2).

The formative process, based on exploratory teaching (Canavarro et al., 2012), occurred in three stages, as indicated: i) introduction with an initial task (IT); ii) carrying out the PLTT in small groups (SG), and iii) discussion and systematization in plenary sessions managed by the two teacher educators. In the first stage, the PTs individually solved the IT, consisting of five school-level mathematical tasks, accompanied by questions to reflect on the difficulties of basic education students in solving such them. Then, the classes were divided into SGs, with three in the first cycle ($PG1_1$, $PG2_1$, and $PG3_1$) and five in the second cycle ($PG1_2$, $PG2_2$, $PG3_2$, $PG4_2$, and $PG5_2$) so that they could discuss the resolution of each of the PLTTs (T1 and T2) autonomously in an online environment.

The PLTTs revisited the mathematical tasks solved in the IT that involved rational numbers, matrices, and functions and, based on them, explored the algebraic structure of groups. Each PLTT presented questions based on practice records (protocols of students from basic education, teaching manuals, vignettes, and class reports) that made up each of the three parts of the PLTT, namely: the first had protocols with resolutions of mathematical tasks seen in IT, carried out by basic education students; the second presented protocols of teaching manuals that address mathematical concepts from an academic point of view, such as definitions and properties; and the third presented videos and class reports that involved the mathematical ideas addressed in the previous parts, constituting what was called “practice cases.”

In total, four cases of practice were addressed, two of which will be analyzed and discussed in this article. Each SG developed each PLTT in asynchronous meetings held on the TEAMS platform*,* and later, there was a plenary session managed by the educators so that all SGs could share, discuss, and systematize the resolutions. In the first cycle, the plenary sessions took place remotely and in person in the second cycle. At the end of the process, the prospective teachers answered an online evaluation questionnaire on using PLTT in Algebra.

Methodology

This article is part of a qualitative approach from an interpretative social constructivism perspective (Esteban, 2010). It uses the design-based research (DBR) method with the execution of two cycles to enable the design of how to use LPTTs, develop them, and evaluate the results, aiming at the execution of new cycles (Barbosa & Oliveira, 2015).

Data was collected with the collaboration of one of the teacher educators[2], whom we named “Paulista” and who, at the time of the research, had almost a decade of experience teaching algebra in the teaching degree course. Paulista holds a teaching degree and a master’s degree in mathematics and has a PhD in mathematics education; in other words, she has a diversified background that is conducive to the proposal presented for the research, which, in turn, consisted of elaborating, developing, and reflecting on the use of PLTT in an algebra subject in partnership with this article’s researcher and first author.

[2] The teacher educator’s and prospective teachers’ names are fictitious and were chosen with their consent after signing the Free and Informed Consent Form approved by the UFABC Research Ethics Committee, linked to research project number 96044518.4.0000.5594 (CAAE – Certificate of Presentation of Ethical Appreciation). Resolution 466/2012 of 12 December 2012.

PTs participated in each cycle, 15 in the first and 20 in the second—they were named after neighborhoods or cities. Most PTs already had some experience teaching in primary school through practicums and (or) teaching initiation programs.

The data used in the study consisted of videos of the educators' planning and the development of PLTTs with the PTs in the SGs, in addition to the lesson plans. This data is part of a descriptive report detailing the entire process, from which three episodes were extracted to compose the corpus of this article.

With this data available and using the DIAP components presented by Ribeiro and Ponte (2020) (Table 1) and explored in the model proposed by Trevisan et al. (2023) (Figure 1), we assumed the theoretical frameworks adopted in this article to outline a set of categories that could help us identify how the discussions took place between the participants (Table 2) and relate such discussions to the role of the educator and the PLTTs used. The categories organized to support our analyses were constructed based on Bardin (2016):

