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Revista de Educação em Ciência e Tecnologia

Mathematical modelling in the early years from a historical-cultural perspective

Modelagem matemática nos anos iniciais na perspectiva histórico-cultural

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Abstract: This paper aims to discuss the role of teacher's mediation in Mathematical Modelling (MM) activities in the early years of Elementary School from the perspective of Historical-Cultural Theory, based on research carried out at master's level. The corpus was composed by MM activities described in two dissertations available in the Digital Library of Theses and Dissertations, from which five tasks were selected for analysis. They were developed with classes from the 2nd and 4th years of Elementary School and were analyzed according to Discursive Textual Analysis, thus obtaining three categories, Mediation regarding the theme, Mediation regarding the problem and Mediation regarding validation. The analysis highlighted the importance of teacher mediation and interaction among students during implementation of MM activities as promoter of learning and development of higher psychic functions.

Keywords: Mathematical modelling, early elementary school, historical-cultural theory.

Resumo: Este artigo[1] tem como objetivo discutir o papel da mediação do professor em atividades de Modelagem Matemática (MM) nos anos iniciais do Ensino Fundamental sob a perspectiva da Teoria Histórico-Cultural, a partir de uma pesquisa realizada em nível de mestrado. O *corpus* foi composto por atividades de MM descritas em duas dissertações disponíveis na Biblioteca Digital de Teses e Dissertações, das quais foram selecionadas cinco tarefas para análise. Elas foram desenvolvidas com turmas de 2º e 4º anos do Ensino Fundamental e foram analisadas segundo a Análise Textual Discursiva, obtendotrês categorias, Mediação em relação ao tema, Mediação em relação ao problema e Mediação em relação à validação. As análisesevidenciaram a importância da mediação do professor e a interação entre os alunos durante a implementação das atividades de MM como promotora

[1]This paper was supported by Capes/PROAP and PRPGEM.

para a aprendizagem e o desenvolvimento das funções psíquicas superiores.

Palavras-chave: Modelagem matemática, anos iniciais, teoria histórico-cultural.

Introduction

Mathematical Modeling (MM)[2]is a pedagogical alternative that addresses themes and problems in the students' context, involving economic, social and cultural issues, and in addition to the learning of mathematical content, enables development of students' critical sense(Burak, 1992, 1994, 2010, 2017), which can be developed from the early years.

The initial milestone of MM in the early years of Elementary School in Brazil was explained by Burak (1987, 1992). The author is considered a precursor of MM in the early years, having guided different MM works in the early years of Elementary School, covering the four grades, from the 1st to 4th, with different themes, such as painting a classroom, building a vegetable garden, among others.

According toBurak (1992, p. 62), MM is "[...] a set of procedures whose objective is to construct a parallel to try to mathematically explain the phenomena present in daily lives of human beings, helping them to make predictions and take decisions". Regarding development of MM in the classroom, Burak (1992, 1994, 2010) points out two guiding principles: 1) based on the interest of the group of people involved; and 2) obtaining information and data, whenever possible, in the environment where the group's interest is located. Furthermore, he presentsfive stages: choice of theme; exploratory research; problems survey; problem-solving anddevelopment of mathematical content within context of the theme; and critical analysis of the solutions.

The author highlights that more emphasis should be placed on the process than on the construction of mathematical models in the early years of elementary school, and that MM can be used not only to teach mathematical content, but also to develop critical thinking, especially when it is implemented in an interdisciplinary way and with situations that involve the students' context, also can be based on non-mathematical themes.

For Almeida and Dias (2004, p. 7), MM is "[...] an alternative for teaching and learning school mathematics, which can provide students with opportunities to

[2]MM, as part of theoretical basis of research carried out, is due to the selection of research corpus, whose object of analysis is pedagogical activities (tasks) of two dissertations. Emphasis was placed on Burak (1992, 1994, 2010, 2017) due to its connection with the earlyyears, and on Almeida and Dias (2004) because of the use of three moments, elaborated by them when developing MM activities.

identify and study problem situations in their reality, awakening greater interest and developing more critical and reflective knowledge [...]". The authors report that MM has potential for teaching and learning mathematics in the classroom, and that it can be developed gradually from three moments, allowing students, organized into groups, to become familiar with the context.

In the first moment, the teacher presents a problem situation to all students and carries out deductions, analyses and the construction of the mathematical model with them.In the second, a problem situation and a set of information are presented and students, divided into groups, formulate simplifying hypotheses, construct the mathematical model and validate it.In the third moment, in groups, students develop MM activity based on a problem situation chosen by them and the teacher follows the entire process.At this point, students are responsible for conducting all MM activities.

According to Almeida and Dias (2004), forwarding of MM activities has proven to be appropriate in classroom practice at different education levels, as well as the appropriation of new knowledge. Then,"[...] students both reframe concepts already constructed and construct new ones when faced with the need to use them" (Almeida &Vertuan, 2011, p. 5). In this process, students acquire autonomy, critical thinking and consolidate mathematical learning.

Moreover, it is important to consider the Historical-Cultural Theory, whose main precursors,Vygotsky[3], Luria andLeontiev (2006),highlight the development of higher psychic functions (HPS) based on historical-dialectical materialism developed by Marx (2013) and Engels (2000).

