



Study on ESMATE in light of Depth Hermeneutics

Estudo sobre o ESMATE à luz da Hermenêutica de Profundidade

Pedro Dell’Agnolo Busarello¹

<https://orcid.org/0009-0003-9022-9182> 

Emerson Rolkouski¹

<https://orcid.org/0000-0001-7961-4715> 

1. Universidade Federal do Paraná, Curitiba, Brasil. E-mail: pedrobusarello@ufpr.br; rolkouski@ufpr.br

Abstract: This article presents an analysis of the ESMATE Program, a public policy implemented in El Salvador with the aim of improving the country’s mathematics education. The research methodology employed is Depth Hermeneutics (DH), an interpretative method that seeks to uncover the hidden structures of meaning in texts and social practices through three stages: socio-historical analysis, formal or discursive analysis, and interpretation/reinterpretation. To that end, a brief socio-historical overview was conducted regarding the spatial and temporal characteristics of El Salvador, focusing on its geographical, political-administrative, and educational contexts, as well as on the implementation of ESMATE. In addition to analyzing the Program, the study describes its main components and provides an account of the teaching materials produced by ESMATE. Finally, the research offers a synthesis of our understandings of this public policy, highlighting its intentionalities and internal inconsistencies.

Keywords: mathematics education, public policy, mathematics textbook, ESMATE, El Salvador.

Resumo: Este artigo apresenta uma análise do Programa ESMATE, uma política pública implementada em El Salvador com o objetivo de melhorar o ensino de matemática do país. A metodologia de pesquisa empregada é a Hermenêutica de Profundidade (HP), um método interpretativo que visa desvendar as estruturas de sentido ocultas em textos e práticas sociais, articulando três etapas: a análise sócio-histórica, a análise formal ou discursiva, e a interpretação/reinterpretação. Para tanto, foi realizado um breve panorama sócio-histórico das características espaciais e temporais de El Salvador, com foco em seus contextos geográfico, político-administrativo e educacional, bem como na implementação do ESMATE. Além de analisar o Programa, o estudo descreve seus principais componentes e fornece uma descrição dos materiais didáticos produzidos pelo ESMATE. Por fim, a pesquisa oferece uma síntese de nossas compreensões sobre essa política pública, destacando suas intencionalidades e inconsistências internas.

Palavras-chave: educação matemática, política pública, livro didático de matemática, ESMATE, El Salvador.

Introduction

The study of educational public policies from other countries helps us reflect on the globalized education present in today's world. Moreover, it allows us, in a particular way, to understand aspects of education in our own country, as we will see throughout this article. In this study, we selected, for reasons that will be explained later, the ESMATE Program (Project for Improving Mathematics Teaching in Basic and Secondary Education¹), implemented in El Salvador.

As justification for the implementation of this project, large-scale assessment data were used. TIMSS² is one such source of data; in its most recent participation, in 2007, El Salvador ranked 32nd out of the 36 participating countries. Additionally, examinations such as PAES³ have also shown low levels of mathematical skills in most of the years in which the test was administered. These data indicate a pattern of lower performance in mathematics when compared to other countries.

In light of this situation, the Ministry of Education of El Salvador (MINED), in collaboration with the Japan International Cooperation Agency (JICA), began developing a new policy in 2015. Based on changes to the curriculum at the time and the development of new textbooks, this new policy was named the “ESMATE Program” and has as its main objective achieving better performance in mathematics through the Japanese educational experience, given that Japan's performance in large-scale standardized assessments has a record of excellence, ranking among the top ten performers according to the IEA (2004, 2008, 2012 & 2016).

According to JICA official data⁴, one of the actions aimed at improving the quality of education is the adjustment of the curriculum in order to strengthen meaningful learning. In this approach, students are expected to tackle problems based on their own experience and thus enhance their learning. Accordingly, teachers must change their way of teaching, becoming facilitators — individuals who

¹ All translations presented in this article were carried out by the authors.

² Trends in International Mathematics and Science Study.

³ Learning and Aptitude Test for High School Graduates of El Salvador. Interested readers may obtain information about the types of Mathematics problems included in this test in Candray (2022).

⁴ Available at: <https://www.jica.go.jp/Resource/project/english/elsalvador/004/outline/index.html>.

guide students in their pursuit of knowledge rather than being viewed as holders of all knowledge who simply transmit it to learners.

After analyzing El Salvador's mathematics curriculum to determine a better sequence of content and avoid repetitions throughout the eleven years of compulsory schooling, the development of ESMATE began. This project, as we will examine in greater depth later, consisted of the creation of teaching materials, the establishment of active learning, and the provision of teacher support.

MINED planned teacher training sessions and seven teacher reflection meetings (to date), with the aim of familiarizing educators with ESMATE and enabling them to share their experiences and results during each quarterly period. Additionally, on-site visits are conducted to verify the use of the materials and the implementation of the strategies defined by ESMATE.

The cooperation between countries with markedly distinct educational characteristics, the development of standardized textbooks along with curricular revision, and the monitoring visits motivated us to take ESMATE as our object of study. This investigation is carried out through Depth Hermeneutics⁵ (DH), which will be presented in the next section.

Depth Hermeneutics

Depth Hermeneutics, proposed by the British sociologist John Brookshire Thompson (1995), has been used in the analysis of textbooks, particularly in Mathematics Education (Azevedo, 2017; Lopes, 2021; and Souza, 2023). Among the desirable features of this analytical approach is the consideration of socio-historical aspects, which enables a contextualized understanding. In this study, we focus on the ESMATE teaching material, which, however, cannot be fully understood without considering the public policy in which it is embedded.

