ALEXANDRIA
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Teaching limit functions of a real variable: a systematic literature review exploring difficulties, challenges and the implementation of tasks

Ensino de limite de funções de uma variável real: uma revisão sistemática de literatura explorando dificuldades, desafios e a implementação de tarefas

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Abstract: The teaching of limits of real functions of a real variable in some undergraduate programs and other courses in the areas of Exact and Natural Sciences requires progressively complex tasks for students to consolidate their knowledge and apply it in new mathematical contexts or in their field of work. That said, this study, part of a doctoral thesis, presents a Systematic Literature Review aimed at critically analyzing the existing knowledge on tasks in the teaching of limits of real functions, exploring difficulties, challenges, and evaluating the implementation of tasks according to the Task Design Criteria (TDC). Thus, adopting a qualitative approach based on the analysis of relevant documents (theses and dissertations), we identified, among other obstacles, difficulties in understanding the formal definition of limit and challenges related to prior knowledge. In the analysis of the proposed tasks, we verified whether the TDC were met; we observed researchers' efforts to produce different types of tasks, predominantly closed tasks such as exercises and problems, which demonstrated good cognitive demand and promoted interaction and open thinking.

Keywords: Task design, Limit of functions, Earning difficulties and challenges.

Resumo: O ensino do limite de funções reais de uma variável real, em algumas licenciaturas e outros cursos das áreas de Ciências Exatas e Naturais, requer tarefas progressivamente complexas, para que os alunos consolidem o conhecimento e o apliquem em novos contextos matemáticos ou na sua área de atuação. Isso posto, trazemos neste estudo, parte de uma tese de doutorado, uma Revisão Sistemática da Literatura, que teve como objetivo analisar criticamente o conhecimento existente sobre tarefas no ensino de limites de funções reais, explorando dificuldades, desafios e avaliando a implementação das tarefas aos Critérios de Desenho de Tarefas (CDT). Por meio de uma abordagem qualitativa, com base na análise de documentos (teses e dissertações), identificamos entre outros obstáculos, dificuldades na compreensão da definição formal de limite e desafios relacionados aos conhecimentos prévios dos estudantes. Na análise das tarefas propostas, verificamos se os CDT foram cumpridos e observamos os esforços dos pesquisadores em produzir diferentes tipos de tarefas. O predomínio de tarefas fechadas, como exercícios e problemas, de notável exigência cognitiva, que estimulam a interação e a abertura do pensamento.

Palavras-chave: Desenho de Tarefas, Limite de funções, Dificuldades e desafios de aprendizagem.

Introduction

The discipline of Differential and Integral Calculus (DIC) is a curricular component present in undergraduate Engineering, Technology, and some Natural Sciences Teaching degrees, which allows students to build knowledge through different tasks with varying degrees of abstraction, ranging from simpler to more complex tasks. Furthermore, this knowledge contributes to the assimilation of content from other disciplines (Silva, 2019).

Lima (2012), based on research about the history of the Calculus curriculum component, noted the need for this discipline to undergo a didactic change, "taking into account the mathematical maturity of students, the course in which the discipline was inserted, and the profile of the professional they wished to form" (Lima, p.7). This fact culminated in the creation of the Differential and Integral Calculus discipline, which still followed an analytical character, with a gradual change in the didactic-pedagogical approach.

The current model in Brazil regarding the DIC discipline in these undergraduate courses has European influence. It was initially implemented at the University of São Paulo (USP) in 1934, intended for Pure Mathematics training. In it, the contents and concepts of this discipline were addressed with high levels of systematization and rigor of more specific algorithmic and algebraic practices, and assessments based on such approaches (Bertolazi, 2017; Lima, 2012).

It is known that student performance is frequently related to these subjects' prior knowledge. However, many enter university without an understanding of mathematical foundations, which becomes evident when they face the need to interpret mathematical language.

For example, when trying to understand the concept of limits, some students have difficulty $\lim_{x \to 0} \frac{\sin x}{x}$, calculating when they don't understand the basic concepts of trigonometry and limits. This difficulty can be reflected in the inability to discern whether a certain procedure is correct or not, compromising the resolution of this limit and other problems involving limits with a higher degree of difficulty, which exemplifies the importance of a solid mathematical foundation for the academic development of this content (Lochhead, 1995).

An alternative we found to minimize such facts is to think about the tasks produced by the teacher. In this sense, problems of this nature can be minimized through tasks produced or selected by the teacher. The first ones can focus on basic knowledge, which constitutes the pillars for understanding the main theme of the class.

The subsequent tasks can be dedicated to the content itself. However, Gusmão (2019) emphasizes that it is not enough to simply replace exercises that emphasize task repetition, typically of the "calculate" type, with investigative problem-solving: "Mathematics teaching needs to vary tasks, diversify their types, and give opportunities for students to know other ways of learning it" (Gusmão, 2019, p. 4).

Studies on mathematical tasks, focusing on student responses and teacher work, highlight their central importance in teaching and learning processes. Hiebert and Wearne (1997) emphasize that student learning is largely shaped by the tasks offered by teachers.

According to Gusmão and Font (2021), mathematical tasks play a fundamental role in students' cognitive development, knowledge assessment, approximation to mathematics, stimulation of convergent and divergent thinking, learning of mathematical concepts and representations, expansion of students' mathematical knowledge, promotion of creative processes, and improvement of teachers' didactic-mathematical competencies.

Among the contents covered in Calculus, the study of limits of real functions is often considered complex due to its abstract nature and the need to understand fundamental concepts such as proximity of values and asymptotic behavior of functions. In this sense, both students and teachers face challenges when addressing this topic, as it demands a solid understanding of the basic principles of Calculus and an ability to think abstractly.

In this sense, we understand that the conception of differentiated tasks, concrete examples, practical applications, and a gradual approach to the concepts and properties of limits can favor the gradual distribution of complexity, constituting a means that may minimize the difficulties presented by students. Thus, this work aims to conduct a systematic review of dissertations and theses produced in the last ten years (i.e., from 2013 to 2023) that address studies on the use of tasks in the teaching and learning process of limits of real functions, and to understand the difficulties, challenges, and to what extent the characteristics of the tasks proposed in the analyzed material fulfill the CDT.

Methodological Path

This research is bibliographic in nature, more specifically, a Systematic Literature Review (SLR). According to Ramos, Faria and Faria (2014), a systematic review is applied to identify research on a specific subject through criteria, methods, and selection of bibliographic sources, with rigor, in order to generate reliability in the work developed.