Leontiev (1978) emphasizes that the individual is not born human but becomes human through experiences in society, in activities that consist of needs, motives, objects and actions. More recent investigations carried out by researchers of Historical-Cultural Theory revisit it, such asDavídov (1988), Martins (2011) and Martins et al. (2016). For them, in human activities, intrapsychic relationships are constituted from interpsychic relationships. In these interpsychic relationships, the individual creates and appropriates increasingly elaborate tools that lead to human development, that is, to development of higher psychic functions (HPF).

Costa andTuleski (2016) emphasize that tools created for man's subsistence work (hunting, agriculture, etc.) are related to instruments designed to appropriate historically accumulated knowledge, as heritage of humanity. Thus, "[...] it was the unity between concrete instruments and abstract instruments (signs) that led to the

[3]In literature, the author's name appears in different ways: "Vigotski, Vygotski", "Vigotsky" and "Vygotsky". In this paper, the written form is preserved according to each referenced work. However, when dealing with interpretations made in this research, the term is standardized as Vygotsky.

emergence of the human race [...]", freeing man from his biological limitations(Costa &Tuleski, 2016, p. 9).

It is important to highlight that, although there are researchers with the same theoretical basis, Martins (2016) uses the term tools, and Costa and Tuleski (2016), the term instruments to refer to the means used for the appropriation of human knowledge or for the construction of some object of subsistence and that mediate human relations in their various activities.

Thus, in this paper, the term instrument is used to refer to objects externalized and objectified by human beings and that can be materialized into tools, or in the case of schools, into pedagogical resources. They are subdivided into material and symbolic instruments, materialized in material pedagogical resources such as pens, boards, paper, slides, etc., or in symbolic resources such as texts, originating from textual genres (Bakhtin, 2016) based on symbolic instruments.

Instruments are mediated by the relationship between thought and language (Davídov, 1988; Vigotski, 2009) in the activity carried out by human being and led to the creation of other forms of activity.

Within the school context, the nature of activity occurs through the learning of knowledge systematized throughout history, which is internalized and forms the subject's consciousness (Leontiev, 1978). Therefore, it is understood that schooling exerts a great influence on the subject's understanding and performance in the world, since learning and human development are not results of maturation, on the contrary, they are promoted socially.

To delve deeper into the subject and clarify the role played by teacher in psychic development, Vygotsky (1991) determines two levels of development: Real Development Zone (RDZ), which is the level of a child's development at which his or her mental functions have already been established as a result of certain developmental cycles,andPotential Development Zone (PDZ), which is the child's level of development determined by the ability to solve problems under the guidance of an adult or in collaboration with more experienced peers. It is worth noting that the PDZ represents the space between the knowledge appropriated by the student and the knowledge that will be acquired by him/her with teacher's mediation, when organizing the teaching activity.

This organization, according to Moura etal. (2010, p. 208),is explained "in the search to organize teaching, resorting to the articulation between theory and practice and which constitutes the teacher's activity, more specifically, the teaching activity", called by the authors as the Teaching Guiding Activity (TGA).

In TGA, the teacher's role is to mediate between what the child already knows and what he or she needs to learn, through instruments materialized in pedagogical resources, with a view to the appropriation of scientific concepts, and eventually for development of higher psychological functions.Then, mediation is considered as:

- Relationship between people through language and objects, these called concrete and symbolic material instruments (Vygotsky et al. 2010);
- Helping development of psychic activities through concrete and abstract instruments (Costa &Tuleski, 2016);
- Process of intervention of an intermediate element in a relationship (for instance, between one concept and another) (Oliveira, 2001);
- Interaction and discussion to promote autonomy, critical thinking and a closer bond between teacher and students (Burak, 1994);
- Process of intentions (with beginning, middle and end) in the organization of classes in TGA, through content (Moura et al.2010).

In that regard,the Curricular Framework for Early Childhood Education and Elementary School in Paraná(Paraná,2018) shows the need to use pedagogical teaching resources to systematize and consolidate students' mathematical knowledge, converging with Moura (2001), when he states that it is up to teacher to prepare these pedagogical resources in his practice.

Based on these assumptions, the research aims to understand the role of teacher mediation in MM activities in the early years of Elementary School.Thereunto, a documentary study was carried out based on two dissertations:*The uses of language in Mathematical Modeling activities in the early years of Elementary School*[*Osusos da linguagemematividades de ModelagemMatemáticanosanosiniciais do Ensino Fundamental*] (Tortola, 2012) and*Mathematical Modeling in the Early Years of Elementary School: Possibilities for Teaching Mathematics*[*ModelagemMatemáticanosanosiniciais do Ensino Fundamental: Possibilidades para o Ensino da Matemática*] (Jocoski, 2020). The following sections present the method, data analysis and research results, and finally, the conclusions.

Method

The qualitative documentary research, based on Creswell (2007) and Bogdan and Biklen (1994), occurred through identification of 14 research studies published in

the Digital Library of Theses and Dissertations (BDTD in its Portuguese acronym)[4]. A reading of abstracts, objectives and theoretical basis of each publication was carried out with the aim of identifying the theme “Mathematical Modeling in the early years of Elementary School” with a focus on students in the classroom context.

Once this was done, the dissertations by Tortola (2012), entitled *The uses of language in Mathematical Modeling activities in the early years of Elementary School*, and by Jocoski (2020), *Mathematical Modeling in the early years of Elementary School: possibilities for teaching mathematics*, were selected.The selection is justified by detailed description and the number of activities contained in each of them, which was sufficient to compose the research corpus.