The object of study in Depth Hermeneutics (DH) is symbolic forms. For Thompson, a symbolic form is anything produced by human beings with some intention, and understanding it requires knowledge of the context and the spatial–

⁵ We became acquainted with DH through the Oral and Mathematics Education Research Group (GHOEM), and thus the works produced by this group became our main references. As its name suggests, GHOEM began in 2002 as a group formed by mathematics education researchers who used Oral History in their work. Later, the group — as well as its objectives — expanded, incorporating an interest in school culture and the role of mathematics education within this context. As a result, its methodological approaches broadened, and the group currently embraces seven distinct yet interconnected research lines, one of which is the analysis of textbooks.

temporal aspects in which it was produced. To better define what constitutes a symbolic form, Thompson highlights five aspects:

1. Intentional aspect: Every symbolic form carries a desire of “wanting to say” something;

2. Conventional aspect: Symbolic forms are human expressions manifested through technical means that follow conventions in an attempt to promote communication;

3. Structural aspect: Symbolic forms have internal elements that are coherently structured rather than merely juxtaposed;

4. Referential aspect: Every symbolic form has an object about which it says something;

5. Contextual aspect: As stated earlier, every symbolic form has an intentional dimension. This intention is not random or disconnected from the world; rather, it exists through relations within social contexts.

Based on the five aspects proposed by Thompson, ESMATE can be considered a symbolic form, given that it: has the intention of improving mathematics teaching in El Salvador; is manifested through its produced textbooks and the proposed curricular changes; includes studies on the proposed transformations in mathematics education in the country; possesses an object about which it speaks — namely, the teaching of mathematics in El Salvador; and, finally, does not have a random intention, since it seeks to intervene in mathematics education with the objective of improving performance on national and international standardized assessments.

Thus, we understand ESMATE as a symbolic form and, therefore, it can be analyzed through DH. We now proceed with the presentation of Depth Hermeneutics. DH proposes three levels of analysis of symbolic forms:

1. Socio-historical analysis: Its goal is to reconstruct the social and historical conditions of production, circulation, and reception of symbolic forms. According to Thompson (1995), five aspects should be examined whenever possible in this stage of the analysis:

Spatial–Temporal Situations: Refers to the spatial and temporal characteristics of the context in which symbolic forms are created and received by the target audience;

Fields of Interaction: Relates to the reconstruction of the environment in which institutions are formed and influence one another. It is the interactional space of the

work — its creators, editors, and readers — along with the power dynamics between them and their interlocutors within the cultural context in which they are situated;

Social Institutions: When considering social institutions, it is necessary to investigate the influence of schools, families, and society of the time on the production and adoption of symbolic forms;

Social Structure: Involves identifying and analyzing the stable asymmetries and differences that characterize social institutions and fields of interaction, along with the relations that maintain a set of stable rules governing the social fabric;

Technical Means of Construction and Transmission: Refers to how symbolic forms manifest in the concrete world, including the technologies used in their production, the materials employed, among other aspects.

2. **Formal analysis:** At this stage, the objective is to study the object itself — that is, to examine the structure of the symbolic form. Thompson (1995) presents several possibilities for investigation:

Semiotic analysis: Examines the internal characteristics of the structure of a work, its constituent elements, and their interconnections;

Syntactic analysis: Explores how symbolic language is structured to express its underlying intention;

Narrative analysis: Evaluates the specific narrative of a story and the development of its plot;

Argumentative analysis: Assesses the coherence of the work; in the case of a textbook, this includes the organization of themes, the structure of presentation for each topic, and its internal consistency;

Conversation analysis: Investigates linguistic interaction dynamics in concrete contexts where they occur.

In the case of the formal analysis undertaken for ESMATE, we consider semiotic, syntactic, and argumentative analyses to be the most coherent approaches.

3. **Reinterpretation:** Together with the socio-historical and formal analyses, this stage seeks to relate contexts and elements in order to construct meaning for the symbolic form.

In this section, we have outlined, in general terms, Depth Hermeneutics as an analytical tool and identified ESMATE as a symbolic form, considering the five

aspects proposed by Thompson. In the following section, we will continue with the socio-historical and formal analyses of ESMATE.

Socio-Historical Analysis

For this analysis, we use the year 2015 as a reference point, since it was during this period that ESMATE began to be implemented. The study is structured into two analyses: one concerning the geographical, political-administrative, and educational contexts of El Salvador, and the other regarding the history and implementation of ESMATE.

Contexts

Geographically⁶, El Salvador is a Central American country with a total area of 21,000 km², divided into fourteen departments and 262 municipalities⁷. Its population is approximately 6.2 million inhabitants.

With regard to the political-administrative context, the organization of the country is guided by the 1983 Constitution, promulgated during the Salvadoran Civil War, a conflict between the Army and the guerrilla group Farabundo Martí National Liberation Front (FMLN), which lasted from 1979 to 1992. With the end of the war in 1992, the Constitution was modified to facilitate the peace agreements that brought the conflict to an end.

Public administration in El Salvador is headed by the President of the Republic, who is supported by ministers responsible for specific areas such as education, health, transportation, economy, among others. These ministers have the authority to propose laws, which must be evaluated by the Legislative Assembly. In the field of education, policies are centralized within the Ministry of Education (MINED), with no delegation of responsibilities to municipalities or departments.

In the educational context, teacher training in El Salvador underwent a significant reform in 1998, as part of the changes introduced by the 1995 reform. In this process, MINED resumed its role as a regulator of teacher education,

⁶ For this part of the analysis, we used Candray's (2018) master's thesis as a source.

⁷ In 2024, the territorial organization underwent a modification, reducing the number of municipalities from 262 to 44.

establishing study plans, admission and completion requirements, as well as mandatory regulations for universities offering teacher training programs.

Teacher training programs, called “professorados”, prepare educators for Basic and Secondary Education in different specializations, such as Mathematics and Chemistry. In 2013, MINED updated these study plans, introducing a new generation of guidelines for teacher education. To enter a “professorado”, candidates must hold a high school diploma, achieve a minimum score on high school examinations, complete an entrance course, and pass a psychological test. To graduate, the student must obtain a grade of seven or higher in all course subjects and also on the exit examination administered by MINED (El Salvador, 2012a).