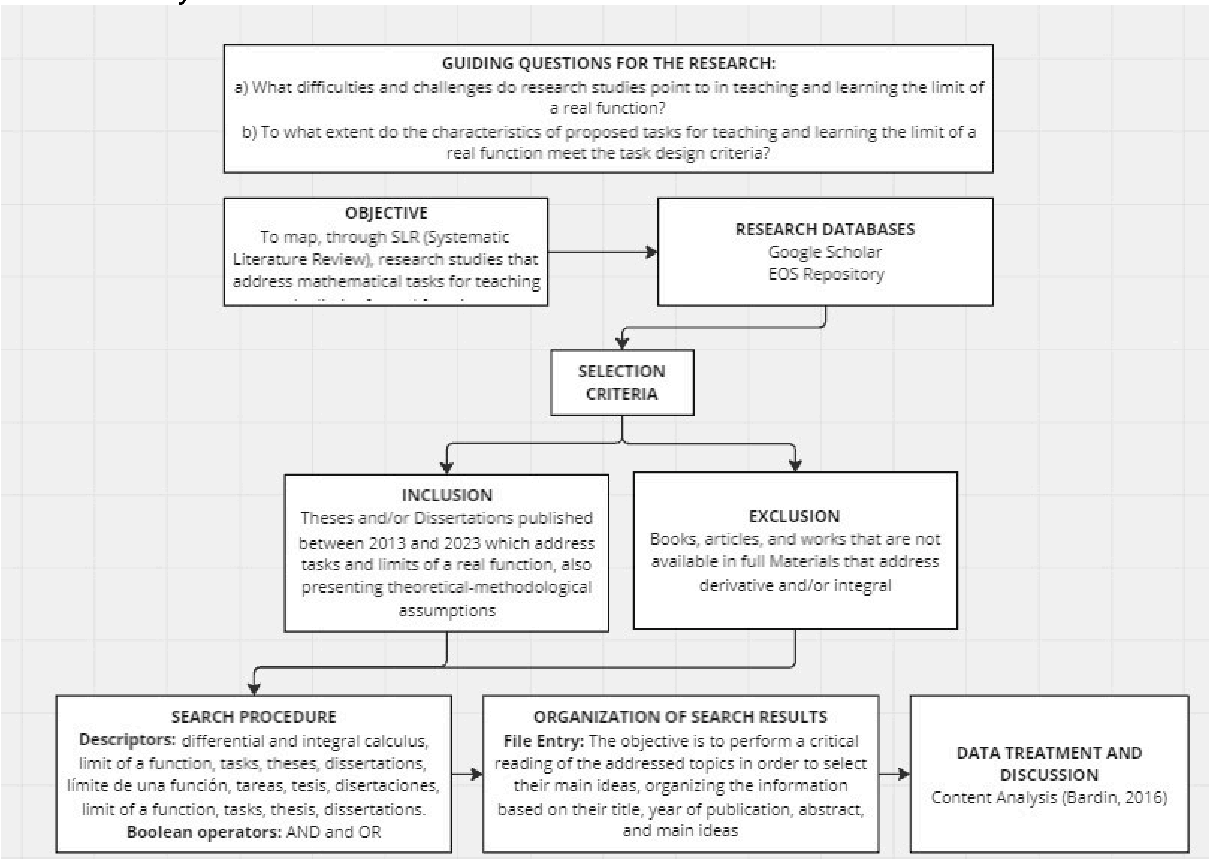
According to Costa and Zoltowski (2014, p.57), the review research problem can be broken down into some parts that aim to facilitate the search and organization of the results found. In this sense, we developed the following questions as guiding elements for the investigation process:

a) What difficulties and challenges do the studies point out in teaching and learning the limit of a real function?

b) To what extent do the characteristics of the proposed tasks for teaching and learning the limit of a real function meet the task design criteria?

According to Ramos and Faria (2014), there is a protocol to be followed in any SLR, which can be visualized in figure 1.

Figure 1
Protocol for Systematic Literature Review



Note. [Image description] Flow diagram with white background and black text, detailing steps of a systematic literature review (SLR) about tasks in teaching limits of a real function. At the top, in a rectangle, are the guiding questions: a) What difficulties and challenges do studies point out in teaching and learning the limit of a real function? b) To what extent do the characteristics of tasks proposed for teaching and learning the limit of a real function meet task design criteria? Below is the objective, highlighted in another rectangle: Map through SLR research that addresses mathematical tasks for teaching limits of a real function. This objective is connected by arrows to two boxes: On the right: Research databases – Google Scholar and EOS Repository. Below: Selection criteria, connected to two blocks: Inclusion: Theses and/or dissertations published between 2013 and 2023 that address tasks and limits of a real function, also presenting theoretical-methodological assumptions. Exclusion: Books, articles and works not available in full; materials addressing derivatives and/or integrals. Following are two more stages: Search procedure: descriptors in Portuguese, English and Spanish; Boolean operators: AND and OR. Organization of search results: Critical reading notes and summary of main ideas. Finally, the last stage is data processing and discussion, based on Bardin's (2016) content analysis. [End of description].

Figure 1 illustrates the Systematic Review Protocol outline, highlighting the essential elements of the process. We specify the review objective, followed by the work selection bases.

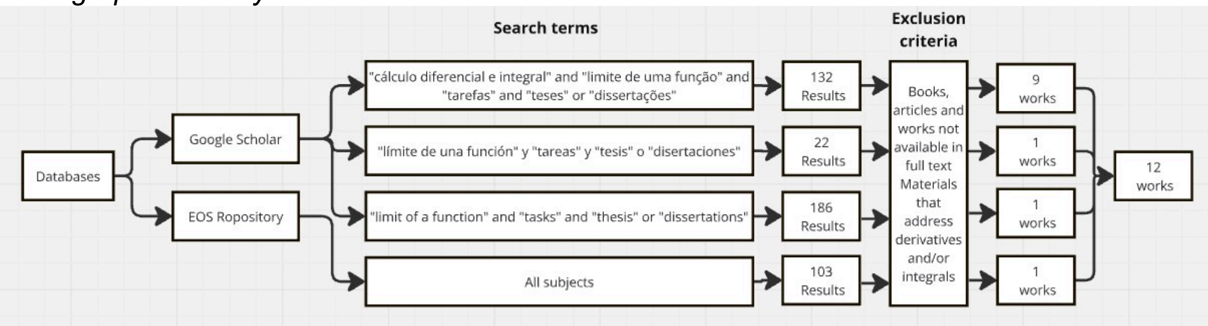
We chose to use the Google Scholar database, as it is an online academic research platform that provides access to studies, and the Repository of the Onto-semiotic Approach to Knowledge and Instruction (EOS)[1], which is the main theoretical foundation for this work. The EOS Repository compiles various works that articulate diverse theoretical approaches and models about research involving

[1] Theoretical basis of the first author's ongoing research.

anthropological and semiotic assumptions about mathematics and its teaching in Mathematics Education investigations (EOS, 2023).

The selection criteria, descriptors, and Boolean operators used in the searches were also specified, providing transparency to the method, proceeding to a planned organization of search results, making it possible to outline strategies for data processing, providing a view of the adopted methodological process, as shown in figure 2.

Figure 2
Bibliographic Survey



Note. [Image description] Process diagram with white background and black text, detailing database search results and exclusion criteria for a systematic review. On the left side, a block titled Bases lists vertically: Google Scholar; EOS Repository. From each base, arrows lead to different search terms (in Portuguese, Spanish and English): 1. Google Scholar: "cálculo diferencial e integral" and "limite de uma função" and "tarefas" and "teses" or "dissertações" (132 results). "límite de una función" y "tareas" y "tesis" o "disertaciones" (22 results). 2. EOS Repository: "limit of a function" and "tasks" and "thesis" or "dissertations" (186 results). "All subjects" (103 results). To the right of results, an Exclusion Criteria block eliminates: Books, articles and works not available in full. Materials addressing derivatives and/or integrals. Numbers of remaining works after exclusion shown next to each result set: 9 works (first search). 1 work (second search). 1 work (third search). 1 work (fourth search). At diagram's end, a centralized block shows total: 12 works. [End of description].

The selection in the EOS Thesis Repository covered all subjects within Differential Integral Calculus, as the database allows different mathematical terms to be included in their specificities. The research began with 103 theses, with two addressing limits of real functions, but only one fit the period criterion (2013 to 2023) as one work was dated 2008.