Table 2

Categories for analysis

Component	***Category***	***Indicators***
Mathematical and didactic discussions	Sociomathematical norms (DMD-Sm) (Elliott et al., 2009)	- Share, question, justify, and/or elucidate confusions regarding the mathematical task and mathematical concepts
Argumentation and justification	Initial structures (AJ-De) (Elliott et al., 2009)	- Mention mathematical definitions, concepts, or ideas to find a justification.
	Types of test by test schemes (AJ-Pr) (Aguilar & Nasser, 2012)	- Explain the use of symbols (based on external elements); - Exemplify to justify (empirical); - Use mathematical arguments analytically (analytical).
Language Mobilized	The use of mathematical langua (LI-Ma) (Morgan et al., 2014; Lorensatti, 2009)	- Use natural or native language to explain mathematical ideas; - Use symbols, terms, and nomenclature considered in a mathematical environment.
	Use of didactic language (LI-Di) (Morgan et al., 2014)	- Mention/create situations with appropriate language for teaching and learning based on the school context.
Dialogic communication	Mathematical discourse (DI-Ma) (Sfard, 2008; Heid-Metzuyamin et al., 2015)	- Manipulate mathematical symbols and use human actions to present ideas or take a position (ritual speech). - Experiment with paths, use feedback and open questions to present ideas, or take a position (exploratory speech).

	The pedagogical discourse (DI-Pe) (Nemirovsky et al., 2015)	- Narrate or describe a teaching case projecting the participants' fictional vision (grounded narrative). - Evaluate the use of practices in a hypothetical situation (evaluative).

Source: Prepared by the authors.

Results

We present our results through three episodes. The first, "*Previsão de discussões* [Anticipating discussions]," shows how the graduates anticipated some discussions. The following two episodes, "*Desvendando a inversão*" [Unraveling the inversion] and "*A incrível simetria do professor Lambarildo*" [Teacher Lambarildo's incredible symmetry], reveal some of the PTs' discussions when solving the two PLTTs during autonomous work in the SG.

Episode 1: Anticipating discussions

In planning, the teacher educators listed discussions that could emerge from the practice records. They highlighted some purposes of the PLTTs in question:

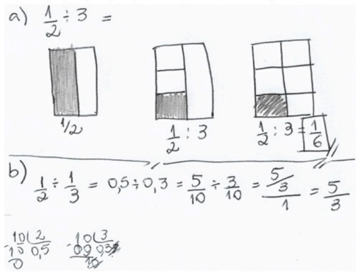
> Researcher - *I tried to write an assignment* [mathematics] *that falls into 'half times a third,' which is very classic* [...], *and based on my classroom experiences, I put together this protocol* [shown in Figure 2]. [...] *I found an exercise that asked me to divide one half by three, and the result was* [...] *to observe the process in a geometric way* [pictorial]. *Then I thought: if a student from* [basic education] *had contact with this process, he or she could multiply using this process* [...] *The idea is to discuss with [undergraduate] students that [...] when you need to go to the procedure, using a drawing or justifying the procedure.* .[$P_1$1, 2021]

As a teacher educator, when preparing PLTT-1, the researcher outlined mathematical purposes related to the neutral and inverse elements (reciprocal) implicit in dividing fractions. She anticipated the discussions among the PTs based on the mathematical task and the practice logs presented in the PLTT (DMD-Sm).

Furthermore, PLTT-1 had records in several languages, as shown in Figure 2.

Figure 2

PLTT-1 math task and practice logs

Mathematical Task 1 with protocol 1 used in the 1st part of TAP-1:	*Definition of the Group algebraic structure presented in the 2nd part of TAP-1*
Renato intends to put ½ liter of soda in several glasses. a) What will be the measurement in liters of soda if Renato divides it into three equal glasses? b) Is it possible to fill 2 glasses with a capacity of ⅓ liter? Justify. 	A mathematical system consisting of a non-empty set G and an operation on G is called a Group if this operation satisfies the following axioms: (i) Associative $(a * b) * c = a * (b * c)$, whatever $a, b, c \in G$; (ii) Existence of the neutral element There is an element $e \in G$ such that $a * e = a = e * a$ whatever $a \in G$; (iii) Existence of symmetric For every $a \in G$, there exists a' such that $a * a' = e = a' * a$. If, in addition to these axioms, the axiom of commutativity is fulfilled, the Group will be called a commutative or abelian Group.

Source: Research data.