Teacher training programs are regulated by the laws LES/2004 and LGE/1996, and each specialization, such as the Mathematics professorado, follows specific guidelines, as established in Agreement No. 15-1046, issued by MINED in 2012. This agreement defines the objectives of the program, the profile of the Mathematics teacher, and the criteria for evaluation and graduation. According to this profile, a Mathematics teacher must be reflective, critical, and capable of transforming reality, with scientific and pedagogical mastery of the content (El Salvador, 2012a, p. 11).

The program’s curriculum is oriented toward problem-solving, the practical application of mathematics in daily life, and the connection of this subject with other areas of knowledge. The curricular structure includes 28 subjects distributed across three areas: mathematical content, psychopedagogical and didactic subjects, and teaching practica. The program also includes seminars on topics such as human rights, equality, and inclusive education (El Salvador, 2012b, pp. 10 and 13). Each subject has an average workload of 100 hours, totaling approximately 2,800 hours for the complete program.

El Salvador’s educational system, according to LGE/1996, organizes “formal education” into different levels: Early Childhood Education, Preschool Education, Basic Education, Secondary Education, and Higher Education. Curricula are determined by MINED and expressed through the Study Plans. Mathematics is primarily approached through problem-solving, and its competencies include the development of logical-mathematical reasoning, communication through mathematical language, and the application of concepts to everyday life (El Salvador, 2008, p. 9). The mathematics curriculum seeks to promote critical and logical thinking, encourage the formulation and validation of conjectures, and apply

mathematical knowledge to practical problem-solving. The Curricular Foundations establish that mathematics should prepare students to identify and solve everyday problems, develop observation skills and the ability to organize information, and apply mathematical knowledge in practical contexts (El Salvador, 1997, pp. 47–56).

History and Implementation

Initially, when ESMATE was created, El Salvador did not have a national policy for the production and distribution of textbooks that encompassed all public schools. Thus, the project focused on developing and distributing these materials.

Regarding the history of the partnership between El Salvador and JICA, the “Comprendo” Program (“I Understand”), carried out between 2006 and 2009, stands out. This program sought curricular reform by adopting a competency-based approach. In the case of mathematics, didactic texts were developed. However, when the materials were ready for distribution, a political administration change prevented their delivery at that time. When distribution finally became possible, it was noted that teaching staff had not received guidance on how to use the materials, which was identified as a weakness of the project.

The implementation of ESMATE took place in two phases: the first involved the conception and execution of the project, while the second refers to the consolidation of the initiative.

The first phase of ESMATE, conducted between 2015 and 2021, began after the signing of an agreement between El Salvador and JICA. During this period, five Japanese specialists visited the country to provide support for the curricular reform. This process included an analysis of mathematics curricula, which identified several gaps: overloaded study programs; repeated content; documents tied to programmed instructional time rather than actual classroom time; inconsistencies in content sequencing; and imbalances among content blocks (El Salvador, 2022–2023).

Based on this analysis, the development of new curricular plans began. Specialists from El Salvador and Japan worked together to build curriculum grids that would guide teaching units, defining initial problems, performance indicators, and the prerequisite knowledge required. These materials were reviewed by the Japanese specialist, who provided suggestions and recommendations. The outcome was a preliminary version of the curricula and the creation of new textbooks for grades 1 through 9, as well as exercise books.

In 2017, a pilot plan was implemented in which the material was validated in 24 schools, where classroom observations, workshops, and meetings with teachers were carried out to collect suggestions and corrections regarding the new curricula. Based on the feedback obtained, improvements were made to the curricular units and teaching materials. In the same year, MINED published the Mathematics Study Program for the Third Cycle of Basic Education⁸ and implemented the ESMATE Module in the National Teacher Training Plan, integrating the new guidelines into school pedagogical practices. The nationwide distribution plan for the materials was also devised.

The distribution of teaching materials for the 7th, 8th, and 9th grades took place in 2018, along with a process of integrating mathematics teachers in the third cycle. The curricular reform also included teacher training programs at universities, featuring workshops and updates to licensure and pedagogy courses. Between 2018 and 2019, impact evaluations⁹ were conducted using the randomized controlled trial (RCT) methodology in 250 schools, which showed improvements in student performance in the 2nd and 7th grades. Additionally, in 2019, educational materials for other grades were incorporated.

The final stage of the first phase coincided with the Covid-19 pandemic, in 2020, which required adaptations to teaching strategies. ESMATE developed educational videos, virtual classes, and websites with content for all grade levels. In the same year, the second edition of textbooks for grades 1 through 9 was released. With the resumption of in-person activities, the ESMATE team returned to conducting classroom observation visits.

In 2021, with the renewal of the agreement with JICA, the second phase of ESMATE began. This phase aimed to improve educational materials based on evidence, focusing on learning assessment and the collection of relevant information. For now, efforts are directed toward analyzing the results of the 2021 and 2022 diagnostic tests “Conociendo Mis Logros” (“Knowing My Achievements”), in partnership with the National Directorate of Educational Evaluation. A virtual questionnaire was also created for teachers and students to identify curricular progress and challenges in the teaching of mathematics.

⁸ In El Salvador, Basic Education is divided into three cycles: the first corresponds to grades 1, 2, and 3; the second to grades 4, 5, and 6; and finally, the third to grades 7, 8, and 9.

⁹ These assessments are available on the website: <https://data.mendeley.com/datasets/vmf6bfm2n/1>.

The second phase also continues the work of the first, including the development and monitoring of the use of educational materials, as well as providing support to teachers. The main actions of this phase are divided into four components: the development of a national educational research strategy to assess learning; the creation of a third edition of materials based on evaluation results; ongoing support for teachers in using these materials; and the possibility of sharing the ESMATE experience with other countries.