In the first Google Scholar search, we used the terms and Boolean operators "differential and integral calculus" and "limit of a function" and "tasks" and "theses" or "dissertations", yielding 132 results. However, most of these works were articles or books. After applying inclusion and exclusion criteria, we identified 9 specific works on the limit of a real function. It's worth noting that including the terms "thesis" and "dissertation" aimed to filter, as most initial Google Scholar results showed articles and books.

In Google Scholar, the second selection used the terms and Boolean operators "límite de una función" y "tareas" y "tesis" o "disertaciones", obtaining 22 results. However, only one work met the inclusion and exclusion criteria.

The third Google Scholar selection used terms and Boolean operators "limit of a function" and "tasks" and "thesis" or "dissertations", reaching 186 results, with only one work meeting inclusion and exclusion criteria.

Following these procedures, we reached a set of 8 theses and 4 dissertations, which were the focus of our dedication during the analysis phase. The list of these works is presented in Table 1 and organized by the order they were identified in the databases.

Table 1

List of Works selected for the study

Code le	Title	Author	Year	Type	Institution	Theoretical Assumption
01	Design of tasks on the partial meanings of the notion of limit in functions of a variable	Daniela Andrea Araya Bastias	2022	T	University of Los Lagos	Didactic Suitability that proposes the Ontosemiotic Approach (EOS) as well as task design criteria.
02	Active learning methodologies and their contribution to teaching Differential and Integral Calculus	Liviam Santana Sources	2021	T	University of Brasilia - UnB	phenomenological method, Didactic Engineering
03	A look at the concept of limit: constitution, presentation and perception of teachers and students about their teaching and learning.	Maria Bethânia Sardeiro dos Santos	2013	T	Pontifical Catholic University of São Paulo	Anthropological Theory of Didactics, Records of Semiotic Representation, Bakhtin's theory
01	An exploratory study on the conceptual image of university students regarding the concept of function limit	Maria Alice de Vasconcelos Feio Messiah	2013	D	UFPA	Teoria de Tall e Vinner (1981) e Vinner (1991)
04	The notion of limits: a study of the didactic organization of a digital training path	Osnildo Andrade Oak	2022	T	Federal University of Bahia State University of Feira de Santana	Anthropological Theory of Didactics, Didactic Engineering
05	Semiotic Representation Registers in	Aécio Alves Andrade	2021	T	Cruzeiro do Sul University	Theory of Semiotic Representation Registers, by

	Learning Real Function Limits					Raymond Duval, Didactic Engineering
06	Teacher documentation for limit remote teaching in exact sciences courses	Danilo dos Santos Christo	2022	T	Pontifical Catholic University of São Paulo – SP	Theory of Conceptual Fields (TCC) and the theory of the Documentary Approach to Didactics (ADD)
07	Teaching and Learning limits at the Secondary Level in Lebanon	Najwa Riad Thabet	2015	T	Lebanese American University	Piaget's cognitive theories of learning; APOS Theory
02	The construction of the concept of limit through problem solving	Matheus Marques de Araujo	2020	D	State University of Paraíba	Romberg-Onuchic Methodological Model
03	Understanding of the concept of limit by students of exact science courses	Ronaldo Dias Ferreira	2021	D	Pontifical Catholic University of São Paulo	Theory of Registers of Semiotic Representation (TRSS) and Theory of Advanced Mathematical Thinking (PMA), Didactic Engineering
08	Three Studies on Limit Discourse: a communicational approach	José Alves de Oliveira Neto	2021	T	Federal University of Bahia State University of Feira de Santana	Sfard's Cognitive Theory (2008)
04	Teaching Function Limits by Activities	Weber da Silva The type	2017	D	State University of Pará	Didactic Engineering

From the analyses, we concluded that most research was defended in Brazil (D01, D02, D03, D04, T02, T03, T04, T05, T06), with three universities presenting more than one completed work - Pontifical Catholic University of São Paulo - PUC-SP (D03, T03, T06), Federal University of Pará - UFPA (D01, D04), and a partnership between Federal University of Bahia-UFBA and State University of Feira de Santana- UEFS (T04 and T08). Additionally, research T01 was defended in Chile and T07 in Lebanon. The main theoretical assumptions addressed in the research include Semiotic Representation Registry Theory (D03, T03, T05), Anthropological Theory of Didactics (T03, T04), and Didactic Engineering (D03, D04, T04, T05). We also highlight the OSA-Ontosemiotic Approach T01 as the basis for the first author's ongoing doctoral research.

From this path that guided the review protocol, it was possible to have a view of the methodological rigor in selecting and analyzing works related to the theme in question, besides understanding all approaches adopted in using tasks for teaching Calculus. This contextualization allows a holistic view of the conducted research, highlighting methodological nuances and theoretical approaches employed. Understanding the path taken was fundamental to situate the contributions in the academic context and identify emerging gaps or trends.

The review was also examined in light of Content Analysis (Bardin, 2016). We conducted pre-evaluation of selected works, based on SLR stages for elaborating the analysis corpus, floating reading of texts, and preparation of material to be interpreted. Then, with all materials separated and organized according to table 2, we performed material exploration, establishing a priori analysis categories.

According to Bardin (2016), categorization plays a fundamental role in research, providing an organized structure for data analysis and interpretation. Results were distributed into two categories, aiming to understand both the "Driving Difficulties in Studying the Limit of a real function" and "Tasks Proposed by Authors".

To analyze the characteristics of proposed tasks and their effective compliance with Task Design Criteria (TDC), we based on Gusmão and Font (2020) and present these criteria and their indicators in table 5.

Table2

Relationship between task design criteria and indicators

Design Criteria of Tasks	Indicators
Nature	Open (infinite responses, multiple responses, no response, admits subjectivity etc.). Closed (normally a single and objective answer).
Cognitive Demand	Tasks must meet different learning objectives, leading students to solver to develop different cognitive and metacognitive skills (mastery of content knowledge, broader reflection on the solution of the problem, etc.).
Interactivity, attraction, fun, inclusion	Tasks must involve solvers in work that gives them pleasure, the desire to continue solving, and that raises their self-esteem and confidence to feel included and capable of solving.
Challenges	Tasks must allow openness in the approach, presenting several solutions or representations; provide forms of reversible, flexible, decentered thinking, as opposed to inflexible thinking centered on a single point of view.
Creativity, originality, authenticity	Tasks must encourage the use of different alternatives, an original solution, which can be applied in other contexts, and demonstrate creativity

Source: (Gusmão & Fontes, 2021).

In the article "Task Study and Design Cycle" by Gusmão and Font (2021), table 5 stands out, presenting the task design criteria proposed by these authors.

This table served as an essential tool for analyzing and understanding the structure and approach of tasks adopted in each research.

In the following sections, we detail the obtained results, addressing these two categories, which provided an analysis of the current panorama of research working with tasks for teaching the Limit of a real function.

Driving Difficulties in the Study of Limit of a Real Function

In analyzing the theses and dissertations in our corpus, while seeking to understand the difficulties and challenges in teaching limits of a real function, we identified the need to create a posteriori subcategories. We adopted the process where "the conceptual title of each category is provided only at the end of the operation" (Bardin, 2016, p.15). Thus, the subcategories were defined as follows:

a) learning obstacles - pointed out by theorists who supported the analyzed research;

b) difficulties cited by other authors in the analyzed research;

c) difficulties arising from the analyzed research.