The mathematical task is accompanied by an interpretation with a drawing of "bars," and the formal definition of the algebraic structure of groups was presented in the 2nd part of the PLTT-1

Still in the teacher educator's speech, she describes how to use vignettes in the PLTT:

> Researcher: *Here comes the video* [with transcription] [...] *The idea is they* [PTs] *see that if the teacher* [from the vignette] *had used the neutral element, inverse element, and explained their meaning so that the students understood them, and mathematically, they could see that it is a group: the rational numbers without zero regarding multiplication. And* [in the 2nd part of *the PLTT-1] we will be giving a tip, trying to relate these examples* [from math task] *with this mathematical definition* [of the algebraic structure of groups]. [$P_1$1, 2021]

The practice logs used throughout the PLTT-1 presented several mathematical languages, as they rely on student protocols (Figure 2) and video transcripts (Figure 3), which explore natural language and definitions extracted from textbooks (Figure 2), with a more formal mathematical language (LI-Ma; LI-Di).

Figure 3

Part of the 3rd part of PLTT-1

1st Practical Case: A lesson on dividing fractions During a class intended for the 7th year of Elementary School, the following procedure was presented to carry out the division between two fractions:	
Vignette 1 - Video crop 7:44 to 8:43 	1) Knowing that (Q*, ·) is an example of a group, answer the following questions: a. What does it mean to say, "we invert this operation [multiplication] and to compensate, we have to invert this last fraction"? Indicate the connections with the algebraic structure of the Group presented above. b. How would you use these links to the group structure to explain the procedure presented in the video to your students?

Source: Research data.

Furthermore, the way the educators intended to present the resources selected in PLTT-1 anticipated the establishment of dialogic communication among participants based on mathematical and/or pedagogical discourse while observing and analyzing the vignette, guided by the questions of the PLTT-1, as exemplified in Figure 3 (DI-Ma; DI-Pe).

The objectives related to mathematical knowledge, mentioned by one of the educators, indicate that the PTs are expected to justify the division procedure between two fractions using arguments linked to the concepts of neutral and symmetrical elements (AJ-De; AJ-Pr), which the other educator confirms:

> Paulista - They are the ways of producing meaning. If I am going to work with the reverse the way I told you [in another meeting about the pro-literacy material], I am producing meaning for the reverse. So, I have a number [rational in form a/b] divided by another, if I multiply by the inverse of the denominator above and below [...] if I multiply by the neutral element, I do not change the value, so I am using all mathematical properties and not simply an algorithm [...]. [$P_1$1, 2021]

Paulista foresees the use of mathematical arguments and points to an expected discussion from the PTs that reveals the procedure, while the symmetric and neutral elements can be revealed by rewriting a/b : c/d as a fraction with numerator a/b and denominator c/d with b, d $\neq$ 0, which points to a mathematical discourse.

In turn, when planning how the PLTT-2 vignettes could trigger discussions, the teacher educators set out some didactic purposes for using the two PLTTs:

> Researcher - *There* [on PLTT-1] *we only worked with the expository class* [in the vignettes], *which is something in which they* [PTs] *are still stuck* [...]. *They just looked at the teacher. Here, in the PLTT-2, it's not just about looking at the teacher; they have to think as if they were the teacher. So, there is this transition from one PLTT to another. Here, even though I start by observing Teacher Lambarildo's class* [...], *they have to give answers thinking as if they were in his place.*

Paulista - *Yes! Yes!*
Researcher - *It's them also seeing themselves as teachers. It's not them analyzing what the teacher did. They are putting themselves in the teachers' shoes, I think there is this leap in relation to the practice of the two PLTTs. I observe and criticize. The other one, no, is how are you going to do it*
Paulista - *Yes. 'You critized it, so give me the best solution.'* [$P_1$5, 2021]