MM activities were selected based on reading the activities described in the two selected dissertations. The activities (from now on, represented by the acronyms A1 to A5) from Tortola (2012), thatare A1 - Ring Size, A2 - Student Space in the Classroom, A3 - Measuring a Person’s Beauty, A4 - Fluoride Expenses, and A5, by Jocoski (2020), Slime Recipe. Organization and analysis of data were carried out based on Moraes and Galiazzi (2011), through Discursive Textual Analysis (DTA).

It began with deconstruction of text and fragmentation from the corpus of analysis, and resulting from the identification of activities, three categories, and nine subcategories emerged.This process, characterized by the production of metatexts, according to Morais and Galiazzi (2011), requires constant improvement and organization of writing, as it demands permanent attention to emerging phenomena, careful analysis and detailed interpretation by the researcher.

Data analysis and findings

This section presents the analysis and results of the research through the three categories and nine subcategories that emerged from the data.

[4]BDTD (ibict.br)The Brazilian Institute of Information in Science and Technology (Ibict in its Portuguese acronym) developed and coordinates the Brazilian Digital Library of Theses and Dissertations and is responsible for integrating the theses and dissertations information systems existing in educational institutions.

Table 1

Emerging Data Categories and Subcategories

Categories	Subcategories	Excerpts
Mediation regarding the theme	S1:Choice of theme by teacher	A1.1; A1.38; A1.39; A1.41; A1.42; A2.38; A2.39; A2.40.
	S2: Choice of theme by teacher and student	A3.1; A3.2; A3.5; A3.6; A3.8; A3.21; A3.22; A3.64; A3.65; A3.66; A3.67.
	S3: Exploratory research	A1.2; A1.6; A1.42; A1.43; A2.1; A2.3; A2.4;A3.3; A3.9; A3.10; A3.11; A4.3; A4.8; A4.22; A5.10; A5.18; A5.19; A5.20; A5.54; A5.58; A5.59.
Mediation regarding the problem	S4:Problems integration	A1.3; A1.4; A1.5; A1.7; A1. 8.; A1.9; A1.12; A2.7.; A2.8.; A3.12; A3.13; A3.6; A4.5; A4.6; A4.9; A4.10; A4.11; A4.12; A4.13; A4.17; A4.190; A4.24; A5.12; A5.21.
	S5: Problems survey	A1.10; A1.11; A1.13; A1.14; A1.16; A1.17; A1.20; A1.21; A2.2; A2.5; A2.6.; A2.9; A2.15; A2.17; A3.1; A3.18; A3.19; A3.27; A3.29; A3.37; A3.38; A4.4; A4.11; A4.13; A5.11; A5.31.
	S6: Problem-solving	A1.23; A1.24; A1.26; A1.27; A1.28; A1.29; A1.30; A1.31; A1.32; A1.43; A1.48; A2.11; A2.13; A2.14; A2.19; A2.20; A2.21; A2.22; A2.23; A2.26; A2.36; A3.39; A3.58; A4.38; A5.26; A5.47.
Mediation regarding Validation	S7: Critical analysis of solutions	A1.33; A1.34; A1.37; A2.18; A2.24; A1.25; A3.25; A3.30; A3.34; A3.35; A3.44; A3.47; A3.49; A3.50; A4.16; A4.33; A4.38; A4.48; A5.27; A5.41; A5.60.
	S8: Interpretation of results	A1.35; A1.36; A1.40; A1.45; A2.10; A2.33; A2.34; A2.35; A2.37; A3.4; A3.31; A3.33; A3.36; A3.43; A3.45; A3.46; A3.48; A3.51; A3.54; A4.41; A4.42; A4.43; A5.28; A5.50;
	S9: Validation and reflective discussions	A1.44; A1.45; A1.47; A1.48; 1.49; A1.50; A2.27; A2.29; A2.30; A2.31; A2.32; A2.38; A3.40; A3.41; A3.42; A3.52; A3.53; A3.55; A3.56; A3.57; A3.59; A3.60; A4.47; A4.50; A5.45.

Source: the authors.

Table 1 summarizes the categories listed in the first column; followed by the subcategories (S1 to S9) in the second column, and the excserpts[5], third column.They were used as a basis for discussion of all categories resulting from the research.

Mediation Regarding the Theme

In the first category, entitled*Mediation regarding the theme,*the activities took place according to the proposal by Almeida and Dias (2004), following the three moments, based on activities: I*) carried out exclusively by teacher; II) carried out by teacher and students; III) conducted by student with teachermediation*.The first category culminated in the Subcategories S1, S2 and S3. Activities that are part of

[5]Excerpts were coded according to data obtained from the five activities, for example: A1.15. S1, which means excerpt number 15 of activity A1 and subcategory S1.

S1, Choice of Theme by Teacher, are A1 – Ring Size and A2 – Student Space in the Classroom.

In the first subcategory (S1), the action of research teacher was addressed (Tortola, 2012), when presenting the theme to students and providing the necessary information for discussion, as can be seen in the following excerpts:

> ● Teacher began the class with some questions regarding the theme*Rings*(A1.1.S1);
> ● The excitement identified in their responses and their willingness to study indicates their acceptance of the invitation made by the teacher to investigate the topic *Ring Sizes*. (A1.42.S1);
> ● The theme*Student space in the classroom*was chosen because it encompasses a context that is directly associated with the students' school environment (A2.38. S1).