As for the funding of the program, resources were provided by the State of El Salvador and by JICA. The State financed the hiring of professionals who joined the project's technical team and paid for the printing and distribution of educational materials — which represent the highest associated costs. JICA financed the visits of Japanese specialists and office equipment such as computers, printers, photocopiers, software, among others (El Salvador, 2022–2023), in addition to short-term training and postgraduate courses in Japan for professionals from El Salvador's Ministry of Education (MINED).

Formal Analysis

This part of the study is composed of two stages: a presentation of the components of ESMATE, and an analysis of the teaching materials, with a stronger focus on Unit Three of the 8th-grade materials, which corresponds to the teaching of linear functions.

To select this content, we considered the following points: linear functions are often the first type of function introduced to students, forming a basis for more complex concepts such as quadratic and exponential functions. Thus, analyzing how they are addressed helps determine whether the material builds a solid foundation; such functions can be observed in everyday life in situations involving proportionality, constant growth, among others; and finally, understanding this content requires students to manipulate equations and unknowns, which is important for the development of algebraic thinking.

Apresentação dos Componentes

As mentioned earlier, ESMATE was created in 2015 as a collaboration between MINED and JICA. The program consists of three main components¹⁰: the development of teaching materials, active learning time, and support provided to teachers.

Development of Teaching Materials

With the aim of structuring the initiative, new educational materials were developed, primarily intended for students and teachers, but also for the training of future professionals. Among the materials produced, the most notable are textbooks, exercise books, and teacher guides.

To date, 11 textbooks have been developed, covering grades 1 of primary education through grade 2 of upper secondary education. These books have the following main characteristics: they adopt a problem-solving approach; each page corresponds to a 45-minute lesson; each lesson focuses on a single performance indicator; and lessons are structured in four stages (initial problem, solution, consolidation, and exercise). Furthermore, the books cover 80% of the hours specified in the study plan¹¹, with the remaining 20% reserved for assessments, reinforcement, and other activities.

The exercise books, designed for students from grades 3 to 9, support the consolidation of the content worked on in class. Each page of the exercise books corresponds to a lesson in the textbook, and at the end of each page, there is a space for the signature of a parent or guardian to ensure monitoring of the activities completed at home.

The methodological guides were created to assist teachers in using the textbooks and exercise books. Their main objectives are to guide lesson planning, offer practical methodological suggestions, and “propose specific strategies for students to achieve the performance indicators and develop mathematical competencies” (El Salvador, 2019b, p. 5).

To achieve these objectives, the guide provides an annual plan, quarterly or periodic tests, and a final assessment. In addition, each unit is introduced with a

¹⁰ For this part of the analysis, we used Madrid’s (2024) work as a source.

¹¹ Thus, for the final years of basic education — 7th, 8th, and 9th grades — of the 200 instructional hours in the school year, 160 hours are reserved for the use of the textbook.

description of how its content relates to other units and includes a detailed plan specifying the number of hours required to teach it. The guide also explains the didactic sequence of each lesson, proposes a format for using the chalkboard, presents the image of the corresponding textbook page, and provides the solutions to all the problems presented.

Support Provided to Teachers

To provide technical assistance to teachers, several activities were planned. Among them, annual pedagogical reflection sessions, conducted in 2018 and 2019, stand out, comprising three sessions focused on basic education and four on upper secondary education, as well as the monitoring of the use of educational materials through classroom visits. These visits, carried out by principals, pedagogical managers from the departmental education offices, and project technicians, follow a structured protocol to guide the observation of mathematics lessons.

According to legislation in El Salvador, only school leadership teams are authorized to conduct classroom visits within their respective institutions. For this reason, these professionals are expected to grant permission to the project's technical team, which may, on certain occasions, be accompanied by an ESMATE representative, to observe classes or perform other visits throughout the school year.

Each year, approximately 100 school visits are conducted to verify whether teachers are following the stages established by the methodological guide's strategy. Recently, the observation protocol was replaced by an application that systematizes the information collected during the visits. The application includes five sections: school, class, schedule, content mastery, and data about each teacher. It also monitors the time teachers and students dedicate to different activities, such as content explanation and individual or group problem solving. At the end of each visit, the observer prepares a report for each teacher, assessing their performance according to the established guidelines.

Active Learning Time

In managing learning, the project emphasizes the importance of the effective use of time in schools. To this end, it establishes an annual plan, a lesson structure, and guidelines for the use of materials, with the goal of ensuring that each student

actively participates in the learning process. The initiative encourages students to work autonomously, while teachers act as facilitators of this process. Family involvement is considered essential — especially in the case of young children — to support learning, particularly through the use of exercise books at home.

The strategy proposes that part of the lessons be dedicated to autonomous work in problem solving and that class time be used efficiently, avoiding “lost lessons.” In this context, the project adopts the motto “160-20-20,” which allocates 160 annual instructional hours for each student, 20 minutes per lesson for individual or group activities using the textbook, and 20 minutes dedicated to the use of exercise books at home.

Furthermore, the methodological guide developed to support teachers provides detailed guidance for lesson development, ready-made assessments, and a “chalkboard plan,” which suggests a specific way of using the chalkboard in each lesson based on the textbook.

Although the guide is described as “a flexible proposal that can be improved and adjusted as needed by teachers” (El Salvador, 2019b, p. 6), progress regarding curricular coverage and the sequence of each lesson is monitored through classroom visits and discussed during pedagogical reflection sessions.

Presentation of the Teaching Materials

We begin this presentation with the textbook¹², which contains a total of 196 pages. On its back cover, there is a message to students, from which we highlight the following paragraph: “This book contains several problems and activities through which you will be able to develop your reasoning and improve the mathematical skills that will be very useful for solving everyday situations.” (El Salvador, 2019c).

Next, the book presents an explanation of its structure, beginning with the description of the “icons” that divide each page/lesson into specific sections: initial problem, solution, conclusion, example, and the exercise section. Following this, the material explains the “complementary information,” which is divided into additional information, prior knowledge, and tips. These elements aim to facilitate the learning of the content.

¹² As mentioned earlier, we used the 8th-grade textbook from basic education.

