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Brazilian GDP as an efficient proxy for the return of market portfolios in the CAPM

PIB brasileiro como carteira de mercado eficiente no modelo CAPM

PIB brasileño como cartera de mercado eficiente en el modelo CAPM

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Abstract
In the financial market, there are often periods of divergence between macroeconomic data (GDP, unemployment) and stock market performance, especially as reflected by stock indexes, which often do not faithfully portray the national economic scenario. The objective of this study was to test the efficiency, in the "mean-variance" sense, of portfolios producing returns that track GDP as proxies for the market portfolio in the CAPM. We compared them with the efficiency of the main Brazilian stock market index, the Ibovespa, for this purpose. To measure the efficiency, multivariate regression analysis was used in a sample of 148 firms, over a period of 10 years, 2009 to 2018. The results showed that none of the portfolios mentioned, along with the Ibovespa, were representative of the Brazilian market according to the CAPM. However, despite not meeting the stipulated efficiency conditions, the Ibovespa was the most suitable measure of the market portfolio.
Keywords: Efficient Market Portfolio; GDP; CAPM

Resumo
No mercado financeiro, nota-se a existência de períodos em que há divergência entre os dados macroeconômicos (PIB, desemprego) e os dados do mercado acionário, principalmente os índices de ações. Questionando se o Ibovespa, retratam fidedignamente o cenário econômico nacional. O objetivo desta pesquisa foi testar a eficiência no sentido 'média-variância', de carteiras inspiradas na composição do PIB, como *proxies* para a carteira de mercado no modelo CAPM. Comparando-as com a eficiência do índice Bovespa para o mesmo objetivo. Para isto foi utilizada a análise de regressão multivariada em uma amostra de 148 empresas, num período de 10 anos, 2009 a 2018. Os resultados mostraram que nenhuma das carteiras apontadas, tanto quanto o índice Bovespa, são eficientes como representante do mercado brasileiro segundo o CAPM. No entanto, apesar de não cumprir as condições de eficiência estipuladas, o Ibovespa se apresentou como a medida mais indicada para a carteira de mercado.
Palavras-chave: Carteira de mercado eficiente; PIB; CAPM

Resumen
En el mercado financiero, a menudo hay períodos de divergencia entre los datos macroeconómicos (PIB, desempleo) y el desempeño del mercado de valores, especialmente como se refleja en los índices bursátiles, que a menudo no reflejan fielmente el escenario económico nacional. El objetivo de este estudio fue probar la eficiencia, en el sentido de "varianza media", de carteras que producen retornos que rastrean el PIB como sustitutos de la cartera de mercado en el CAPM. Los comparamos con la eficiencia del principal índice bursátil brasileño, el Ibovespa, para este propósito. Para medir la eficiencia, se utilizó un análisis de regresión multivariante en una muestra de 148 empresas, en un período de 10 años, de 2009 a 2018. Los resultados mostraron que ninguna de las carteras mencionadas, junto con el Ibovespa, eran representativas del mercado brasileño según al CAPM. Sin embargo, a pesar de no cumplir con las condiciones de eficiencia estipuladas, el Ibovespa fue la medida más adecuada del portafolio de mercado.
Palabras clave: Cartera de mercado eficiente; PIB; CAPM

1 Introduction

In 2019, the Brazilian stock market closed the year at the highest level in the history of its main index, the ibovespa, at 117,802 points. That fact could at first glance be taken as indication of an economy growing by leaps and bounds. However, the macroeconomic data showed otherwise. The main economic metric, gross domestic product (GDP), was negative from 2014 to 2016, followed by weak growth through 2019 (Figure 1).

With the resumed albeit modest GDP growth in 2017, the ibovespa broke its previous record of 85,365 points, and generally continued to rise thereafter, despite the mediocre GDP performance described above.