Learning Obstacles

When exploring the difficulties and challenges in various analyzed works, the first identified category refers to learning obstacles. Given the peculiarity of these obstacles, we will opt for a brief presentation, based on Brousseau (1983), to elucidate the understanding of their nature and impact in the context of teaching limits of a real function.

In 1976, Guy Brousseau introduced the concept of epistemological obstacle in Mathematics Didactics, building on the notion of obstacles proposed by Bachelard in 1938. As we began the review, confusion arose between the terms "difficulties," "challenges," and "obstacles" which was clarified upon realizing that, according to Brousseau (1983), an obstacle is not simply a lack of knowledge, but rather of specific knowledge. This knowledge generates appropriate responses in a determined context; however, outside this context, it produces incorrect answers. Additionally, this knowledge is resistant to contradictions and to the assimilation of more refined knowledge. Brousseau (1983) emphasizes the importance of investigating epistemological obstacles related to mathematical concepts, as well as didactic methods to help students overcome these barriers. Since then, various researchers have conducted studies addressing these obstacles.

According to Brousseau (1976), obstacles in the learning process can be categorized into three distinct types: of ontogenetic origin[2], of epistemological origin and of didactic origin[3]. In the scope of this review, we will focus on obstacles of epistemological origin, as this was predominant in the analyses of the examined research.

Epistemological obstacles refer to conceptual conflicts linked to the historical trajectory of Science. They can arise from a lack of in-depth understanding of the content or even from ignorance of its development throughout History, influencing teaching and learning processes. Sierpiriska (1985) emphasizes that these obstacles involve conflicts and problems in the mathematical community during certain periods, directly reflecting on these processes. Furthermore, she emphasizes that such conflicts are essential for the development and understanding of a concept, advocating for historical studies to be conducted in parallel with experimental studies. Table 3 presents the most cited epistemological obstacles in the analyzed research.

Tabela3
Relationship between epistemological obstacles and the research studied

Epistemological Obstacles	Authors Cited	Code.
• "The error of linking geometry to numbers" • “The notion of the infinitely large and the infinitely small” • "The metaphysical aspect of the notion of limit" • "Is the limit reached or not?"	(Corn, 1983)	T01 T03 T04 D04 T05 T06 D03 T08
• Students do not understand the limit as a tool to understand the behavior of functions • The formal notion of limit causes problems for the student, as it considers two distinct processes, one on the variable and the other on the images of the function.	(Artigue, 1995)	T01 T06
• Horror to infinity • Obstacles related to the notion of function • Geometric obstacles • Logical obstacles • Symbol obstacle	(Sierpinska, 1985)	T01 D01 T04 T06 T08
• Obstacles caused by the use of quantifiers such as: "all", "some" among others. • The use of expressions such as "approaches", "comes close", "tends to" lead to the idea that f(x) ≠ L	(Tall & Winner, 1981)	D01 D03

The epistemological obstacle "The error of linking geometry to numbers" (Cornu, 1991, p.159) is addressed in several works, including T01, T03, T04, D04, T05, T06, D03, and T08. This obstacle refers to Greek studies conducted between 400 and 250 BC, focused on solving geometric problems using the method of

[2] Obstacles of ontogenetic origin include specific genetic conditions in students arising from neurophysiological limitations, among others, which are specific to each individual at a moment in their development.
[3] Obstacles of didactic origin are related to the teaching process, mainly in relation to the communication of knowledge and the choice of pedagogical approaches.

exhaustion. Cornu (1991) highlights the lack of connection between the notion of limit and numbers, resulting in an epistemological obstacle. The method of exhaustion, being fundamentally geometric, proves results without dealing with infinity or making an effective transfer to the numerical domain. Similarly, the "Geometric Obstacles" challenge (Sierspinska, 1985, p.52), explored in T01, D01, T04, T06, and T08, manifests in the projection of the circle as a limit of inscribed or circumscribed polygons. This obstacle also involves the conception of the tangent as a variable secant limit, introducing the complexity of changing the meaning of the term "difference" with the change in magnitude. Both obstacles represent substantial challenges in transitioning from geometric to numerical approaches.

The epistemological obstacle "The notion of the infinitely large and infinitely small" (Cornu, 1991, p.160) is addressed in works T01, T03, T04, D04, T05, T06, D03, and T08. This obstacle highlights the controversy in mathematics regarding the possibility of a number being as small as desired without being zero, contrasting the views of D'Alembert, Cauchy, Newton, and Euler. The complexity of these discussions is reflected in teaching the notion of limit, where the symbolic representation of epsilon can generate misconceptions about its nature. As for Sierpinska's (1985) "Horror of infinity" challenge, addressed in T01, D01, T04, T06, and T08, it focuses on the aversion to the "step to the limit" procedure. This method involves approximations and, in some cases, an "incomplete induction," where only a finite number of terms in a sequence is considered to obtain the limit. Additionally, there are epistemological obstacles of algebraic origin, related to transferring methods from finite to infinite quantity algebra, and the dynamic conception of "step to the limit" contrasts with the static approach of the formal notion of limit. Sierpinska (1985) highlights historical aspects, revealing the persistence of these obstacles until the 19th century and the evolution in formalizing the construction of real numbers by Weierstrass, Méray, Cantor, and Dedekind to correct logical misconceptions and solve the problem of the existence of limits for convergent sequences (Sierspinka, 1985).

The obstacles addressed in works T01, D01, T04, T06, and T08, related to the notion of function as discussed by Sierspinka (1985), demonstrate the lack of precision in Cauchy's and D'Alembert's definitions of limit. The author highlights the importance of introducing the general concept of function in the 19th century, emphasizing that this was crucial for establishing a clear formulation of the concept of limit, disconnecting it from geometric and physical intuitions. However, the epistemological obstacle named "The metaphysical aspect of the notion of limit"

(Cornu, 1991, p.161) cited in works T01, T03, T04, D04, T05, T06, D03, and T08, brings up philosophical and metaphysical problems associated with the notion of infinity in real numbers. Cornu (1991) highlights that this difficulty has been perceived by mathematicians from Greek times until D'Alembert, who minimized the importance of the metaphysics of infinity in Differential Calculus. The complexity in understanding the notion of infinitesimal and the infinite cardinality of real numbers, as pointed out by Cornu (1991), becomes an epistemological obstacle for calculus students, hindering the understanding of the concept of limit, especially due to the impossibility of calculating limits using directly familiar methods of algebra and arithmetic.

The epistemological obstacle discussed by Cornu (1991, p. 161), which refers to students' confusion about "whether the limit was reached or not," is addressed in T01, T03, T04, D04, T05, T06, D03, and T08. This challenge is aggravated by the historical divergence in the conception of limit, highlighted by Cornu, where D'Alembert emphasized the unreachability of the limit, while Cauchy argued that the limit could be reached. This ambiguity in the perception of limit is associated with the epistemological obstacle proposed by Cornu (1981) and reinforced by Tall & Vinner (1981) when they state that the use of expressions like "approaches," "gets close to," "tends to" leads to the idea of $f(x) \neq L$.

Artigue (1995) adds perspective by mentioning that students face challenges in understanding limit as a tool for comprehending function behavior, present in T01 and T06. The author agrees with several epistemological obstacles pointed out by Cornu (1991), noting that students view the limit as an unreachable barrier and interpret the convergence process as strictly monotonic. Additionally, she highlights the misconception that the procedure for calculating limits follows an algebraic process similar to other topics, without recognizing the particularities of the real number set. Artigue (1995) emphasizes the importance of understanding the limit process as distinct from common algebraic operations, thus avoiding misconceptions related to continuity[4] (Artigue, 1995, p.112). Both authors converge in addressing obstacles related to the nature of limit and its understanding by students.