Figure 1

Presentation of the Icons

Íconos



La letra P representa el Problema inicial. En el primer momento de cada clase, el estudiante debe pensar una solución a partir de una situación problemática la cual permite introducir el contenido que se va a desarrollar.



La letra S simboliza la Solución. En este segundo momento, el texto propone una o varias formas de resolver el problema planteado.



Con la C de Conclusión se llega a la explicación del contenido. Aquí se relacionan los momentos P y S para explicar con lenguaje matemático la finalidad del contenido.



La letra E representa un ejemplo. A veces es necesario presentar un problema adicional, que permita consolidar el contenido de la clase.



El lápiz representa la sección de problemas y ejercicios.

Source: Textbook (El Salvador, 2019c)

Note. [Image description] Textbook image. It has a rectangular format with a white background. It contains an explanation of each of the icons, which are: Initial Problem – the first stage of the lesson, in which the student must think of a solution based on a problem situation that introduces the content to be developed. Solution – the second stage, in which the text proposes one or more ways to solve the problem. Conclusion – the explanation of the content. Example – an additional problem presented to consolidate the lesson content. There is also an icon with a pencil drawing, which represents the section for problems and exercises. [End of description].

The textbook also provides an explanation of how lessons are distributed. It states that it is composed of 8 instructional units, each subdivided into lessons, and each lesson is composed of different classes. The categorization of the classes is detailed, as exemplified in the title “1.2 Applications of Direct Proportionality.” In this example, the number 1 refers to the lesson and the number 2 to the class. The unit number is indicated by a square positioned along the outer margin of the odd-numbered pages.

Each of these 8 instructional units addresses a specific topic, namely: algebraic operations; systems of first-degree equations with two unknowns; linear functions; parallelism and angles of a polygon; criteria for triangle congruence; characteristics of triangles and quadrilaterals; area and volume of geometric solids; organization and analysis of statistical data. These units use, respectively, 20, 22, 47, 13, 9, 25, 19, and 28 pages to introduce and conclude the content.

As previously stated, we use Unit 3 as the main reference for the analysis of the teaching materials. This unit is divided into three lessons. The first lesson, Linear Function, is divided into 22 classes, namely: recalling the meaning of proportionality; applications of direct proportionality; meaning of the linear function; meaning of the rate of change; rate of change; characteristics of the function $y = ax + b$; relationship between the graph of $y = ax + b$ and $y = ax$; graphical analysis of positive slope; graphical analysis of negative slope; relationship between the rate of change and the

slope of the graph of $y = ax + b$; angular and linear coefficients of the function $y = ax + b$; relationship between table, equation, and graph of the linear function; sketching the graph of the linear function given the angular and linear coefficients; relationship between equation and graph of the linear function; values of y when the values of x are delimited; expression of the function $y = ax + b$ through the interpretation of the graph; equation of the function from one point of the graph and the angular coefficient; equation of the function from two points of the graph; equation of the function from the intercepts with the axes; and finally, two classes dedicated to practice of the content learned.

The second lesson, titled Linear Function and First-Degree Equation with Two Unknowns, is divided into 8 classes: sketching the graph of a first-degree equation with two unknowns; relationship between the graph of the equation $ax + by + c = 0$ and the function $y = ax + b$; graph of the equation $ax + by + c = 0$ based on intercepts; sketching the graph of the equation $ax + by + c = 0$ when $a = 0$; sketching the graph of the equation $ax + by + c = 0$ when $b = 0$; intersection of the graphs of two equations of the form $ax + by + c = 0$; graphical solution of a system of equations of the form $ax + by + c = 0$; and practice of the content learned.

The third lesson, called Application of the Linear Function, consists of 5 classes: applications of the linear function, parts one, two, and three; and two classes dedicated to the practice of the content learned.

The unit begins with a brief historical introduction about the use of the term “function,” along with several real-world applications of this concept, and provides a short overview of what will be studied in the unit. The material then proceeds to the lessons.

Most of these lessons occupy one page, while a few use two pages. All lessons are structured around the initial problem, solution, conclusion, and exercise section, except for lessons 1.1, 3.2, and 3.3, which do not include a conclusion. Lessons 1.6, 1.14, 1.15, and 1.20 also include an example after the conclusion. The practice lessons consist solely of exercises. Below, we present Lesson 1.6 as an example.

Figure 2

Unit 3: Lesson 1.6

1.6 Razón de cambio

P

Observa los datos de la tabla:

| | | | | | | |
|-----|----|----|----|----|----|-----|
| x | 0 | 1 | 2 | 3 | 4 | ... |
| y | 20 | 18 | 16 | 14 | 12 | ... |

- Expresa y como una función lineal de x .
- Si x toma el valor de 6, ¿cuánto vale y ? Y si x toma el valor de 9, ¿cuánto vale y ?
- Calcula la razón de cambio de y respecto a x .
- Compara la razón de cambio con el valor de a en el resultado del literal a). ¿Qué concluyes?

S

a) Al observar $x = 0$, $y = 20$ y cada vez que x aumenta una unidad y disminuye 2, entonces al expresar y en función de x , se tiene $y = 20 - 2x$, lo cual es equivalente a $y = -2x + 20$.

| | | | | | | |
|-----|----|----|----|----|----|-----|
| x | 0 | 1 | 2 | 3 | 4 | ... |
| y | 20 | 18 | 16 | 14 | 12 | ... |

$\xrightarrow{+1}$ $\xrightarrow{+1}$ $\xrightarrow{+1}$ $\xrightarrow{+1}$
 $\xrightarrow{-2}$ $\xrightarrow{-2}$ $\xrightarrow{-2}$ $\xrightarrow{-2}$

b) Para determinar el valor de y , se analiza la variación de los valores que se reflejan en la tabla, tal como se muestra en la figura. Mientras x aumenta una unidad, y disminuye 2; por tanto:

$$\text{Si } x = 6, y = 20 - 2(6) = 20 - 12 = 8.$$

$$\text{Si } x = 9, y = 20 - 2(9) = 20 - 18 = 2.$$

c) Se toman los valores en dos momentos y se determina el cambio en las dos variables:
 Variación en x : $4 - 1 = 3$. Variación en y : $12 - 18 = -6$.