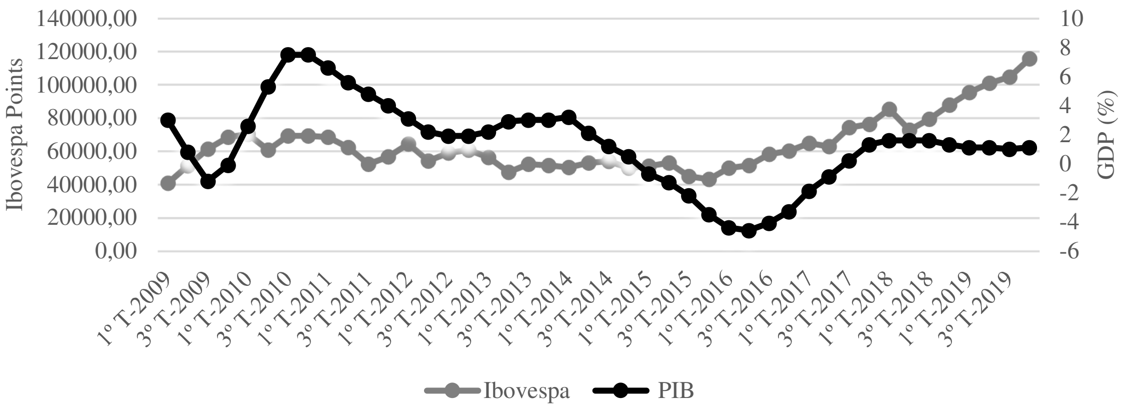

Figure 1: Ibovespa x GDP
Source: Comdinheiro and Ipeadata

Researchers such as Fama (1981, p.545), Kaul (1987, p. 254) and Barro (1990, p. 130) have already addressed the issue of the relationship between macroeconomic variables and the stock market indexes, indicating that the equity market returns can presage the performance of economic variables, such as the level of investments, since the stock market is the main source of financing of firms. Therefore, an increase of equity funding should cause an increase in the level of investment, and thus raise industrial output, as reflected in GDP.

The same relation can be expected to operate in the opposite direction, whereby an increase of GDP would lead to higher corporate profits, and consequently higher stock prices. With respect to causality between the two variables, various Brazilian researchers, such as Nunes and Silva (2005), Silva and Coronel (2012), Bernardelli and Bernardelli (2016), Ribeiro, Leite and Justo (2016), Machado et al. (2018), Bernadelli and Castro (2020), have found significant causality from GDP to the ibovespa. Other authors, however, have found causality in the opposite direction (positive or negative) in certain periods, among them Pereira, Araújo and Iquiapaza (2020), who reported strong cointegration between the variables, with high forecasting power of GDP on the ibovespa.

The same results have been reported in studies of foreign markets, such as Fama (1990, p. 1107), Schewert (1989, p. 1256), Asprem (1989, p. 607), Chiang and Doong (1999) and Pilinkus (2009, p. 889). But this relationship is by no means unanimous in the academic community. For instance, some Brazilian authors, such as Nunes, da Costa Jr. and Seabra (2003), Nunes, da Costa Jr. and Meure (2005), Groppo (2004, p. 75), Silva Jr., Menezes and Fernandes (2011), Monteiro et al. (2013), and Carvalho and Sekunda (2020) have found no significant relation between the ibovespa and GDP, so that the stock market is not a good forecaster of the level of domestic output, and vice versa. The same results were found previously in foreign markets by Kwon and Shin (1999), Omran (2003, p.372), Dufour and Tessier (2006) and Gan, Young and Zhang (2006).

Silva Jr., Menezes and Fernandes (2011) stressed that in emerging economies, the causality behavior differs from that in developed economies, observing that stock market variables have a closer relationship with productivity. In counterpoint, the supposition is that the stock market performance anticipates changes in the future cash flows of firms, and hence the future levels of economic activity.

For Brazil, another point to be analyzed within the problem described is the composition of the ibovespa. It reflects the performance of a theoretical portfolio of 76 stocks. In the period from 2009 to 2018, only 21 firms were among the 10 with the greatest trading volume. Of this group, eight firms (Bradesco, ItaúUnibanco, Petrobras, Vale, [B]³, Itaúsa. Banco do Brasil and Ambev) were in the top ten in at least half of the years, and three had that status in all years (Bradesco, ItaúUnibanco, Petrobras). The same would have been true of Vale, but the tragic tailings dam breach in Mariana at the end of 2015 drastically reduced trading of the company's shares. Of the set of eight firms, it is important to mention the cases of Petrobras, Vale and ItaúUnibanco, which represented 22.784%, 17.431% and 11.616% of the ibovespa, respectively. Besides this, other firms' shares were only among the 10 most traded in specific cases, impelled by relevant occurrences.