From these excerpts, it is possible observe that, in the action of choosing the theme, even without direct participation of students, Tortola (2012) demonstrated concern in promoting acceptance of the theme, as seen in *A1.42.S1.* He selected a theme related to the students' context, directly associated with school environment, and encouraged discussions that enabled development of democratic dialogues in the classroom.

This action corroborates with Vygotsky (1991, p.129),by stating that "[...] thought itself is generated by motivation, that is, by our desires and needs, our interests and emotions".Then, by providing an environment for dialogue, Tortola (2012) allowed the student to develop thought, and consequently enhanced the development of higher psychological functions. In other words, these approaches awaken a "[...] greater interest and develop a more critical and reflective knowledge in relation to the contents of Mathematics" (Almeida & Dias, 2004, p. 7).

Second Subcategory (S2) refers to the *Choice of Theme by Teacher and Student and* is part of the second moment by Almeida and Dias (2004). Excerpts below are from activity A3 – Measuring a person's beauty, and it enablesto observe the actions that denote the choice of theme between teacher and student.

> ● To promote interaction and provoke a discussion with students about the subject, the teacher presented some slides with photos of famous people, such as singers, hosts of tv shows, and football players, etc. (A3.5.S2);
> ● Everyone was excited, some just raised their hands as requested, others spoke loudly and even commented on their opinions (A3.6.S2);
> ● [...] we hand out a sheet with information about the topic (A3.64.S2);
> ● [...] the class was organized into 8 groups with 3 or 4 students each (A3.65.S2).

The students' participation in choosing the theme suggested by the teacher is notable, in addition to the organization of the class into groups, as highlighted by Almeida and Dias (2004). Teacher's mediation in the process of presenting the theme was significant for acceptance and participation in choosing the theme,

converging with the assumptions by Burak (1992, 2017). Through these excerpts, the importance of working in groups and through a theme becomes clear, as it allows the student to study phenomena in various dimensions, biological, psychological and social, enabling discussions and playing a fundamental role in development of student autonomy (Burak, 2017).

Mediation instruments used by Tortola (2012) through pedagogical resources, such as a sheet with information to mediate discussions, a table with indications to work on measures and a sheet with information about the topic, have as a starting point the students' PDZ, by bringing to the classroom what the student can learn through the more experienced pair, in this case, the teacher. In that regard, Burak (1992, 2017) highlights that discussing the topic and investigating it contributes to mathematics teaching, especially in the early years, and in addition, to development of critical thinking.

Burak (1994) also presents the teacher as a mediator throughout the entire process of discussions and development of the activity, and the importance of the relationship between teacher and student in the learning process.Asbahr (2016) highlights that affective relationship is enhanced for the organization of pedagogical work. In that regard, Moura (2001) highlights that in TGA, the act of planning is done in such a way that it leads students to interact mediated by a meaningful and motivating activity.

Subcategory (S3) refers to Exploratory Research, the second stage of conducting a MM activity in the classroom, according to Burak (2010, 2017), which occurs after choosing the theme to be researched.The following excerpts list moments in which students adopt an attentive, judicious and committed attitude towards research.Tortola's mediation (2012) directed students' attention through discussions to some selected information, according to their objectives, articulated with the content printed, a priori, on the information sheet. Below, some excerpts.

- [...] students selected information they considered useful to study the problem, the words used were taken from the sheet with information about the rings (A1.43.S3);
- [...]teacher suggested that students watch an excerpt from the video "Donald in the land of Mathmagic" (A3. 10.S3);
- Students searched for information and interesting facts on the internet, which they wrote down on a blank paper sheet that was given to them (A4.3. S3).

From the excerpts presented, the main path chosen by Tortola (2012) to conduct exploratory research on the activities A1, A2, A3 and A4 was dialogue. Moura (2001, p. 25) emphasizes the primordial role of language for Vygotsky and highlights that "[...] the word is like the sign par excellence, responsible for the

cultural development of subjects. The word is the fundamental cultural mediation sign, responsible for transformation of the subject's natural intelligence functions into higher functions".Then, within the school context and from TGA,the teacher plays a significant role, since the learning of scientific concepts, when properly organized, produces a greater effect.

Organization presented by Jocoski (2020) in activity A5 provides development of students' autonomy, as well as learning from others who are more experienced.Here are some excerpts, for example:

- Some children brought from home different names given to slimes (A5.18.S3);
- [...] children often asked their parents for help, especially when they needed to bring in research notes that were requested from them about Slime. The parents were very helpful with the children's research [...] (A5.54.S3).

From the excerpts listed in activity A5 by Jocoski (2020), the first principle based on Burak (2010, 2017) related to the group's interest can be observed.Family participation, when required in activities sent home in the research process, greatly enabled the emotional bond between teacher and student as well as the collective participation of all.Exploratory research was made possible by the internet, interviews, videos and family members, and teacher used pedagogical resources effectively, enhancing development of higher psychological functions.