Utilizando la expresión **Razón de cambio** = $\frac{\text{Variación en } y}{\text{Variación en } x}$; se tiene Razón de cambio: $\frac{-6}{3} = -2$.

d) Al comparar la función $y = -2x + 20$, con la forma de la función lineal $y = ax + b$, se tiene que $a = -2$, en donde se puede concluir que la razón de cambio es igual al valor de a .

C

En la función lineal $y = ax + b$, la razón de cambio es constante y es equivalente al valor de a , es decir:

$$\text{Razón de cambio} = \frac{\text{Variación en } y}{\text{Variación en } x} = a$$

Considerando la expresión para determinar la razón de cambio se tiene:

- Variación en y** = $a \times$ (**variación en x**), es decir, que el aumento en y es proporcional al aumento en x .
- El valor de a es equivalente al aumento de y cuando x aumenta una unidad.

E

Para cada una de las siguientes funciones lineales, realiza:

- Identifica la razón de cambio.
- Determina el valor de y , cuando $x = 4$.

a) $y = 3x - 5$

b) $y = -2x + 3$

Solución.

a) Para la función $y = 3x - 5$

1. Razón de cambio: 3

2. Valor de y , cuando $x = 4$:

$$y = 3(4) - 5 = 12 - 5 = 7$$

b) Para la función $y = -2x + 3$

1. Razón de cambio: -2

2. Valor de y , cuando $x = 4$:

$$y = -2(4) + 3 = -8 + 3 = -5$$

E

Para cada una de las siguientes funciones lineales, realiza:

- Identifica la razón de cambio.
- Determina el valor de y , cuando $x = 6$.

a) $y = 2x - 7$

b) $y = -3x + 4$

c) $y = \frac{1}{2}x + 1$

50

Source: (El Salvador, 2019c)

Note. [Image description] Image of page 50 of the textbook, corresponding to Lesson 1.6 of Unit 3. It has a rectangular format with a white background. This page of the textbook addresses the topic of rate of change. In the "Initial Problem" section, there is a table with corresponding values of x and y . Based on this table, students are asked to represent y as a linear function of x , determine values of y for values of x that are not in the table, calculate the rate of change of y , and compare this rate of change with the slope obtained in the first question, as well as state their conclusions based on this comparison. A possible solution to these questions is then presented. The explanation of the lesson content follows, stating that the rate of change is equal to the variation in y over the variation in x , which is equal to the slope of the linear function. Finally, the page presents exercises for students, which provide additional linear functions and ask them to identify the rate of change and calculate the value of y for a given value of x . [End of description].

From the image above, we can gain a clearer understanding of how a lesson in the program is structured. We now begin the analysis of the exercise book.

The exercise book contains a total of 228 pages. On its back cover, there is a message to students, from which we highlight the following paragraph: "This book contains multiple problems and activities through which you will be able to develop your reasoning and improve mathematical skills that will be very useful for solving everyday situations." (El Salvador, 2019a).

Next, the exercise book presents an overview similar to that of the textbook: an explanation of the “icons” that divide each page into review, conclusion, and the exercise section; an explanation of the “complementary information”; and a description of the “lesson distribution.” On the following page, the table of contents is presented, and the exercises for Unit 1 begin immediately thereafter. It is also worth noting that at the end of each lesson there is a space for students to record how much time they used to answer the questions, and at the end of each unit there is a self-assessment sheet. As an example, we present the page corresponding to Lesson 1.6 of the exercise book.

Figure 3

Unit 3: Lesson 1.6 of the Exercise Book

1.6 Razón de cambio

R 1. Identifica las ecuaciones que corresponden a una función lineal.

a) $y = -2x - 3$ b) $y = -3x$ c) $y = 2x^2 - 3$ d) $y = -\frac{1}{x}$

2. Un empleado de una venta de celulares tiene un salario base de \$100 mensuales más una comisión de \$5 por cada celular vendido (el salario total del empleado es el salario base más la comisión).

a) Llamando x al número de celulares vendidos y y al salario del empleado, completa en la tabla los datos que hacen falta.

| | | | | | | | | | |
|---------------------------|-----|---|---|---|-----|----|----|----|-----|
| x (número de celulares) | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | ... |
| y (dólares) | 100 | | | | 140 | | | | ... |

b) ¿Cuánto recibiría de salario si vende 15 celulares? ¿Y si vende 20?

c) Determina la razón de cambio tomando los resultados del literal b.

d) Expresa y como una función lineal de x .

C En la función lineal $y = ax + b$, la razón de cambio es constante y es equivalente al valor de a ; es decir,

$$\text{Razón de cambio} = \frac{\text{Variación en } y}{\text{Variación en } x} = a$$

Considerando la expresión para determinar la razón de cambio se tiene:

- **Variación en $y = a \times$ (variación en x)**; es decir, que el aumento en y es proporcional al aumento en x .
- El valor de a es equivalente al aumento de y cuando x aumenta una unidad.

P Para cada una de las siguientes funciones lineales:

1. Identifica la razón de cambio.

2. Determina el valor de y , cuando $x = 4$.

a) $y = 2x - 3$ b) $y = -2x + 3$ c) $y = 3x - 5$

d) $y = \frac{1}{2}x + 3$ e) $y = \frac{3}{2}x + 1$ f) $y = -\frac{3}{2}x - 1$

¿Cuánto tiempo necesité para resolver los problemas?