The "logical obstacles" addressed in works T01, D01, T04, T06, and T08 reveal fundamental challenges in understanding the concept of limit. Sierspinka (1985) emphasizes the need to consider quantifiers for complete comprehension,

[4] When Artigue (1995) mentions "continuity", she refers to mathematical continuity, but not in the specific sense of continuous functions. Instead, it highlights how students may mistakenly treat the process of calculating limits as an ordinary algebraic operation, without considering the particularities and specific rules associated with limits.

highlighting that natural and intuitive language often lacks formality and symbols, resulting in an imprecise definition of limit. Moreover, the lack of clarity in the dependency between the neighborhood of the point where the limit is calculated and that of the point that is the limit, as observed by Cauchy, contributes to this complexity. In parallel, the author emphasizes the importance of quantifier order in the limit definition, associating it with studying the limit through the function graph. However, the "symbol obstacle" presented by Sierpinska (1985) highlights the ambiguity and loss of meaning resulting from using the "" notation. Although this notation facilitated algebraic operations, its association with algebra can confuse students, revealing a historical conflict in the search for a universal algorithm to solve infinite equations in the context of Differential and Integral Calculus.

Cornu (1991) emphasizes that analyzing these epistemological obstacles offers an opportunity to improve teaching and learning processes. By recognizing these challenges, teachers can enhance their pedagogical practices, helping students overcome them and understand them as essential components of the notion of limit. In the next section, we continue discussing the Difficulties cited by authors in the analyzed research.

Difficulties Cited by Authors in the Analyzed Research

In this section, we will address the difficulties identified in both international and national studies. Table 4 presents a synthesis of these difficulties.

Table 4
Difficulties listed in international and national surveys

Students' difficulties	Authors Cited	Code.
• Solve non-routine tasks; • Understand specific characteristics of the notion of limit; • Calculate limits that contain indeterminacies; • Understand what the function limit and function are; • Establish/understand the meaning of the quantifiers involved in defining limits;	Juter (2006)	**T03** **D01**
• Solve non-routine tasks; • Understand specific characteristics of the notion of limit; • Calculate limits that contain indeterminacies; • Understand what the function limit and function are; • Establish/understand the meaning of the quantifiers involved in defining limits.	Corica; Other (2009)	**T03**
• Master algebraic manipulations; • Discern function limit and succession limit;	Cottrill et al. (1996)	**D01**
• Understanding the limit of a function at a point as a static notion that does not differ from the value of the function at the point, i.e. $f(x) = f(a);f(x) = f(a);$ • Present a superficial notion of the inequalities involved in the formal definition of function limits.	Jordan (2005)	**D01**
• Understanding the limit in a partial way: as a border; as	Zuchi (2005)	**D01**

being unattainable; as being an approximation; • Understand that the function will not always have a limit at a certain point; • Understand that the function is not always defined at a point so that it has a limit at that point.; • Understand continuity and its relationship with the existence or not of a limit.		
• Understand the relationship between the notion of infinity, abstraction, basic mathematics and practical application of limits; • Understand mathematical language; • Understand concepts of functions and inequalities.	Nair (2010)	**D01**
• Understand that the function is not always defined at a point so that it has a limit at a certain point; calculate infinite limits and limits at infinity; • Understand the conditions for the existence or not of limits and their relationship with lateral limits;	Santos (2005)	**T03**
• Understand the specific mathematical language of limits; • understand the limit beyond an algebraic procedure that results in a number.	Nunes (2001)	**T03**
• Apply Basic Mathematics concepts: comparing negative numbers, operations with fractions	Robert (1982)	**T03**
• In sequences they do not consider the variable character of n; • Make a mental representation of convergence appear;	Celestino (2008)	**T03**
• Understand sequences • Discern the meaning of the terms “being limited” and “having limits”; “the sequence is limited” and “the sequence has a limit”	Abreu and Reis (2011)	**T03**
• Understanding the rigorous definition of limit and continuity proved meaningless for students; • Associate continuity with the existence of limits.	Fernández and Mora (2019)	**D02**
• Use mathematical notation appropriately; • Properly use algebraic procedures and manipulation; • Understand the concept of limit.	Cabral e Baldino (2008)	**D04**
• Understand and apply limits due to the way it is approached, that is, through pure mathematics.	Rezende (2003)	**T04**
• Apply essential mathematical calculation ideas and problems that should be consolidated during Basic Education. • Understand the formal definition of limit	Burigato (2019)	**T04**

Works T03 and D01 addressed Juter's (2006) research, which points out the following difficulties presented by students: solving non-routine tasks; understanding specific characteristics of the notion of limit, such as deciding whether the function can reach the limit value and/or determining what the definition components represent; calculating limits, for example, $\lim_{x \to 1} \frac{x^2-1}{x-1}$; establishing/understanding the meaning of quantifiers involved in the limit definition; arising from certain confusions between function limit and function; from the fact that individuals' concept images about the notion of limit proved inconsistent with its definition.

Research T03 addresses various studies on difficulties faced by students when dealing with concepts of function limits and sequences. Corica and Otero

(2009) identify difficulties in calculating limits and manipulating algebraic expressions, while Santos (2005) highlights students' confusion in conceiving limit as a real number and excessive pursuit of algorithmic techniques. Nunes (2001) observes problems in comparing negative numbers and understanding infinite sets. Robert (1982) points out difficulties with the notion of sequence convergence and considering the variable character of n. Celestino (2008) identifies confusion about the concepts of limit and bounded sequence. Abreu and Reis (2011) find that students have difficulties with rigorous notation[5] of limits and continuity, associating these concepts mainly with graphic-geometric intuitions.

Dissertation D01 addresses various studies that identify difficulties faced by students with the concept of limit. Cottrill et al. (1996) report that many students have a static view of limit, confusing it with the function value at the point, and present vague notions of inequalities involved in the formal definition. According to Jordaan (2005), students see the limit as an unreachable boundary[6], a dynamic process, and believe that the function must be defined at the point to have a limit. Zuchi (2005) highlights difficulties in understanding limit due to its relationship with the notion of infinity, mathematical language, and the transition from intuition to formal definition. According to Nair (2010), students often confuse the roles of x and f(x), believe that the limit does not exist if the function is not defined at the point, and have difficulties with infinite limits and indeterminations, focusing only on direct substitution and simplification processes.

Thesis T04 addresses research by Rezende (2003) and Burigato (2019), highlighting epistemological difficulties in teaching Calculus. Rezende points out that the main source of difficulties in higher education is the omission of fundamental ideas and problems of Calculus in basic education, suggesting the importance of introducing these concepts early. Burigato focuses on the (ε,δ) definition, highlighting that students encounter difficulties with implicit notions in expressions due to their complexity and difference from previous education, mixing modules, inequalities, functions, and quantifiers, and reinforces the need to introduce the concept of limit in High School.

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[5] Term used by the author.

[6] Definition 1: Let $a \in R$ and $r > 0$ be a positive real number. The open ball of radius r and center a is the set $B(a; r) = \{x \in R : |x – a| < r\}$.
Definition 2: Let $X \subseteq R$. We say that $a \in X$ is an interior point of X if there exists $r > 0$ such that $B(a; r) \subseteq X$. The set of all interior points of X is called the interior of X and denoted by int(X).
Definition 3: Let $X \subseteq R$. We say that $a \in R$ is an exterior point of X if there exists $r > 0$ such that $B(a; r) \cap X = \emptyset$. The set of all exterior points of X is called the exterior of X and denoted by ext(X).
Definition 4: Let $X \subseteq R$. We say that $a \in R$ is a boundary point of X if a is neither an interior point nor an exterior point of X. The set of all boundary points of X, denoted by Front(X), is called the boundary of X.