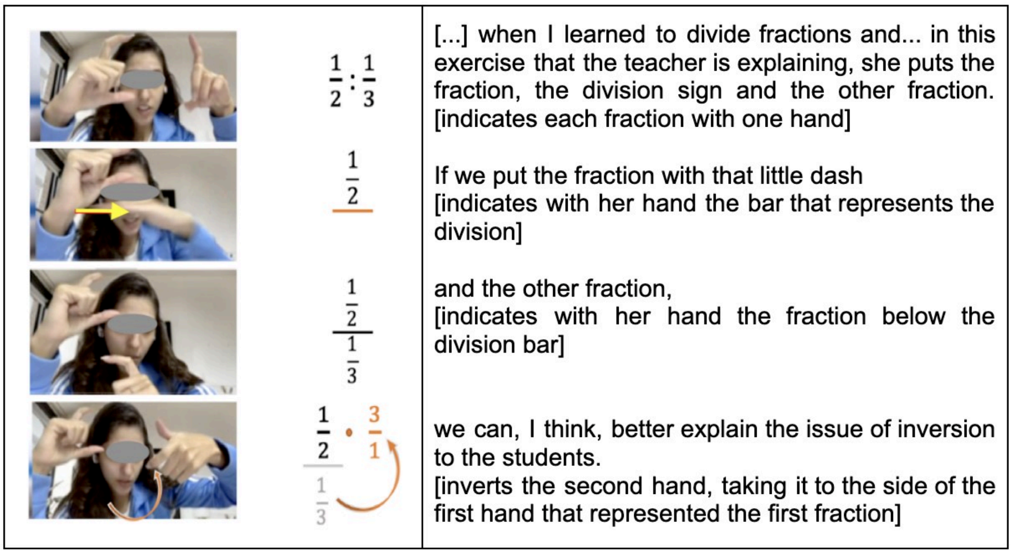
The teacher educators state that the PLTTs must support the PTs in making decisions related to the practical case situations, which requires changing from an observant and reflective perspective to a critical and constructive perspective between the development of the two PLTTs. For this to happen, prospective teachers must see themselves as teachers through dialogical communication that uses mathematical and didactic discourse.

Episode 2: Unraveling the inversion

PLTT-1, "Mundo Paralelo" [Parallel world], used practice logs seen in Figures 2 and 3 to promote a discussion about the connections between the division of fractions and algebraic properties. When exploring the PLTT issues inspired by vignette 1 (Figure 3), PT Itaquera, a $PG1_1$ member expressed herself with gestures (Figure 4):

Figure 3

Gestures, illustration, and speech by PT Itaquera



Source: Research data.

While Itaquera *narrated a teaching case* she experienced (*when I learned fractions*), she *shared* her experience with the procedure (DMD-Sm). She used gestures to *explain and convince* on the inversion of the fraction 1/3 by "transforming" the division into a multiplication (AJ-Pr). Even though such an explanation does not establish the connection with the algebraic structure, as requested by PLTT-1, Itaquera sought to elucidate the real meaning of the use of the symmetrical element, hinting at the use of didactic language since she *mentioned a situation using language appropriate for the classroom* (LI-Di). She went on, but presenting new examples for the procedure:

> Itaquera - *Because when we're dividing, we say that, I don't know, 2/3 divided by 5, we say that there's a '1' under the 5 for the students, and we talk about this inversion. I think it would be easier to understand this inversion than leaving just that sign* [with colon] *of the division* [SG11-T1, 2021].

To *exemplify and justify* the procedure (AJ-Pr), Itaquera reflected on the didactic language to be used in the classroom (LI-Di) and reinforced the importance of mathematical symbols to represent division, *evaluating the use of practice in a hypothetical situation* (DI-Pe).

Based on Itaquera's reasoning, the other $SG1_1$ members engaged in answering the PLTT-1 questions (Figure 3).

> Moema - [...] *as for inverting, what she means [the teacher in the vignette] in the question, it is to explain what happens when we invert.*
> Itaquera - *Why it reverses and the result is still the same...*
> Moema - *And the connection regarding multiplication, why the group, from what we saw in the last activity* [2nd part of PLTT-1- Figure 2] *has a relationship with operations, doesn't it? [...] there is* a *question for the operations for the rationals to be closed* [...] *I believe we can provide this justification to indicate the connection with the algebraic structure of groups, but as for the explanation of the inversion of operations for multiplication, I wouldn't know how to explain it* [$SG1_1$-T1, 2021].

PT Moema *mentioned concepts and ideas to justify* (AJ-DE) and find the connection with the algebraic structure of groups and, thus, justify the inversion of the fraction and the change of operation (from division to multiplication) based on the concepts covered during algebra classes and presented in the 2nd part of the PLTT-1 (operations, inverse (symmetric), and rational numbers). After some discussions, PT Capão Redondo presented new arguments to complete the ideas that were put forward:

> Capão Redondo - *It would be more or less like this: the neutral element of multiplication would be 1* [...] *and when we invert an element, the* $\boldsymbol{a}$ *of the group finds the* $\boldsymbol{a'}$*, which, in the case of rationals in relation to multiplication,* $\boldsymbol{a'}$ *is the inverse fraction. And using the inverse fraction of* $\boldsymbol{a}$*, we can build an algorithm* [procedure] that allows us to calculate the inverse operation [division].
> Mooca - *And if we multiply a fraction by its inverse, it will give us the neutral element, which is 1.*
> Moema - *It is possible to make this relationship* [...] *relate to the algorithm, as it builds the entire operation. I believe that is all. I can only make this connection* [$SG1_1$-T1, 2021].