Unidad 3

55

Source: (El Salvador, 2019a)

Note. [Image description] Image of page 55 of the exercise book, corresponding to Lesson 1.6 of Unit 3. It has a rectangular format with a white background. This page of the exercise book presents the explanation of the lesson content and several exercises: first, some functions are presented and students are asked to identify which ones are linear. Next, a contextualized exercise is provided, which can be solved using the same reasoning presented in the initial problem of the textbook. Finally, additional exercises ask students to find the rate of change and the value of y when $x = 4$ for six linear functions. [End of description].

Finally, regarding the methodological guide, it is divided into two volumes, of which we present Volume 1, which contains a total of 276 pages. On its back cover, there is a message to teachers, from which we highlight the following paragraph: “The main objective of this resource is to provide concrete and precise guidelines for the development of the lessons of this subject and to achieve the development of logical-mathematical thinking among Salvadoran students.” (El Salvador, 2019b).

Next, the table of contents and an introduction are presented. In this introduction, the objectives of the methodological guide¹³ are described, along with five essential starting points for the implementation of this material: the fundamental importance of learning mathematics; the fundamental role of the teacher and the active role of the student; lesson sequencing as an authentic learning experience; decisive harmony with school management; and student learning at home through the use of the exercise book.

At the end of the introduction, we highlight the following statement: “Following this participatory dynamic, it is considered important to regard these materials as a flexible and improvable proposal, where the teaching staff should make the adjustments, they deem necessary to support student learning.” (El Salvador, 2019b).

The guide then explains how the three main components of the program function¹⁴ and presents the structure of the textbook¹⁵. In the next section, titled “Structure of the Methodological Guide,” an annual plan is provided, indicating which units should be covered in which months, along with the content within these units, their corresponding textbook pages, and the respective pages in the methodological guide. At the end of the annual plan, it states that “to develop all the established content, the schedule shown must be followed.” (El Salvador, 2019b). This section also introduces the unit sections (unit competency, relationship and development between the previous and subsequent grades, unit study plan, essential points of each lesson) and the elements of the methodological guide pages (lesson performance indicator, lesson sequence within the unit, lesson objective, and solutions to the textbook exercises). Finally, it is stated that an example of the unit test is provided in order to:

¹³ These objectives have already been described in the section “Development of Teaching Materials.”

¹⁴ These were also described in the section “Development of Teaching Materials.”

¹⁵ Already presented in this same section.

[...] measure both the level of student understanding and the extent to which teachers have achieved the unit objective. If performance is low on some problems, teachers should think about how to improve it and, at the same time, try to ensure that this low performance does not become an obstacle to future learning. (El Salvador, 2019b, p. 14).

The next section provides guidance for the development of a Mathematics lesson based on problem-solving. To achieve this, the guide offers pedagogical recommendations for lesson development. At this point, a procedure for teachers is presented, containing eight steps, which involve the student learning process, the assistance process, and points that must be considered when providing support. Additionally, the guide presents ten important points to consider in facilitating learning. Among these, we highlight in particular: the use of the chalkboard, for which a structure is provided to guide how it should be used (this structure can be seen at the bottom of Figure 4); and guidance on how to assess and move around the classroom, in which it is stated:

Guidance should be provided as follows: if the number of students experiencing difficulty is fewer than five, provide individual guidance; otherwise, it is better to offer another type of explanation, such as: plenary explanation, group explanation, checking the correct answer together, among others. (El Salvador, 2019b, p. 21).

In the following section, the exercise book is presented along with its structure¹⁶, objectives, and instructions for its use. Finally, the last section presents the unit tests, quarterly tests, and final tests. At this point, the importance of administering the tests is emphasized, as well as the purpose and function of each type of test, and it reminds teachers that “you must make use of the tests included in this MG¹⁷, which aim to collect valuable information related to the learning reality, both acquired and not acquired.” (El Salvador, 2019b).

Lastly, we present the page corresponding to Lesson 1.6 of the methodological guide as an example.

¹⁶ Already presented in the current section.

¹⁷ Methodological Guide.

Figure 4

Unit 3: Lesson 1.6 of the Methodological Guide

Indicador de logro
1.6 Resuelve situaciones mediante el análisis de la razón y comparación con la ecuación de la función.

Secuencia
Ya definida la ecuación de la función lineal y la razón de cambio, para esta clase se compara el valor de α en la ecuación $y = \alpha x + b$ con la razón de cambio de la función para establecer la igualdad entre ambas cantidades.

Propósito
 ①, ② Modelar la situación presentada en el Problema inicial mediante una función lineal y determinar la razón de cambio entre los valores.
 ③ Establecer la igualdad entre el valor del coeficiente de la variable x en la ecuación de una función lineal y el valor de la razón de cambio de la misma.

Fecha: U3 1.6

| | | | | | |
|-----|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 |
| y | 20 | 18 | 16 | 14 | 12 |

P a) Expresa y como función lineal de x .
 b) Si $x = 6$, ¿cuál es el valor de y ? ¿Y si $x = 9$?
 c) Calcula la razón de cambio y compara este resultado con el valor de α .

S a) Si x aumenta una unidad entonces y disminuye 2; $y = -2x + 20$
 b) Si $x = 6$, $y = -2(6) + 20 = 8$
 Si $x = 9$, $y = -2(9) + 20 = 2$
 c) Razón de cambio: $\frac{12-18}{4-1} = -2$
 $\alpha = -2$, por tanto la razón de cambio y el valor de α son iguales.

E Identifica la razón de cambio y calcula el valor de y cuando $x = 4$:
 a) $y = 3x - 5$
 Razón de cambio: 3
 Si $x = 4$, $y = 3(4) - 5 = 7$
 b) $y = -2x + 3$
 Razón de cambio: -2
 Si $x = 4$, $y = -2(4) + 3 = -5$

R a) $y = 2x - 7$
 Razón de cambio: 2
 Si $x = 6$, $y = 2(6) - 7 = 5$
 b) $y = -3x + 4$
 Razón de cambio: -3
 Si $x = 6$, $y = -3(6) + 4 = -14$

Tarea: página 55 del Cuaderno de Ejercicios.