Figure 2 shows the variation in the participation of the group of 10 stocks most traded in the ibovespa, representing a minimum of 40.769% and maximum of 61.901% of the index in the period between 2009 and 2018, or an average of 51.707%.

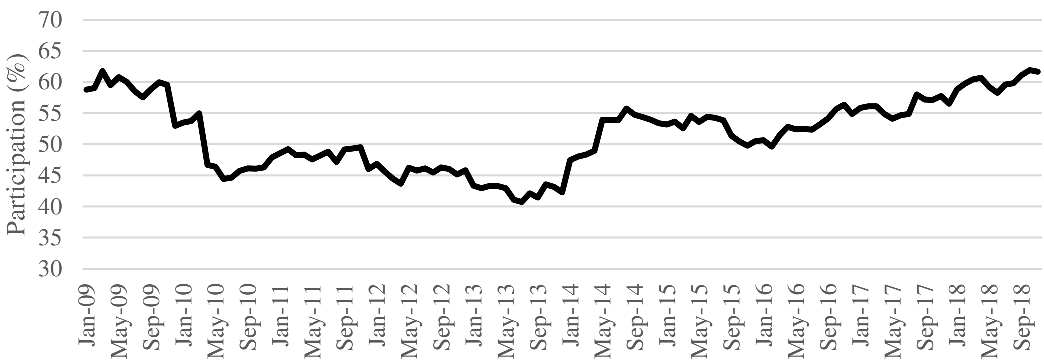

Figure 2: Sum of the weights of the 10 most traded stocks of the ibovespa
Source: research data

Based on these observations and seeking to shed light on the uncertainty about the relationship between GDP and the ibovespa, in this study we focus on the following question: Can the ibovespa be employed as an efficient market proxy for companies in the Brazilian market? To respond to that question, we test the efficiency of capital market portfolios, adopting the composition of the GDP as proxies for the market portfolio of the CAPM, comparing them with the efficiency of the ibovespa.

The importance of this study is its investigation of the gap due to high concentration of the ibovespa, which in turn is an indicator of the average performance of the stocks with greatest trading volume and representation in the Brazilian Stock Exchange, [B]³, adopting the composition of the index at the end of each month of the 10 stocks with greatest weight in forming the index. In this respect, we use the capital asset pricing model (CAPM), which has the basic proposition of denoting a portfolio of assets that efficiently represents the market to which the model is applied (known as the "efficient market portfolio"), with the purpose of representing the return of all the assets in the market while maximizing the expected returns.

Finally, we propose three market portfolios, basing their formation on the sectors composing GDP, and divided into: (i) trading volume (GDP-volume); (ii) presence (GDP-presence); and (iii) equal weighting (GDP-number), with the aim of testing their ex-ante efficiency by comparing them with the efficiency of the ibovespa. Therefore, this study differs from that of Araujo, Farjado and Di Tavani (2006), since they used GDP to denote the dividend paid by a hypothetical portfolio, but without presenting an efficient market portfolio for the CAPM.

2 Theoretical framework

Since 1952, the modern theory of portfolios of Markowitz has been used as a pillar of asset pricing models. The main and most widely used asset pricing model was and still is the capital asset pricing model (CAPM), as developed and improved by Sharpe (1964, p. 425) and Lintner (1965, p. 587).

To estimate the CAPM, it is necessary to estimate a market portfolio that covers all the assets of the market analyzed. However, Roll (1977, p. 130) stated that "[t]he theory is not testable unless the exact composition of the true market portfolio is known and used in the tests. This implies that the theory is not testable unless *all* individual assets are included in the sample." He further listed two other problems regarding the efficiency of the market portfolio. First, the proxy can have mean-variance efficiency even if the true market portfolio does not. Second, the most reasonable proxies will be highly correlated with each other and with the true market, whether or not they are mean-variance efficient.