Based on Moema's mathematical concepts, Capão Redondo *used symbols, terms, and nomenclature considered in a mathematical environment* to construct its justification (AJ-Pr) but did not achieve an analytical argument capable of outlining the nuances of the procedure under scrutiny. One reason for this might be associated with *ritual speech* because the members of $SG1_1$ *manipulated mathematical symbols and used human actions to present ideas,* which might have limited the argument (DI-Ma).

At the end of the formative process, prospective teachers evaluated the use of PLTT through an online form, and prospective teacher Butantã, na $SG1_1$ member, shared his experience:

> Butantã- The advantage of doing the activities [mathematical tasks], first individually, then in groups and then in plenary sessions, makes us realize the different ways of thinking in those exercises [in those tasks]. And often people notice something you hadn't noticed at first and this interaction helps with reflection. (Butantã, Evaluation, 2021).

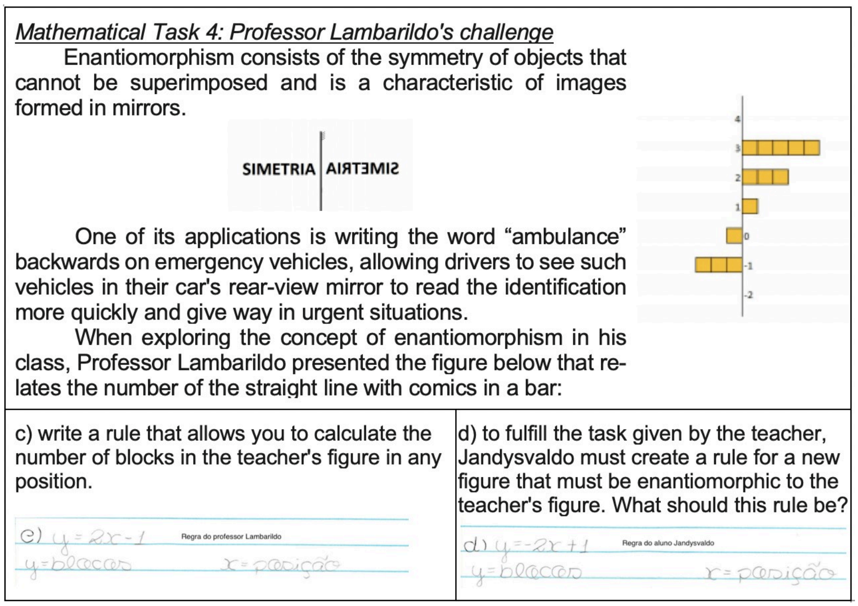
Butantã's assessment shows that the interaction in small groups and plenary sessions helped the participant perceive other ways to solve mathematical tasks and identify possible connections that were not evident when she faced the challenges individually. Furthermore, he mentioned the stages of exploratory teaching as an environment that facilitated interactions and reflections throughout the process.

3rd episode: Teacher Lambarildo's incredible symmetry

PLTT-2, "Mundo Antagônico" [The antagonistic world], revisited the three IT mathematical tasks that involved the content of functions and presented protocols with solutions from basic education students, as exemplified in Figure 5:

Figure 5

Math task 4 and student protocols - 1st part of PLTT-2.


Mathematical Task 4: Professor Lambarildo's challenge

Enantiomorphism consists of the symmetry of objects that cannot be superimposed and is a characteristic of images formed in mirrors.

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One of its applications is writing the word "ambulance" backwards on emergency vehicles, allowing drivers to see such vehicles in their car's rear-view mirror to read the identification more quickly and give way in urgent situations.

When exploring the concept of enantiomorphism in his class, Professor Lambarildo presented the figure below that relates the number of the straight line with comics in a bar:

c) write a rule that allows you to calculate the number of blocks in the teacher's figure in any position.

c) y=2x-1 Regra do professor Lambarildo
y=blocos x=posição

d) to fulfill the task given by the teacher, Jandysvaldo must create a rule for a new figure that must be enantiomorphic to the teacher's figure. What should this rule be?

d) y=-2x+1 Regra do aluno Jandysvaldo
y=blocos x=posição

Source: Research data.