In the next section, we highlight the difficulties arising from activities resulting from the research analyzed in this review.

Difficulties Arising from Analyzed Research

The third subcategory consists of difficulties encountered by the study authors themselves in implementing the tasks. These difficulties were synthesized in table 5.

Table 5
List of difficulties presented by students listed in the research studied

Difficulties	Code.
• Understand the limit as a tool; • Understand content and concepts related to mathematical functions. • Understand the definition of Limit; • Reproduction of concepts.	**T02**
• Arising from students' negative view of the Calculus discipline	**T05**
• Arising from gaps in basic mathematical content not consolidated during Basic Education, such as factoring; notable products; simplification	**D02**

Source: Elaborated by the authors.

In thesis T02, the author presents conceptual errors in the study of limit, where students showed difficulty in understanding content and concepts related to mathematical functions, in comprehending the definition of limit, and in reproducing concepts. The author of thesis T05 concluded that students' views about Calculus can also negatively interfere with their learning, and this was the main difficulty observed. In D05, the highlighted difficulties were observed by the author throughout the course he taught during his research period. Regarding the absence of prior knowledge of algebra, from the errors, it was possible to evidence that there are open gaps in understanding the concepts of: Factoring; Notable products; Simplification and operations with algebraic fractions; and Indeterminate expressions which were probably the factors that inflamed the difficulties presented by students.

In the next subsection, we address the second category of analysis developed for this SLR, the tasks proposed in the analyzed research.

Table 5
CDT pointed out in each article studied

Code	Nº	Nature			EC	Int	Of the	Typology				AP	Crt
		A	A/F	F				Ex	Prb	AND	SD		
	1			x	x	x	x		x			x	x
	2			x	x	x	x		x			x	x
	3			x	x	x	x		x			x	
	4			x	x	x	x		x			x	
	5			x	x	x	x		x			x	
T1	6			x	x	x	x		x			x	
	7			x	x	x	x		x			x	
	8		x		x	x	x		x			x	
	9		x		x	x	x		x			x	
	10		x		x	x	x		x			x	
	11		x		x	x	x		x			x	
T2	1		x		x	x	x	x				x	x
	2		x		x	x	x	x				x	x
	3			x	x	x			x				
	4			x	x	x			x				
T3	1	x			x	x			x			x	x
	2	x			x	x			x			x	x
	3	x			x	x			x			x	x
D1	1			x	x			x					
T4	1		x		x	x	x		x				
T5	1			x	x			x		x			
T7	1			x	x			x					
	2			x	x			x					
	3		x		x				x				
	4		x		x				x				
D2	1			x	x			x					
D3	1			x	x						x		
	2			x	x						x		
	3			x	x						x		
	4			x	x						x		
	5			x	x						x		
	6			x	x						x		
D4	1			x	x						x		

Caption: A = open; **A/F** = open and closed, **F** = closed; **EC** = Cognitive Demand; **Int** = Interactivity; **Of the** = Challenges; **AP** = Opening of Thought; **Crt** = Creativity; **Ex**= Exercises**; Prb**=Problems**; AND**= video lesson**; SD**= Didactic Sequence.

In research T01, Araya (2022) presented a chapter with six Epistemic Configurations (EC): Limit as approximation in Greek mathematics; Limit in the conception of Indivisibles; Newton's Intuitive Notion of Limit; Leibniz's Idea of Infinitesimals; Pre-formal Limit Conceptions; Weierstrass's notion of limit. In another chapter, the author presented the CDT. The detailed proposal was validated by both researchers (doctoral student and advisor professor) and by the teacher, who applied the tasks to a group of 11 students belonging to a mathematics teacher training program at a Higher Education Institution. The tasks were implemented at the beginning of the Differential Calculus course.

The tasks outlined by study T01 are predominantly closed in nature, with only a few open-ended exceptions, and were worked on in groups. They demand good cognitive requirements, foster interactivity, and are challenging, adopting the typology of problems and incorporating different forms of representation, such as graphs, geometric figures, tables, and GeoGebra software as a didactic resource. Although the tasks were designed with varied forms of representation and clear objectives, most students found difficulties in answering them, with some tasks receiving only partial responses from one group. Notably, an intuitive task recorded satisfactory performance from students. It's important to note that the researcher of thesis T01 offered solutions for all proposed tasks, and after implementation, specialists analyzed the tasks, and they were (re)designed. In Figure 3, we present a closed-nature task present in the research.

Figura 3

Tarefa de natureza fechada contida na pesquisa

Tarea N°1: Triángulo de Sierspinka

Considere un triángulo equilátero, trace sus medianas y elimine el triángulo del centro, repite el proceso anterior con los tres triángulos restantes, y después con los nueve triángulos restantes, tal como se muestra en la siguiente figura.

Figura 4.1: Triángulo de Sierspinka (creación propia)

1. Si el triángulo tiene lado a, determine las áreas de la figura obtenida en la iteración 1,2,3 y 4.
2. Determine el área de la figura obtenida en la iteración n.
3. Según los datos obtenidos en (1) y (2) completa la siguiente tabla:

Tabla 4.1: Tabla de Áreas

$N°$ de iteración	Cantidad de triángulos	Área de la figura
1		
2		
3		
4		
n		

Source: (Bastias, 2022).

Note. [Image description] Image of a task written in Spanish, with white background. The task presents the following instruction: "Consider an equilateral triangle, draw its medians and create the center triangle. Repeat the previous process with the 3 remaining triangles and then with the 9 remaining triangles, as shown in the following figure." Below, there are three sequential figures: 1. An equilateral triangle with blue edges and white background. 2. The same triangle with a smaller blue triangle in the center. 3. The triangle with three more blue triangles added around the central triangle. Right after the figures, three questions are presented: 1. If the triangle has side a, determine the areas of the figure obtained in iterations 1, 2, 3, and 4. 2. Determine the area of the figure obtained in iteration n. 3. Use the data obtained in 1 and 2 to complete the following table: The table contains three columns: "Number of iterations", "Number of triangles", and "Area of the figure". [End of description].

Figure 3 illustrates a closed task, as the area of the figure in each iteration is requested and only one solution is obtained. We highlight the search for generalization by requiring the area in the nth iteration, promoting a more comprehensive approach to the process.

In thesis T02, in Fontes' (2021) study, before starting classes with active methodologies in the investigated class, students performed two tasks aimed at familiarizing themselves with GeoGebra and working on changing from algebraic to geometric register and vice versa. The activities were conducted in the computer laboratory, in pairs, due to the limited number of available computers.

In thesis T02, 4 tasks were worked on: the first two deal with functions, both having open and closed nature, good cognitive demands, promoting interactivity, being challenging, presenting exercise typology, and using GeoGebra as a didactic resource. The last two tasks addressed limit, closed problem-type tasks. Figure 4 shows one of the tasks from T02 of closed nature, problem type.

Figura 4

Tarefa de natureza fechada do tipo problema

Eduarda montou uma equipe para participar do Torneiro de Robótica da Semana da Física na UEG. Ao realizar os testes, a equipe constatou que a velocidade média do carrinho foi de 2,5 m/s. Com os conhecimentos físicos e matemáticos, a equipe conseguiu obter uma fórmula para o espaço percorrido pelo carrinho: $s(t) = \left(\frac{t}{3}\right)^3$, sendo s o espaço percorrido e t o tempo.