In counterpart, Guermat (2014, p.27) showed that the CAPM is really applicable. He based his study on Kandel and Stambaugh (1995) and Kan, Robotti and Shanken (2013), using estimation by ordinary least squares (OLS) and generalized least squares (GLS), and by assuming the absence of hidden variables, seeking the condition to maximize R^2 in the CAPM. This was only possible when the market portfolio was efficient and there were no omitted variables.

In turn, Pereira and Laurini (2020) applied a different approach to test the efficient portfolio of the CAPM, by comparing the applicability of the estimators of the generalized method of moments (GMM) and generalized empirical likelihood (GEL) method. They identified a change in the results according to the model applied, whether or not the sample was finite. They found the infinite samples to be most nearly efficient, while the efficiency of the finite samples was "super-rejected".

Consequently, it is reasonable to believe that the applicability of the CAPM is subject to the premises of the user and the features of the market to which it is applied. For financial market participants, a solution for this limitation of scope of the market portfolio is to use the index with greatest representation of the market, which in Brazil is the ibovespa.

Therefore, bringing this discussion to the Brazilian scenario, researchers have investigated the efficiency of the ibovespa as an efficient market portfolio. Volpe (2010, p.52) compared the ibovespa, IBrX (Brazil Index), FGV-100 (index computed by Fundação Getulio Vargas), a market-value-weighted portfolio and an equally weighted portfolio, the latter two proposed by the authors. They found that in the period from 1996 to 2009, all the indexes obtained results very close to each other and were not efficient. Hagler and Brito (2007) tested the efficiency of the same indexes, through the Sharpe-Lintner and Black versions of the CAPM in the period from 1989 to 2003, also finding that none of the indexes were efficient. In turn, Silva and Motta (2002) applied the Black version of the CAPM between 1986 and 2001 and found that the ibovespa was not efficient over the entire period, but was efficient in shorter windows. Araújo, Fajardo and Di Tavani (2006) applied the method of Hou (2002, p. 3) to compare a hypothetical portfolio having return equal to GDP with the ibovespa, employing the Sharpe-Lintner and Black versions of the CAPM between 1991 and 2002, and concluded that the hypothetical portfolio was not efficient.

2.1 Gross domestic product (GDP)

As defined by the Brazilian Institute of Geography and Statistics (IBGE), GDP is a measure of economic growth that represents the sum of all the wealth produced in a region (city, state, country) in a determined time frame. Here we use GDP from the supply perspective. This method is based on determining the value added by each productive unit, calculated as the difference between the value of sales and the value of purchases necessary to attain the output.

Table 1:
Composition of GDP from the supply perspective

YEAR		2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
AGRIBUSINESS		149,213	159,932	190.024	200,695	240,29	249,975	258,967	306,655	303,751	297,77
INDUSTRY	**Extraction**	62,705	109,965	162,567	186,191	189,434	184,797	110,773	55,559	94,398	174,155
	Manufacturing	435,277	494,352	515,704	514,021	558,733	597,376	630,813	675,816	690,731	659,664
	Utilities	76,616	92,914	99,219	100,233	92,818	93,975	123,183	143,698	148,519	165,437
	Construction	154,624	206,927	233,544	265,237	290,641	306,946	296,018	275,134	269,193	259,944
SERVICES	**Commerce**	361,914	416,229	478,242	548,339	614,087	676,562	685,708	699,15	715,714	769,365
	Transport	109,403	141,66	165,557	183,063	203,421	227,757	226,5	235,851	250,823	256,085
	Information	122,173	126,542	137,006	148,451	157,509	168,61	176,004	178,982	183,944	187,025
	Finance	187,593	224,561	239,426	260,312	272,571	318,68	365,276	425,476	425,238	405,416
	Real Estate	247,217	274,42	311,38	358,947	419,202	463,781	498,884	526,995	551,972	577,205
	Others	455,85	517,493	589,733	676,669	768,867	867,467	897,888	949,262	1,020,721	1,063,107
	Public Admin.	487,179	537,845	598,059	652,101	746,187	816,808	885,587	945,121	985,873	1,017,943

Source: IBGE

As can be seen in Table 1, of the group of 21 firms that compose up to 60% of the ibovespa, the eight most relevant (Bradesco, Itaú Unibanco, Petrobras, Vale, [B][3], Itaúsa. Banco do Brasil and Ambev) come from only 3 of the 12 sectors of GDP, which are mineral extraction, manufacturing and finance. These groups together represented around 26% of all the wealth created in the country during the period studied, meaning that approximately 74% of the Brazilian economy is not represented in the CAPM when using the ibovespa as a market proxy. In particular, the agribusiness and service sectors should be included since they have relevant concentration in the ibovespa.