Source: (El Salvador, 2019b)

Note. [Image description] Image of page 140 of the methodological guide, corresponding to Lesson 1.6 of Unit 3. It has a rectangular format with a white background. This page of the methodological guide presents the performance indicator of the lesson, the sequence of the lesson within the unit, the purpose of each lesson section, and, finally, a suggested way of organizing information on the chalkboard for this lesson. [End of description].

Reinterpretation

At this stage, the objective is to present a systematization of our perceptions regarding the ESMATE Program and its teaching materials. Based on the analyses carried out, we highlight, as a central point, the discourse that presents the material as “a flexible and improvable proposal, where the teaching staff should make the adjustments, they deem necessary to support the learning of their students.” (El

Salvador, 2019b, p. 6). However, throughout this article, several contradictions regarding this claim can be identified. We begin with the fact that the textbook follows the same rigid structure for all lessons, alongside the numerous recommendations made to teachers that involve lesson organization, chalkboard layout, annual scheduling, and the assessments to be administered.

It is also important to emphasize that such “recommendations” are consistently accompanied by the verb *should*, which conveys an underlying intention that teachers are expected to do what is written. Furthermore, the classroom visits carried out to verify whether teachers are following the steps established by the methodological guide show that there is an observation protocol/application used to evaluate the teacher and the time spent on content explanation and problem solving. Thus, one may question how teachers could make adjustments to the material if so many mechanisms encourage them to strictly follow what is written.

Considering now the problem-solving approach, we examine how this approach is understood in the teaching material. In this regard, we agree with Diniz (1988), who states:

A problem is any situation in which the individual confronted has no guarantee of obtaining a solution through the use of an algorithm, and all relevant knowledge must be combined in a new way to resolve the issue. (Diniz, 1988, p. 15).

Examining the problems in Unit 3 of the 8th-grade textbook, we noticed that the majority of them do not correspond to this view, since few of the problems presented to students deviate from an algorithmic pattern. In most lessons, students need only follow the reasoning presented in the “initial problem” and/or “example” sections to solve the proposed tasks. Thus, we argue that the teaching material confuses the idea of problem with exercise. According to Dante (1991):

Exercise, as the name suggests, serves to practice a specific algorithm or process. The student reads the exercise and extracts the necessary information to practice one or more algorithmic skills. Problem – process [...] is the description of a situation in which something unknown is sought and no algorithm is previously available that guarantees its solution. (Dante, 1991, p. 43)

Reflecting on the message to students in the textbook — “This book contains several problems and activities through which you will be able to develop your reasoning and improve mathematical skills that will be very useful for solving everyday situations.” (El Salvador, 2019c) — we understand that such activities do

not fully achieve this purpose. On the contrary, they merely teach students to solve predetermined exercises.

Finally, we emphasize the issue of the material working with only one performance indicator per lesson. When we examine how the lessons analyzed were divided, we observe an excessive subdivision of content. For example, there is a lesson specifically for “graphical analysis of positive slope” and another for “graphical analysis of negative slope.” Such subdivision compromises conceptual connections and restricts teacher autonomy in creating more contextualized and interdisciplinary activities. This form of pedagogical control disregards teacher training, implicitly assuming that teachers lack autonomy to plan their instruction.

Therefore, although we view ESMATE as a well-intentioned proposal, given that studies were conducted regarding the changes to be implemented, we believe that greater coherence is needed between the objectives proposed and what was ultimately delivered. Adjustments are required that value teacher autonomy and promote the construction of genuinely active learning.

Final Considerations

In this study, we analyzed the ESMATE teaching material through a more contextualized perspective, seeking to understand it within the public policy in which it is embedded. For this purpose, we proposed interpretations based on the theoretical and methodological framework presented by Thompson (1995), Depth Hermeneutics.

In the first two sections, we examined the geographical, political-administrative, and educational contexts of El Salvador and how ESMATE was implemented, along with the historical background of the program. We also presented the components of ESMATE and conducted an analysis of the teaching materials, examining the internal characteristics of the book’s structure, exploring the underlying intentions of its symbolic language, and considering the coherence of the work. Finally, in the third section, we sought to connect the first two in order to construct meanings for the program. In doing so, we observed that although ESMATE appears to be well intentioned, it exhibits contradictions in both its implementation and the teaching materials produced.

We observed that this study enables us to establish connections with the growing platformization of education in our own country. One of the initial movements

in this direction allows us to draw a parallel between ESMATE materials and the RCO+Aulas system made available in Paraná by the State Department of Education. This program originated in the context of the Covid-19 pandemic, during which schools closed abruptly. As a response, the government of the State of Paraná created the Aula Paraná project to ensure free access to teaching materials and lessons for all students, both through broadcast television and digital applications.

After the pandemic, the Aula Paraná material was incorporated into the Online Class Register (RCO). This material began to be used in several schools across Paraná in face-to-face classes and became mandatory in many of them. Thus, criticism of this material lies not only in the fact that it contains numerous inconsistencies and inaccuracies — given that it originated in an emergency context — but also in its mandatory use and the standardization of teaching, which ignores the individuality and learning processes of each student.

Although the material is presented as something flexible — being described as “a planning module available in the Online Class Register (RCO). In it, teachers can find specific lesson plans for the subjects and grades they teach, with pedagogical suggestions and methodological guidance.” (Government of the State of Paraná, n.d.) — there is evident pressure for teachers to use these materials. Furthermore, the slides available for classroom use already include pre-established times for each stage of the lesson, which limits teacher autonomy to adapt the content as necessary. This logic of rigid planning and time control is also present in ESMATE, revealing, in both initiatives, a tendency toward controlling teachers’ pedagogical freedom.

Thus, as stated at the beginning of this article’s introduction, we hope that the results of this study enable a better understanding of globalized education currently present in other countries and in our own, particularly in the State of Paraná. Moreover, by identifying such contradictions in ESMATE, this research may contribute to the improvement of the Program itself and of public policies with similar objectives.

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