Mediation Regarding the Problem

Second category, entitled *Mediation regarding the problem*, refers to the stage in which students identify and formulate problems related to the themes investigated by them, or a priori, prepared by teacher, following the three moments (Almeida & Dias, 2004). It refers to the stage in which students identify and formulate problems based on the information and data collected according to the group's interest, in accordance with the principles presented by Burak (1992, 1994, 2010, 2017), which aims to respond to the problems raised and the doubts that arise during the research, through mathematical content and possible mathematical models. However, in the stage study of early years, Burak (1994, p. 53) highlights that "[...] the teacher must be more concerned with the process than just creating mathematical models, because at this stage, the mathematical structures are still in process of construction by child".

Mediation carried out by teacher is significant, as it allows the student to strengthen skills from a critical perspective, favoring the elaboration of problem

situations based on the data collected (Burak, 2017). The teacher's role is to act in the PDZ between what the student already knows how to do on their own and what they are about to learn through mediation.Then, it is understood how relevant the role of the teacher is as responsible for teachingorganization. When planning and preparing classes, acting as a mediator of pedagogical resources used, the teacher works on the student's development potential (Moura, 2001).In this perspective, the category is composed of three Subcategories, S4, S5 and S6.

Subcategory (S4), related to *Problem Integration*, refers to the phase in which students are guided by a specific problem and seek to understand and become familiar with the theme investigated through data collection and information about it. The following excerpts show discussions raised by Tortola (2012) and Jocoski (2020):

- After this initial discussion, each student was given a sheet with information about the topic *Ring Size*(A1.4.S4);
- But how do you know what size ring is right for your finger? (A1. 7.S4);
- Upon learning about the problem, the teacher asked students if they could imagine how to solve it (A1.12.S4);
- Teacher asked about the beauty of the same person in a photo, some classmates raised their hands and others didn't, that is, a person who is beautiful to one classmate may not be to another (A3. 6.S4);
- Student 5: How many fluoride applications are sent here to the school? (A4.19.S4);
- [...]decided to create a WhatsApp group to forward photos, videos, messages, questions and other issues to the children's guardians about the activity that would be carried out during the research (A5.12.S4);
- Downloaded images of different types of Slime; children commented on their colors, textures and ingredients used in each image viewed (A5.21.S4).

Examples show that discussions are initiated with the aim of familiarizing students with the problem, based on information collected and on the group's interest, according to Burak (2010, 2017). These problems do not need to be related to mathematics, but to an interest in research.For the author, at this stage it is necessary for the teacher to assume the role of mediator with the aim of contributing to delimitation of a subject, by leading students to become aware of the problem.

This corroborates the concept of learning based on Historical-Cultural Theory, highlighting its social nature and attributing significant importance to the role of others in development (Vigotski, 2009). In data listed, the discussions make students show interest and participation, developing critical and reflective knowledge in relation to the mathematics content, enabling decision-making (Burak, 1992, 2010).

Interaction is essential in understanding the problem during MM process, as it is the phase that involves selecting and annotating data, which contributes to finding possible answers to the problem.The main mediation resource used by Tortola (2012) and Jocoski (2020) is clearly visualized in the mental actions and in

discussions development, actions carried out by the teacher and previously thought through teachingorganization (Moura, 2001).

Subcategory (S5) refers to *Problem Survey*. According to Burak (2010, 2017), it is the teacher's role to encourage students to raise questions based on information they collected and discuss whether it is related to mathematics or not and then develop the research problem.The author emphasizes that, by obtaining so much new knowledge about the subjectstudied, students define the themeto be researched.For Burak (2010, p. 53), this stage "[...] requires effort and reflection by students and teachers; favors understanding a given situation; encourages the student's active participation in discussions and elaboration".

At this stage, the first steps are identified to develop in the student the ability to transform everyday situations into mathematical or non-mathematical situations, with teacher's mediation throughout the process of carrying out the activity (Burak, 2017). Therefore, in activities A1, A2, A3 and A4 by Tortola (2012) and A5 by Jocoski (2020), it was noted that the actions were thought out and guided by teacher, as suggested by Moura (2001), through information and questions about the problem, as explained by the excerpts below:

- How to determine the size of a ring? (A1.10.S5);
- Based on information obtained, a problem was formulated: "what is the cost of fluoride applications in our school?" (A4. 4.S5);
- They commented that for the next class, they would bring a Slime made by them, recipes or drawings if they couldn't take photos (A5.11.S5);
- We started playing the video about the biggest Slime in the world, by YouTuber *Tio Lucas*, and they watched the video presentation closely and asked some questions about it (A5.31.S5).

From these excerpts, it was found that mediation carried out by teachers was relevant. Burak (2017, p. 12) highlights that "[...] teacher's action, as a mediator, is significant because it provides the opportunity to strengthen the student's abilities to review data collected from a critical perspective, favoring the construction of problems or problem situations based on discussion of data collected".It is observed that Tortola (2016) and Jocoski (2020) raised the problems through discussions based on the questions, which is in line with what Burak (2017, p. 15) states, that with "[...] a generic character, it requires effort and reflection by students and teachers". It is highlighted, from these excerpts, the teacher mediation occurring in the PDZ, in agreement with Vygotsky (2007).