2.2 Sharpe-Lintner version of the CAPM

In this study, we selected the Sharpe-Lintner version of the CAPM, because it includes borrowing and lending at the risk-free rate, which is the same for all investors. The Brazilian economy has an asset class that yields the risk-free rate: government bonds called Treasury Financial Notes (Letras Financeiras do Tesouro, LFTs), whose yields are tied to the benchmark interest rate, the Selic rate. In this sense, the Selic rate can be defined as the risk-free rate, since any excess return requires investing in riskier assets. Besides this, their risk depends on the government's capacity to pay, which is considered to have zero risk.

Fama and French (2007) stressed that tests of the CAPM are based on three implications of the relationship between expected return and the market beta implicit in the model. First, the expected returns of all assets are linearly related to their market betas, and no other variable has marginal explanatory power. Second, the risk premium is positive, meaning that the expected return of the market portfolio exceeds that of assets whose returns are correlated with the market return. And third, in the Sharpe-Lintner version of the model, the assets not correlated with the market have expected returns equal to the risk-free interest rate, and the premium of the beta is the expected market return minus the risk-free rate. Therefore, based on these observations, and assuming the existence of N risky assets, we have the representation denoted by Equation 1.

$$E(R_i) = E(R_f) + [E(R_m) - E(R_f)]\beta_{im} \qquad \forall\, i = 1, \dots, N \tag{1}$$

Where: $E(R_i)$ = expected return of asset i; $E(R_f)$ = expected return of the risk-free asset; $E(R_m)$ = expected return of the market; $[E(R_m) - E(R_f)]\beta_{im}$ = risk premium; and, β_{im} = market beta of asset i.
Therefore, as described by Campbell, Lo and MacKinlay (1997), we assume the existence of N risky assets with return R_{it}, such that $i = 1, \dots, N$, and one risk-free asset with return R_{ft} for each period. Equation 1 can be rewritten in matrix form as Equation 2. With $Z_i = [E(R_i) - E(R_f)]$ e $Z_m = [E(R_m) - E(R_f)]$.

$$Z_t = \alpha + Z_{mt}\beta + \epsilon_t \tag{2}$$

Where: Z_t = an $(Nx1)$ vector of excess returns of assets i in relation to the risk-free asset in period t; Z_{mt} = the excess return of portfolio m in relation to the risk-free asset in period t; β = the $(Nx1)$ vector of betas; and ϵ_t = the $(Nx1)$ vector of the error term of asset i in period t, independent and identically distributed (iid) over time.

If a given portfolio m is efficient in the Sharpe-Lintner version, then the condition can be represented according to Equation 3.

$$E(Z_t) = \beta \cdot E(Z_m) \tag{3}$$

Satisfaction of that condition implies the restriction of the intercepts as null, without statistical significance for the model, presented under the null hypotheses, as shown in Equation 4.

$$H_0: \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = \cdots = \alpha_n = 0 \quad \text{and} \quad H_1: \text{qualquer } \alpha_i \neq 0 \tag{4}$$

Hypothesis H_0 is violated if and only if some linear combination of the intercepts α is different from zero, i.e., if and only if some portfolio of N risky assets has an intercept different than zero when regressing their excess returns on the excess returns of portfolio m.