Furthermore, the 2nd part of PLTT-2 explored some properties of the algebraic structure of groups, extracted from abstract algebra books, and the 3rd part presented a class report, accompanied by two vignettes to support the discussion in the SGs. All were designed for the PTs to answer the following question: *How would you use the concept presented by Teacher Lambarildo to address symmetry in basic education?*

Figure 6

A register of Teacher Lambarildo's class – 3rd part of PLTT-2

1st Practice Case: Professor Lambarildo's class

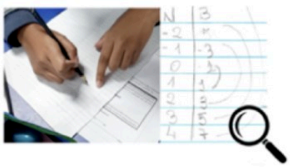
Professor Lambarildo led an 8th grade class through exploratory teaching, implementing mathematical task 4. He started the class, introduced the task, allowed students to work in small groups to solve it, and finally, systematized the ideas presented by the students. students after the presentation of the resolutions.

The class led to many discussions and one of them dealt with the use of the "-" sign. Vignette 4 shows part of a dialogue between the teacher and one of the students while the students discussed the task in groups.

The vignette 3, which is part of the introduction to mathematical task 4 taught by Professor Lambarildo, provides a glimpse of what happened in this class.

Vignette 3: Explanation by Professor Lambarildo

Vignette 4: Dialogue with a student



Source: Research data.

Concerning the report presented and the associated issue presented in the PLTT, SG1_1 presented the following discussion:

> Moema - *We can associate the concept of symmetry with a type of group* [...]
> Capão Redondo - *Enantiomorphism, you mean? Related to symmetry.*
> Moema - *Yes. It could be.*
> Capão Redondo - *I think enantiomorphism is the operation, right? And group* [set] *would be the groups* [sets] *of functions, images, whatever.*
> Moema - *No. What I'm saying is that there's probably a group* [...] *that speaks of symmetrical. Not about functions*. [...] *It's just that in the vignette, he* [Professor Lambarildo] *talks about symmetry, not about function.*
> Butantã - *Symmetrical elements, right?*
> Moema - *That is it. So much so that the position appears in the student's notes, of course. But the main discussion is about the elements being symmetrical, although the positions are not the same.* [...] *It's just that in the vignette, he [professor Lambarildo] is talking about symmetry. He is not talking about function itself* [SG1_1-T2, 2021].

When starting the discussions, PTs seek to *present ideas related to the mathematical task* and *mention concepts* originating from it (enantiomorphism, functions) in an attempt to connect them to the mathematical ideas presented in PLTT-2 (properties involving symmetric elements, operation) (DMD-Sm; AJ-De). Even though the PTs had not realized until then that the functions addressed by the mathematical task could be part of an additive group of functions, they raised suspicions to justify the relationship between enantiomorphism and the existence of a symmetric element. And the discussion went on:

> Capão Redondo - *Maybe if we used the fact that the inverse of the inverse is the figure itself, which is a consequence of here* [referring to the properties] [...]
> Mooca - *But would you use this to deal with symmetry?*
> Capão Redondo - *Right! Because those properties are related to...* [symmetry]
> Moema - *Explain through properties. Of course you will use consistent, exact examples as a background. But we can use different language. Because the teacher, in this case, uses body language* [...] *he shows it through his body* [$SG1_1$-T2, 2021].

Guided by the question, the PTs discussed how they would teach the concept of symmetry using a natural language connected to the context of the task *(the inverse of the inverse is the figure itself)* (LI-Ma) to indicate the property involving the symmetrical element (a')' = a*.* They still *evaluated* the example the teacher gave and recognized that he used gestures (*body language*) as an appropriate language for teaching the concept at stake (DI-Pe; LI-Di). We also observed that $SG1_1$**,** in the PLTT-2 resolution *experiments with ways of presenting their ideas, and positions itself,* as seen in Mooca's speeches (*But you would use this to deal with symmetry),* which characterizes an exploratory discourse (DI-Ma).

In the second cycle of the PLTT development with a new PTs' class, an SG presented other arguments to connect the idea of symmetry presented in the mathematical task and the algebraic structure of groups.