1. Considerando essas informações:

a) represente o espaço percorrido pelo carrinho por tabela e graficamente.

b) qual será a velocidade do carrinho no instante 5 segundos após largada?

2. A equipe também obteve a fórmula para a velocidade (v) do carrinho em função do tempo (t), que é $v(t) = \frac{t^2}{9}$,

a) represente graficamente a função.

b) essa fórmula obtida para a velocidade está correta?

Source: (Fontes, 2021).

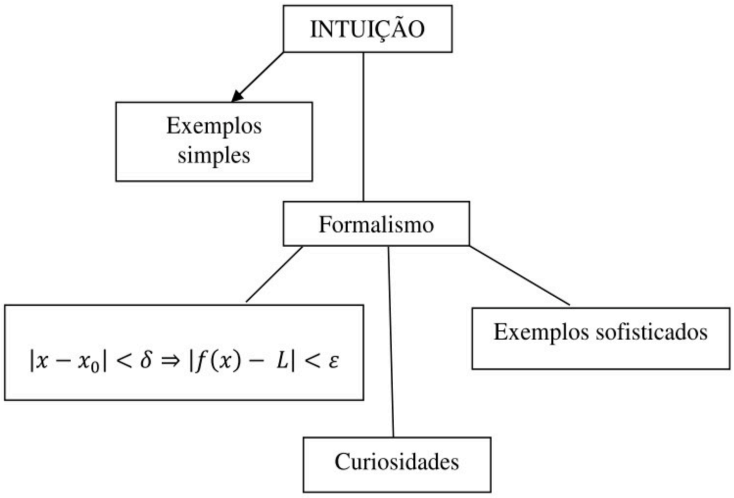
Note. [Image description] Image of a task written in Portuguese, with white background and centered text. The statement presents the following problem: Eduarda assembled a team to participate in the Robotics Tournament during Physics Week at UEG. When conducting tests, the team found that the average speed of the cart was 2.5 meters per second. With physical and mathematical knowledge, the team managed to obtain a formula for the distance traveled by the cart: $s(t)=(t/3)^3$, where s represents the distance traveled and t the time. The task is divided into items: 1. Consider this information. a. Represent the distance traveled by the cart in table and graphical form. b. What will be the speed of the cart at the instant 5 seconds after the start? 2. The team also obtained the formula for the speed (v) of the cart as a function of time (t), which is $v(t)=t^2/2$. a. Graph the function. b. Is this formula obtained for speed correct? The image includes simple elements, with text predominantly in black, and some mathematical details highlighted in italics and with appropriate mathematical notation. [End of description].

It is a simple, closed task of the problem type, where the limit appears applied to the context of studying derivatives.

In Thesis T03, researcher Santos (2013) conducted a study in three stages. Initially, she administered questionnaires to Differential and Integral Calculus professors at the Federal University of Goiás, seeking to build an overview of how these professors approach Calculus during the teaching process. Then, she administered questionnaires to Mathematics Education students to evaluate their understanding of the limit concept. In the third stage, after analyzing the data, she developed a free activity for the education professors, aiming to confirm previous results and provide space for a more open and dialogued approach. This dialogue, during the activity response, revealed additional elements that enriched the professors' understanding of the limit concept and other relevant aspects of the classroom.

The tasks proposed in T03 are open-ended, have high cognitive demand, promote interactivity, are of the problem type, promote open-mindedness and creativity. Figure 5 shows a professor's response to the task of constructing a concept map about the topic of limit.

Figure 5
Concept map constructed by participant



Source: (Santos, 2013).
Note. [Image description] Image with a white background showing a concept map constructed by a participant. The map presents the following elements: the word "Intuition" is centered and connected to two concepts, "Simple examples" and "Formalism". "Formalism", in turn, is linked to three other concepts: "Definition in terms of delta and epsilon", "Sophisticated examples" and "Curiosity". All words are organized in a diagram format with lines connecting the related concepts. [End of description].

We observe that this is an open task, allowing teachers to construct their conceptual maps about limits in various ways. This flexible approach stimulates creativity and diversity in the teaching process.

In the first stage of research D01, knowledge about the concept of function limits was investigated, where the research subjects were 25 university students from Mathematics Education programs at two public universities located in Pará, a Brazilian state. These students had already completed Calculus I. Data collection occurred through questionnaires, and participants took approximately 60 minutes to complete them. It was a closed task, of the exercise type.

The analysis of results was conducted by classifying responses into different categories, aiming to understand the conceptual images evoked by students when dealing with questions related to function limits. These categories were then used to initiate a discussion, supported by the theoretical frameworks presented in the second chapter of the dissertation. It is noteworthy that some questions presented a greater variety of classes due to the diversity of conceptual images evoked by participants.

In the task present in T04, the author simulated a fictional class, with a fictional teacher asking a question about the definition of limit, in the first part, with 19 items (statements) from 19 also fictional students, simulating a response for each student according to the teacher's question. The responses were handwritten in pen, scanned, and projected (multimedia projector – Datashow) to students in a Calculus

II class, so they could judge the truthfulness of the responses, which produced the analyzed data. This scenario was constructed so that students would feel in the position of evaluators of the responses, rather than being evaluated, at the time of experimentation. This is an open and closed task, with high cognitive demand, stimulates iteration, is challenging, promotes open-mindedness and creativity.

In T05, data were produced from the application of 12 questions to research subjects across 6 classes, in which most were closed questions of the exercise type and few were of an open nature, of the problem type. The classes were recorded and analyzed.

In Thesis T07, two formative Tests were conducted, developed by the class teacher as formative assessment instruments, during the first year of study, and constructed to evaluate students' ability to solve problems related to the concept of limits. Both tests were analyzed with the aim of identifying types of student errors, which reflect specific difficulties and obstacles that students may have encountered during the sessions. The tests were also analyzed with the objective of evidencing the evolution of students' conceptual understanding between the first test and the second. The first test was applied immediately after completing instruction on the concept of limit as an object of instruction, while the second was applied several sessions after the introduction of two other related concepts (derivatives and differentials), where the concept of limit is used as a mathematical tool. These tests were closed tasks of the exercise type.

Subsequently, two more tasks were carried out, a Post-test and Questionnaire, applied in the second year of the study in the General Sciences and Life Sciences sections, to analyze students' retention, conceptual understanding, and ability to apply the concept of "function limit," as well as students' perception of the limit concept and their difficulties, almost a year after instruction. The questionnaire includes three subjective questions, which aimed to investigate the meaning that students attribute to the limit concept, as well as the types of difficulties that students may have faced when learning the concept. The post-test included different types of mathematical questions and test items involving limits. It includes: "True or False", multiple choice questions and problem-solving questions. Students were asked, in all test items, to show their work and explain their choices. The questions were developed and categorized based on Cornu's (1991) "cognitive obstacles". These tasks are of an open and closed nature and of the problem type.

In D02, first a course was given by the researcher teacher and students were questioned about the content. A proposal for teaching and learning Limits was presented, consisting of 22 problems, 11 being generators and the others, complementary. It is worth noting that they also addressed in this proposal the concept of Continuity and Derivative, since it is an extension of the concept of Limit. The task is of a closed nature and of the exercise type.