For measurement of the unrestricted estimators, Campbell, Lo and MacKinlay (1997) described four efficiency test statistics, called by them (and here) J_0, J_1, J_2 and J_3. Thus, a Wald test of the null hypothesis can be formulated (4), versus the alterative hypothesis $H_a: \alpha \neq 0$. The test statistic J_0 of the Wald test is defined according to Equations 5, 6 and 7.

$$J_0 = \hat{\alpha}'[\mathrm{Var}(\hat{\alpha})]^{-1}\hat{\alpha} \tag{5}$$

$$J_0 = T\left[1 + \frac{\hat{\mu}_m{}^2}{\hat{\sigma}_m{}^2}\right]\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \sim \chi^2_N \tag{6}$$

$$J_1 = \frac{(T - N - 1)}{N}\left[1 + \frac{\hat{\mu}_m{}^2}{\hat{\sigma}_m{}^2}\right]\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} \sim F(N, T - N - 1) \tag{7}$$

Where: J_0 and J_1 refer to the unrestricted model. In this sense, when the restriction $\alpha = 0$ is inserted, the new estimators α and β result in Equations 8 and 9.

$$\hat{\beta}^* = \frac{\sum_{t=1}^{T} Z_t Z_{mt}}{\sum_{t=1}^{T} Z_{mt}^2} \sim N\left(\beta, \frac{1}{T}\left[\frac{1}{\hat{\mu}_m{}^2 + \hat{\sigma}_m{}^2}\right]\Sigma\right) \tag{8}$$

$$\hat{\Sigma}^* = \frac{1}{T}\sum_{t-1}^{T}(Z_t - \hat{\beta}^* Z_{mt})(Z_t - \hat{\beta}^* Z_{mt})' \therefore T\hat{\Sigma} \sim W_N(T-1, \Sigma) \tag{9}$$

When knowing the unrestricted and restricted estimators, one can use the likelihood ratio tests for the restriction established previously. The test is based on the asymptotic result that under the null hypothesis, -2 times the logarithm of the likelihood ratio has chi-square distribution with degrees of freedom equal to the number of restrictions under H_0, according to Equation 10. Thus, it is possible to test H_0 using Equation 11.

$$J_2 = -2LR \tag{10}$$

$$J_2 = T\log\left(\left[\frac{\hat{\sigma}_m{}^2}{\hat{\mu}_m{}^2 + \hat{\sigma}_m{}^2}\right]\hat{\alpha}'\hat{\Sigma}^{-1}\hat{\alpha} + 1\right) \sim \chi^2_N \tag{11}$$

With the adjustment for finite samples suggested by Jobson and Korkie (1982), one obtains Equation 12.

$$J_3 = \left(T - \frac{N}{2} - 2\right)\left[\log|\hat{\Sigma}^*| - \log|\hat{\Sigma}|\right] \sim \chi^2_N \quad (12)$$

It is important to stress that the test statistics J_1 and J_3 are similar to the test statistics J_0 and J_2. However, J_1 and J_3 are adjusted for finite samples. Therefore, after applying the test statistics J_0, J_1, J_2 and J_3, the test of the efficiency of the market portfolio must be performed separately. The mean-variance efficiency can be tested with a t-test via the Sharp ratio, according to Equation 15.

$$SR_m = \frac{\mu_m - R_f}{\sigma_m} \therefore H_0: Z_m = 0 \quad (13)$$

According to Campbell, Lo and MacKinlay (1997), the excess expected return per unit risk is useful to supply a base for economic interpretation of the tests of the CAPM. The Sharpe ratio measures this quantity. For any asset or portfolio a, the Sharpe ratio is defined as the average excess return divided by the standard deviation of the return.

Testing the mean-variance efficiency of a determined portfolio is equivalent to testing whether the Sharpe ratio of that portfolio is the maximum of the set of Sharpe ratios of all possible portfolios.

In light of the previous discussion, in this study we tested the following hypotheses: H_1: a hypothetical market portfolio based on the composition of GDP, formed by companies from each sector, by itself explains the return of a portfolio of assets; and H_2: a hypothetical market portfolio based on the composition of GDP, formed by companies from each sector, is mean-variance efficient.

3 Methodology

3.1 Research procedures and characteristics of the sample

All the monthly stock prices were obtained from the databases of [B][3] and Comdinheiro. We selected the first closing prices available, with tolerance of five business days. The data for the macroeconomic variables were obtained from the databases of the IBGE and Brazilian Central Bank. Finally, the firms selected to represent the assets in the portfolios were chosen at random, from a total of 70 stocks separated into 10 portfolios according to size of the beta of each one.