> Morumbi - *I thought his idea was cool, but what happens: the way he taught symmetry is for the addition operation, right?*
> Pirituba - *It's true!*
> Morumbi - *Because he says that equal elements must be equidistant from the main axis. And why does it have to be equidistant? Because when you add the two together, you will get the main axis. There would be the 1 and -1, 2 and -2...* [...]. *In the concept he presented,* [...] *we could complement this by addressing symmetry, seeking reference about what this axis of symmetry is, that he chooses* [...].
> Interlagos - *In this case, it is a margin to talk about the number line.*
> Morumbi - *Taking advantage of the concept that he* [the teacher] *presented, I was going to try to show the students that this axis that we were going to take as a reference depends on the operation we are mentioning.* [...] *and this, in this case, would be for the addition operation, therefore, the elements must be equidistant from this axis* [$SG4_2$-T2, 2022].

$SG4_2$ sought *to justify* the enantiomorphism seen in the mathematical task (DMD-Sm) from the *assessment* of the didactic language Teacher Lambarildo used (DI-Pe). PT Morumbi argues about the existence of symmetrical elements in the addition operation, which leads the group to mention the equidistance to the axis of symmetry and the number line as interconnected mathematical elements when exploring symmetry. In this way, $SG4_2$ *mentioned concepts to find a justification* to connect the concept of symmetry and the additive group at stake (AJ-De) and used natural language to treat them, as seen in Morumbi's speech (*Because when you put the two together*, *the result will be the main axis* (a * a' = e) (LI-Ma). This SG's speech was based on *the evaluation of the use of practices*. The SG complements the teacher's actions, *experimenting with ways to position itself* in the face of what was presented by the vignette and asked by the PLTT-2 (DI-Ma).

As in the first cycle, upon completing the development of the PLTTs, the prospective teachers wrote their evaluations of the experience, and discussions were emphasized again:

> Morumbi: The initial task, done individually, brought me face to face with concepts and problems that I was not used to noticing and reflecting on. With the small group, I realized that there were other opinions and views different from mine, which in fact supported the discussions and brought me new knowledge. As for the plenary, the discussions were expanded even further and, as a consequence, expanded learning and new positions too (Morumbi, Evaluation, 2022).

Morumbi states that the discussions in small groups and expanded to plenary sessions allowed him to learn new knowledge while engaging with other members' opinions. This point is also defended by prospective teacher Interlagos:

> Interlagos: The contribution that the discussion had was to reveal our doubts and those of our colleagues, to share and help each other in what each can understand and relate the doubts with possible doubts that will arise from students when we teach (Interlagos, Evaluation, 2022).

Interlagos and Morumbi pointed out that the PLTTs brought concerns that allowed interactions with peers to share ideas and knowledge. In turn, Interlagos highlighted as a positive point that sharing ideas could help in the exercise of the profession, even if this was only idealized until then.

Discussion of the results

In this section, we present the relationships between the episodes and the meanings that constitute each component of the DIAP domain (Ribeiro & Ponte, 2020) based on the model by Trevisan et al. (2023), supported by other references.

In the first episode, the teacher educators established mathematical and didactic purposes by selecting mathematical tasks and practice records that outlined how these purposes would be explored through questions presented in the PLTT (Rodrigues et al., 2018; Jardim et al., 2023a) in order to provide discussions based on argumentation and justification of procedures (Elliott et al., 2009). Furthermore, the PLTT incorporated diverse mathematical and didactic languages through different discourses (Sfard, 2008; Morgan et al., 2014; Heid-Metzuyamin et al., 2015; Nemirovsky et al., 2015) to explore discussions among PTs that led them to identify themselves as teachers (Barretto & Cyrino, 2023; Cyrino & Estevam, 2023). In short, the teacher educators' actions in favor of constituting the PLTTs characterize guidance of these two domains in favor of the DAIP domain (Ribeiro & Ponte, 2020).

In the SGs' discussions, we observed that the educators' expectations were met in the following episodes. In the second episode, there were mathematical and didactic discussions, which were leveraged by the question presented in PLTT-1 (Trevisan et al., 2023; Jardim et al., 2023b), although with some limitation in the expected argumentation, possibly due to the use of ritual discourse (Heid-Metzuyamin et al., 2015). This episode showed how gestural communication may have helped communication between the prospective teachers (Kendon, 2004; Cyrino & Estevam, 2023), which can be better explored in face-to-face formative environments.