The activities proposed in D03 were developed in six meetings (all online), with an estimated time of 120 minutes each, through the Google Meet Platform, due to the new coronavirus (COVID-19). Used value tables and GeoGebra. The tasks are mostly closed in nature, few open and closed, didactic sequence type.

The didactic sequence seen in D04 contains activities, which were divided into 9 activities about function limits, in these activities, and based on the difficulties observed in the a priori analysis phase. The tasks are mostly closed of the exercise type.

Discussion of Results

When revisiting the main results regarding challenges and difficulties related to the topic of limits of real functions of real variables, we observe common points in the three subcategories that we outlined. The difficulties surrounding the formal definition of limit are practically unanimous, including the use of terms, symbols, misconceptions/partial perceptions by students, and issues related to prior knowledge.

Cottrill *et al.* (1996) highlighted students' superficial perception regarding inequalities involved in this definition. Abreu and Reis (2011) observed students' difficulties with both the rigorous definition[7] of limit and continuity. Burigato (2019) addressed challenges related to epsilon and delta quantifiers in the definition of limits. These aspects converge with the logical and symbolic obstacles pointed out by Sierpiriska (1985), discussed in this article. If properly addressed by the teacher, involving students in explaining the use of these terms and symbols, presenting tasks that go beyond the formal proof of the limit, such as manipulating graphs to verify if the quantifiers in the definition are satisfied, students can better understand the definition.

What stands out in the research is students' partial view regarding the concept of limit. Different perspectives were identified, such as the view brought by Jordaan (2005), that they see the limit as a boundary, unreachable or a simple approximation, and the approach brought by Fontes (2021), that they consider it a tool. The author further emphasizes that previously the limit was seen as preparation for

[7] Term used by the author.

Mathematical Analysis, erroneously perceived as the fundamental discipline, which resulted in a limited view of Calculus as a set of procedures and techniques for other disciplines. With appropriate tasks, we believe that teachers can address these partial views, showing that the concept of limit is multifaceted[8] and goes beyond a boundary, something unreachable, an approximation, or a simple tool. This limited understanding represents only one facet of a vast and enriching topic.

The results revealed various distortions and misconceptions by students regarding the concept of limit. Some of these confusions include mixing up function limits with the function itself, the confusion between function limits and sequence limits, identified by Juter (2006). Some students incorrectly perceive the limit of a function at a point as a static notion, without differentiating it from the function's value at that point, as observed by Corica & Otero (2009). Other confusions involve the idea that a function must be defined at a point to have a limit at that point, as pointed out by Jordaan (2005) and Nair (2009). Additionally, there are misconceptions about continuity, such as the belief that a function is continuous only if its lateral limits are equal, as noticed by Nair (2009). We believe that methodological approaches, such as the task design proposed by Gusmão (2021), can be effective in reducing these confusions, especially if directed at teachers, so they can design tasks according to their classes' needs.

We also have results that point to specific difficulties in topics related to limits, such as calculating limits containing indeterminate forms, infinite limits, and limits at infinity, as well as difficulties arising from inadequate formation regarding contents and concepts that should have been consolidated during Basic Education, including algebraic manipulation, functions, inequalities, factoring, notable products, simplification and operations with algebraic fractions, and indeterminate expressions. In such cases, we believe in the importance of exercise-type tasks, providing students with practice for content retention. Gusmão (2021) highlights variety in tasks as crucial, indicating that exercises are not a problem as long as there is diversity in the proposals. Resende (2003) suggests that the essential ideas and problems of Calculus should be introduced in basic mathematics education, a practice adopted in other countries. However, we emphasize that the specific approach to working with the content is fundamental to overcome these difficulties.

The results of the tasks, as evidenced in thesis T01, are promising, presenting well-elaborated tasks. However, the author claims to consider that the results did not achieve the central objective of the research. We believe this perception may be

[8] Pode ser visto por várias perspectivas.

related to the research's target audience and the specific Tasks and high degree of complexity applied. The work was developed with initial formation classes, and in these classes, it is common for students to face various difficulties when activities are complex and specific, requiring greater theoretical and practical background on the topic. And the tasks, although thorough in addressing the topic of limits in various ways, including geometry, infinitesimals, sequences, and function limits, may have caused resistance and difficulties. We believe the results could bring different data and observations if the same tasks had been directed to another audience, such as applied in continuing education courses for teachers who have already encountered such knowledge, or complementary formation courses in the same context, or specifically for teachers who teach Calculus.

Thesis T03 also deserves highlight, as its research participants were professors who teach Calculus courses. The tasks were open-ended and aimed to understand these educators' practices, including the use of the conceptual map presented in the results. Although it is important to apply work with students in these professors' specific classes, we can also infer that such an approach can indicate positive points of knowledge construction when applied with the educators themselves, since by including this audience we also expand the reach of positive results to students. Teachers who don't know or don't allow themselves to work on the topic through diversified activities with focus only on "calculate," bring as a consequence students who also fail to understand the topic in its specificity, and don't know how to apply it in their context of action.

The other works presented diversified tasks, with some incorporating resources such as GeoGebra, but most followed the traditional format of exercise or problem. The results indicate that there is room to improve this scenario through research that adopts task design associated with Didactic Suitability[9], as exemplified in the thesis approach (T01). Incorporating this methodology can provide a more promising diversification in teaching strategies, promoting a deeper and more engaging understanding of the concept of limits.

Final Considerations

From this review, it was possible to analyze and critically synthesize the existing body of knowledge about limit of a real variable, from theses and dissertations about tasks for teaching limit of a real function, in the considered period

[9] Ver Godino (2008).

(2013-2023). We managed to understand the main difficulties related to limits of a real function, categorizing them into subcategories: learning obstacles, challenges found in works cited by the analyzed theses, and difficulties identified in the examined research.

Regarding the analysis of proposed tasks using Task Design indicators, we highlighted common points in the three outlined subcategories. Among these, we emphasize the difficulty in understanding the formal definition of limit, encompassing issues such as the use of terms and symbols, and misconceptions or partial perceptions by students, as well as challenges related to prior knowledge. These elements reflect obstacles that deserve attention in the teaching process of this content.

As for the tasks, we identified a variety, some more comprehensive and diverse than others. However, it's worth highlighting the tasks presented in thesis T01, which adopts task design as methodology and the Onto-semiotic Approach as theory. These tasks stand out for their depth, exploring the concept of limits throughout history, from ancient Greece to contemporary approaches, which provided a comprehensive and contextualized understanding of the topic, enriching the learning experience.

In general, we notice that researchers have made efforts to produce different types of tasks. Most tasks produced in the theses and dissertations seen in this SLR are of a closed nature, with open and closed tasks in smaller quantities and only three produced open tasks. All tasks present good cognitive demand, most promote interaction, half are challenging, the typology varies between exercise and problems type, but there is one that is a didactic sequence and one video lesson. Some promote open-mindedness and few stimulate creativity.

This study presented limitations that deserve consideration. First, the research focused mainly on published academic works, of the thesis and dissertation type, which may result in a partial view of the field, as other journals were not included. The delimitation of the review to the period before the cutoff date may also influence the representativeness of the findings, considering the dynamism of research. Moreover, the complex nature of teaching limits of real functions of a real variable implies multiple factors. These limitations highlight the need for future research that incorporates more diverse approaches.

We still intend to conduct a study focused on the curriculum of the Differential and Integral Calculus Course, seeking Pedagogical Political Projects of courses that offer this discipline and training on the subject of limits working with tasks within study cycles and task design, thus demonstrating that this research, in its initial character, supports others that will still be developed.

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