Actions were thought out and outlined by teacher, according to Moura (2001). Through discussions, interest and motivation in participating in the activity are observed, as follows:

- They said yes and started with some guesses, some said it would fit 38, others, 40, others still, 42, 50, and so new guesses emerged (A2.5.S5);
- “Each person must have one meter of space for themselves”, as Student 35 points out (A2.6.S5);
- [...] one more piece of information was pointed out by Student 2, that “the classroom is occupied by students, teacher and objects... such as chairs, desks, lockers”, thus completing the informationcollection(A2.15.S5);
- They measured and were measured by their colleagues (A3. 29.S5);
- Student 8:How many fluoride applications are given? [...] How much is spent? (A4.13.S5);
- Group 3: What ingredients do we need to make our slime in the classroom? (A5.34.S5).

Reflections and questions raised were crucial for internalization process to occur. Burak (2017, p. 12) highlights that “[...] the quality of action favors improvement of autonomy, formation of a critical spirit in the student that gives meaning and significance to the resolution of problems raised”, generating learning and development, according to Historical-Cultural Theory.Vygotsky (1991, p. 83) highlights that “intrapsychic relationships (individual activity) are constituted from interpsychic relationships (collective activity).”It can be inferred that interaction that occurred between students enhanced the process of searching for information in identifying problems.

Among the excerpts listed above, a dialogical environment is observed that is necessary in the early years amongst children, thus contributing to teaching and learning (Burak, 2010), corroborating the concept of learning through relationships with others, starting from social to individual.

Subcategory (S6) refers to the*Solving problems* stage. According to Burak (1992, 2017), at this stage, the objective is to respond to the problems raised through mathematics.Usually, the focus is on the construction of a mathematical model, but for Burak (2017), p. 12), this is not the priority in the early years of Elementary School, “[...] because in this period of children’s formation they must be taught to construct concepts more than to appropriate formulas and carry out mathematizations that, due to the level of abstraction required, are not meaningful to them”.Therefore, it is important to emphasize and discuss the mathematical concepts that emerge from this process, understanding the teacher as a mediator, acting within the PDZ and enabling learning through mathematical content.Then, activity (Leontiev, 2006), in its constitutive elements, need, motive, objective and action, always mediated by the most experienced pair (Tuleski and Eidt, 2016), is responsible for development of psyche, seen through the lens of Historical-Cultural Theory.

Problem-solving stage allows the teacher, as a mediator, to provide

appropriation of new mathematical knowledge by acting in PDZ, as well as difficulties, and taking them to a higher level of knowledge. Some examples (Tortola, 2012) show it:

- [...] doubt regarding the starting point of measurement: Where to start measuring, from zero or from one? (A1.23.S6);
- Another issue that also needed clarification was regarding millimeters (A1.29.S6);
- Students in general were measuring the classroom without observing the fact that it was necessary disregard the space occupied by lockers or the space allocated to the teacher (A2.19.S6);
- Geometric shapes [...] content explored little in classes until now (A5.26.S6).

The importance of mediation carried out by the teacher based on the PDZ is clear, as doubts and difficulties are rethought through teaching planning to carry out necessary mediations, converging with the meanings by Moura (2001). In that regard, Martins, Abrantes andFacci (2016) state that doubts and difficulties encountered amidst a problem situation are the starting point for the teacher's teaching practice.

From the organization of activities A1, A2, A3, and A4 by Tortola (2012) and A5 by Jocoski (2020), it is possible to observe that the teachers started from the students' difficulties to clarify their doubts. The following excerpts are examples:

- Teacher explained the difference between adopting zero or one as the starting point of measurement, and we usually start from zero, since measurement becomes easier, because what do we do when we measure? (A1.24.S6);
- Students did not see how to explain this relationship through a number, since for them beauty was something essentially personal (A3.39.S6);
- [...] so the teacher suggested they measure how much solution could fit in a small cup (A4. 38.S6).

It is possible to observe that there were discussions mediated by the teacher and encouraged by students, making the teacher's detailed planning evident when presenting strategies used for resolution. In this resolution analysis process, strategies used to produce the mathematical model can be observed below:

- Representations used in the production of the mathematical model of this situation were varied, involving figural, numerical and natural language records (A2.36.S6);
- [...] mathematical model produced by students for this activity was verbally constructed with teacher's help [...] and to construct it, teacher asked students to present their conclusions, group by group (A3.58.S6);
- [...] there was the construction of Slime model from the recipe adopted by children, and which was initially one of the problems raised by some groups (A5.47.S6).

From the excerpts listed, it is reinforced, from the Historical-Cultural Theory perspective, that learning starts from the social environment and, therefore, there is no learning without relationship with the other. Mathematical models obtained involved figures, graphs, tables, recipes and verbal presentations carried out in

groups with the aim of mathematically explaining the problem situations.The main resources used by research teachers were presented through dialogue during discussions in development of activities, which represents one of the preponderant factors for learning, seen from the Historical-Cultural Theory and paraphrasing Vigotski (2004), making man capable of planning his actions, solving problems, encouraging discussions and making decisions, among others.

Mediation Regarding Validation

*Mediation regarding validation*had as its main point the validation of the mathematical model. However, previous analyses have pointed to the importance of the process of appropriation of mathematical knowledge, not the formulation of mathematical models in the foreground, as highlighted by Burak (1994). This category is composed of Subcategories S7, S8 and S9.