In the third episode, we identified mathematical and didactic discussions by connecting school and algebra content through natural language (Lorensatti, 2009; Morgan et al., 2014), which demonstrates that the PTs could internalize the mathematical language presented in the 2nd part of the PLTT-2 and use it in a didactic discussion, although neither of the two analyzed SGs used a mathematical language widely accepted in other mathematical environments (Aguila & Nasser, 2012). Moreover, $SG1_1$ presented distinct mathematical discourses when dealing with different PLTTs, probably related to the use of didactic language and the evaluation of pedagogical discourse, evidenced especially in PLTT-2 (Nemirovsky et al., 2015).

Furthermore, when analyzing some participants' evaluations on the use of the PLTTs, they pointed out that the discussions promoted provided them with new understandings regarding the concepts addressed in the mathematical tasks and reflecting on how to approach them in the classroom, which appears to be a PLOT (Ribeiro & Ponte, 2019).

We noticed that the mathematical and didactic purposes (Ribeiro & Ponte, 2020; Sasseron, 2020) presented by the educators were reflected in the SGs' discussions, involving different mathematical languages and encouraging dialogical communication (Cyrino & Estevam, 2023). These discussions started from sociomathematical norms (Elliott et al., 2009) and contributed to argumentation and justification in different mathematical contexts (Elliott et al., 2009; Aguilar & Nasser, 2012;). Furthermore, the PTs established connections between the mathematical tasks presented and the properties of the algebraic structure of groups autonomously, exploring the meaning of the symmetrical element in different school contexts due to the algebraic structure that connects them, from arithmetic content to geometric content (Wasserman, 2014; Ribeiro, 2016; Zazkis & Marmur, 2018) and were able to have an experience with exploratory teaching as students (Cyrino & Oliveira, 2016; Aguiar et al., 2021).

Conclusions

When seeking to identify potential relationships between discursive interactions, the teacher educator's role, and the formative tasks, and to understand how such relationships promote learning about the teaching of school mathematics in an algebra discipline in a degree course in mathematics, we observe how the components of the discursive interactions among participants (DIAP) domain emerge in discussions between prospective teachers (PTs) and how the educators develop the mathematical and didactic purposes in defining the classroom objectives (Sasseron, 2020; Jardim et al., 2023a).

To answer questions about how discursive interactions between participants in an algebra discipline are anticipated by the teacher educator and leveraged by formative tasks to promote learning about teaching school mathematics, we realized that the discussions in small groups (SGs) presented components of the DIAP domain, which emerged from practice logs and the questions of the professional learning tasks for teachers (PLTTs) that, in turn, had been outlined by the teacher educators. This demonstrates the effectiveness of this resource associated with exploratory teaching in promoting discussions in the context of teacher education (Fiorentini & Oliveira, 2013; Cyrino & Oliveira, 2016; Marins et al., 2021; Cyrino & Estevam, 2023).

To indicate which relationships are established between discursive interactions, the teacher educator's role, and the formative tasks when addressing algebra in a mathematics teaching degree course, we observed that the role of the educator in predicting these discussions and ensuring that they do not become just moments of exchanging ideas without purpose (Sasseron, 2013; 2020) were established through the choices of practice logs and PLTT questions, which directed the autonomous work in the SGs in favor of the PT's reflection on algebra (Sousa & Paiva, 2023), which reinforces the interconnection between the domains of the professional learning opportunities (PLOT) model to facilitate teachers' professional learning (Ribeiro & Ponte, 2019; 2020).

Trevisan et al.'s (2023) model was essential to structure the analysis of the DIAP, although we considered other elements, such as the types of evidence (Sowder & Harel, 1998; Aguilar & Nasser, 2012). Furthermore, the way components are connected indicates that the model can be applied to analyze discussions in initial education, especially when analyzing school mathematics tasks from the perspective of academic mathematics. Therefore, new approaches can be considered, such as understanding the opportunities offered to teachers and the signs of learning presented at the end of the subject.

It is worth highlighting that prospective teachers' transition process from basic school student to an actual teacher is fundamental and requires him/her to get closer to the professional practice that he/she will exercise, which can be facilitated by hypothetical situations directed by educators and supported by resources that aim to improve PT's professional learning (Marcelo, 2009; Fiorentini & Oliveira, 2013; Cyrino & Estevam, 2023).

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