Subcategoria (S7), *Critical analysis of solutions,*is based on criticality and is a unique moment, as it aims to discuss the hypotheses presented throughout the problem-solving process and the solutions found (Burak, 2017). Excerpts below demonstrate the discussions that mediated the problem solutions:

- Regarding the discussion in relation to the problem, students noticed that the measurements follow regularity (A1.33.S7);
- In response to the problem, students obtained values such as 36, 39, 40, 42, etc. (A2.24.S7);
- Students were immersed in the language game, referring to beauty, practiced in their daily lives, in which beauty is seen as something personal and cannot be measured, this meaning of beauty took a long time to be modified (A3.47.S7);
- Children were very creative in their slime illustrations (A5.27.S7).

Active participation of students in discussions surrounding problem-solving is highlighted, presenting moments of critical reflection (Burak, 1992, 2010, 2017). Discursive moments contributed to learning through identification of errors and presentation of ideas, as shown in the following excerpts:

- As for the errors made, they related it to the difficulty in obtaining an accurate measurement [...]. (A1.37.S7);
- Students were suspicious of the answer (A4.38.S7);
- Based on decisions made throughout the activity to add quantities of ingredients to the recipe (A5.60.S7).

Through these group discussions, the teacher's mediation based on the PDZ stands out, addressing the difficulties presented by students. The central importance of teacher as the person responsible for organization throughout this process is evident, which corroborates the TGA proposed by Moura et al. (2010). They

emphasize the planning and re-planning of teaching. It is also emphasized that group discussions, according to assumptions of Historical-Cultural Theory, lead to individual development.

Subcategory (S8), *Interpretation of results*,as Burak (1992) states, is an important step to reflect on results achieved, both mathematical answers and discussions contemplated regarding the activities. The following excerpts are examples:

> ● Contents on determining area, perimeter and units of measurement of length and area, in addition to covering some operational properties, such as the commutative nature of multiplication, and introducing students to interpretation of area from a geometric point of view (A2.10.S8);
> ● Comparing these quotients led Student 27, as well as others, to conclude that results obtained are close to 1.6 or 1.61; as shown by the protocols presented (A3.31.S8);
> ● Students then checked how many milliliters were needed to prepare 155 fluoride applications, by multiplying 8 × 155, obtaining 1240 ml of solution (A4.41.S8);
> ● The child tries to record in the drawing the units of measurement present on the packaging, trying to differentiate the size of pots with the quantity recorded 100g and 200g (A5.28.S8).

It is observed that, in this action, group discussions led to the interpretation of different mathematical concepts and situations present in everyday life, enabling the analysis of a possible response to the problem. Discussions presented are consistent with the students' mathematical repertoire, which required the teacher's intervention to interpret problems and find a model for the situation, as can be seen below:

> ● During the discussion about wearing rings, or about how to determine the size of a ring, natural language prevailed, and communication at that time was strictly oral (A1.40.S8);
> ● Teacher intervention was necessary to give a new meaning to the word in a new context, and in another language game (A2.34.S8);
> ● Beauty is something personal (A3.48.S8);
> ● They calculated how many days of fluoride application there are in the semester by multiplying 6 × 4 – considering that each month has 4 Fridays –, therefore, in 6 months there are approximately 24 days of fluoride application and, therefore, 24 × 90 = 2160 cents are spent on fluoride in this school, that is, R$ 21.90 (A4.43.S8);
> ● They were able to question, research and resolve doubts about mathematical content (A5.50.S8).

Discussions denoted the conclusion of the activity through moments of socialization and validation of mathematical or non-mathematical models for the problem situations.In these moments of socialization, a reflective environment was observed, organized by teacher, with objective of finalizing the activity to validate all the work carried out during the students' actions.

The classroom organization, carried out by research teachers Tortola (2012) and Jocoski (2020), during the five activities analyzed in this study, was through division into small groups. Such an approach aligns with the assumptions of an activity, especially regarding problem-solving and reflective discussions about the

results.This structure resembles the interests of TGA, because according to this perspective, Moura et al. (2010, p. 225) highlight that “[...] ensuring that the students’ study activity takes place primarily within a collective, seeks to implement the principle or law of formation of higher psychic functions developed by Historical-Cultural Theory [...]”. This makes MM a learning-enhancing activity in the early years, enabling development of meanings in mathematics that relate to students’ lives. Furthermore, it facilitates the connection between school mathematics and extracurricular problems experienced by students.

Subcategory (S9), *Validationandreflective discussions,*aims to validate the model, that is, to prove that the model or models produced are intended to respond to the problem motivating the activity. Burak (2010, p. 23) highlights that “a price list in a supermarket is considered a mathematical model”.

Actions belonging to this subcategory aim to socialize and validate mathematical models for the completion of the activity.Teacher’s mediation is preponderant (Burak, 2010), and at this moment, it is the teacher’s role to prepare a reflective environment according to the TGA, proposed by Moura et al. (2010), which promotes learning with a view to emphasizing the importance of finalizing the activity to validate the work performed by students throughout the time of carrying out the MM activity.Excerpts listed in S(9) demonstrate the reports of the research teachers on implementation of socialization of the groups’ results, that is, of the models achieved regarding the MM activities.Below are some excerpts on this subject:

> ● [...] mathematical model produced by students for this activity was verbally constructed with teacher’s help, who guided students in articulating available information and evidence found, and for its construction, teacher asked students to, group by group, present their conclusions (A3.57.S9);
> ● Students did not explicitly construct a mathematical model; this is revealed in their discussions and records through the sequence of operations they performed to solve the problem [...] (A4. 47.S9).

Reports by research teachers about the completion of the work with a view to socializing the results of each group denote more reflective moments. The teacher’s role in mediating these discussions is important, as the aim is to finalize the activity and validate the work done by students, which is aligned with TGA. Validation process with an explanatory character is presented as the main link in activities.

Through discussions, Tortola (2012) and Jocoski (2020) addressed new mathematical concepts, even from a non-mathematical theme, as Burak (1992, 2010) explains. Vygotsky (1991) emphasizes that learning new concepts occurs in the PDZ, based on understanding the student’s real knowledge, and thus the

introduction of scientific knowledge takes place, which contributes to development of higher psychological functions, as observed in the excerpts below:

- Multiplying the length measurement by the width measurement of classroom (A2.38.S9);
- Teacher then showed an example of a division algorithm on the board, as they were used to doing, and showed them the rules for using it [...] (A3.52.S9);
- Numerical language used to solve the activity related to fluoride application (A4.50.S9);
- [...] count one hundred by one hundred [...] Meter, kilogram (A5.45.S9).

These excerptss reveal well-organized teaching, aligned with TGA. Moura et al. (2010) explain that learning of concepts, when adequately organized, produces a greater effect on development of human psyche, especially in the formation of theoretical thinking at this stage of the child's study. Validation took place through presentations of investigations carried out, discussions on the results and critical conclusions about the models produced.According to Burak (2017, p. 14),"[...] the opportunity to redo the thought process about a given situation benefits each student and the group and promotes self-evaluation". It is noteworthy that participation of students in discussions, carried out in a reflective and critical manner, enabled learning and development.

Some Conclusions

In the three categories analyzed, the teacher's mediating action proved to be essential in development of MM activities and learning of students in the early years.

In the first category, *Mediation regarding the theme*, with a motivational character, students explained the reasons for their choices, making connection with their interests clear, which can be personal or group interests. Vygotsky (1991) emphasizes that thought is instituted by motivation, that is, by our interests and emotions. Activity development carried out through groups contributed to a greater bond between teacher and students, also in the relationships between the students themselves. Knowledge shared through discussions on the choice of theme between students and teachers, in a planned and organized way with specific purposes, contributed to development of student autonomy, generating learning and enhancing development.

Another action that enhances the first category refers to the organization and use of pedagogical resources as materialization of instruments. In this TGA, the teacher is primarily responsible for the appropriation of scientific concepts,which

through the relationship established with students, creates an environment conducive to learning.

In the second category, *Mediation regarding the problem*, the teacher sought to raise problems related to the themes and resolve them.From the analysis carried out, the importance of teacher's role as a mediator acting in PDZ became explicit, based on deepening and appropriation of mathematical content, contributing significantly to development of psyche seen through the lens of Historical-Cultural Theory.

The problem-solving phase also converged with principles of TGA, with teacher acting as a mediator of new mathematical knowledge, scientific school concepts and formation of theoretical thinking.Doubts and difficulties encountered amidst a problem were the starting point for teacher's teaching practice, realizing difficulties and rethinking his/her practice and seeking new paths for learning(Moura et al., 2010).

In the third category, *Mediation regarding Validation*, validations related to the mathematical models produced for problems studied in the five activities were identified. Students' participation in discussions surrounding the analyses was evident, both in mathematical discussions and in reflections on reality. In line with principles of TGA, teacher's mediation proved to be extremely important in this process, when he/she prepares a reflective environment with specific purposes, promoting learning, when concluding the activity and validating the work performed by students throughout the entire time of carrying out MM activity.In producing the model for the problem addressed, the students' initial mathematical repertoire was considered, and this enabled learning mathematical content and development of critical thinking on the theme studied.

It is worth noting that the actions that mobilized the three categories provided a response to the general objective of the research previously presented, with an emphasis on teacher mediation, which is crucial for learning and development of students' higher psychological functions.With the aim of organizing teaching, it can be inferred that TGA was present in all activities. In activities where the problems were proposed by students, the theme arose from collective interest, as everyone was motivated to find answers to the problem. When the problem was suggested by teacher, the objective was to familiarize students so that MM activity could be consolidated.

Mediation did not occur directly, but through signs and pedagogical resources, which when considered in the school context considering Historical-Cultural Theory,

highlight the teacher's mediation.It is up to him/her to organize his/her pedagogical practice, using these mediating resources with specific objectives and purposes.The exchange of ideas and opinions mediated between teacher and student enhanced the implementation and consolidation of MM activities.

MM can offer and enhance mathematicsteaching, contributing to understanding new mathematical concepts. In other words, students become more human by appropriating and internalizing the processes of social significance, giving new meaning to what they learn. This promotes development of critical thinking and higher psychological functions through an organized process, as proposed by TGA, with a view to learning and humanizing subjects.

There is no intention of putting an end to the matter, as MM in the early years of Elementary School is a vast field.However, it is expected that this research can contribute to development of other investigations on teacher mediation, considering teacher-student relationships, pedagogical resources as materialization of instruments and TGA in MM.

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Text received on 12/20/2023

Text revised on 05/20/2025

Text approvad on 07/03/2025

Text published on 09/24//2025