All told, we collected data on 148 companies, for a total of 17,244 observations. It was necessary to accept companies with up to 15% missing data so that no sector would be without a representative. The period analyzed was from January 1, 2009 until December 31, 2018, as shown in Table 2. Subsequently, we divided this period into 30-month sub-periods to enable evaluation of the efficiency of the portfolios in the short run. The window was chosen to cover a period of greater stability of the Brazilian economy, with more stable inflation and better monetary equilibrium.

Table 2:
Periods analyzed

Period	Nomenclature	T
01/2009 – 12/2018	Period 0	120
01/2009 – 06/2011	Period 1	30
07/2011 – 12/2013	Period 2	30
01/2014 – 06/2016	Period 3	30
07/2016 – 12/2018	Period 4	30

Source: research data

For each asset in the sample, the compound return was calculated through the natural logarithm. These returns were continuous and were represented by Equation 14.

$$R_{it} = \ln(P_{it}/P_{it-1}) \quad (14)$$

Where: P_{it} = the price of asset i in month t; and P_{it-1} = the price of asset i in month $t-1$.

The risk-free asset established as the parameter to construct the series of excess returns was the monthly overnight Selic rate, obtained from Ipeadata and the Brazilian Central Bank (series 4390). The quarterly series for composition of GDP was the real value from the perspective of supply, obtained from the IBGE in its Quarterly National Accounts System (SCNT). We selected the portfolios of assets based on the study of Gibbons, Ross and Shanken (1989), who tested the efficiency of the zero-beta CAPM by dividing the assets into portfolios according to three criteria: i) 10 portfolios in function of the beta of each asset in relation to the index being tested; ii) 12 portfolios by industrial sector; and iii) 10 portfolios in function of the firms' size.

However, due to the smaller dimension of the Brazilian market in relation to its American counterpart, we selected the portfolios with a smaller number of firms. These portfolios were determined only by the size of their betas. The betas were calculated according to the ratio between the covariance of the asset and variance of the market portfolio. The market portfolio chosen was the ibovespa. The portfolios to be tested against the proposed portfolios were also chosen according to the size of their betas versus the ibovespa. That choice did not influence the efficiency tests, since any portfolio of assets that was checked against the market portfolios needs not to reject the null hypotheses $H_0: \alpha = 0$.

3.2 Formation of the market portfolios

The weight of each of the 12 market segments for assembling the portfolios was established by the arithmetic mean of the composition of GDP in the period analyzed. This method was chosen because of the small dispersion of the variables around the means, as can be seen in Table 3.

Table 3:
Descriptive statistics of gross domestic product (GDP)

Sectors	$\bar{X}$	σ	*M*	*Min*	*Max*	*Assym.*	*Kurt.*
Agribusiness	44.144	0.2273	43.517	41.158	4.893	0.6629	-0.498
Mineral Ext.	25.769	10.289	26.903	0.8865	38.671	-0.2035	-15.848
Manufacturing	110.563	10.997	106.076	96.617	130.595	0.7069	-10.568
Construction	21.442	0.2703	22.666	16.262	24.231	-0.8013	-0.9427
Utilities	48.814	0.6113	51.243	38.073	55.088	-0.5001	-14.762
Industry	206.588	19.444	208.486	183.528	23.268	-0.0007	-1.793
Commerce	111.893	0.3271	112.121	107.114	117.073	0.044	-15.759
Transport	37.388	0.1763	37.803	32.824	39.411	-15.701	16.834
Information	30.345	0.2708	29.449	27.393	36.655	10.921	0.2118
Finance	58.219	0.5163	57.036	51.123	67.889	0.5183	-1.049
Real Estate	78.543	0.5539	7.944	7.062	8.454	-0.2193	-17.987
Others	145.222	0.8519	146.981	133.174	155.744	-0.1348	-1.756
Public Admin.	143.599	0.589	143.754	135.438	150.804	-0.0656	-1.854
Services	605.209	22.331	604.649	575.682	631.994	-0.0696	-18.476

Source: research data

The Industry and Service sectors represent the sum of the participations of their subsectors. Therefore, we only used the participations of the subsectors to form the weights of the stocks per sector. Also, Agribusiness did not have any subdivision.

The 148 firms were allocated among the segments manually, since the sectors forming GDP are different than those of the [B][3]. Therefore, we separated the companies according to their main business activity regarding generation of value. The public administration subsector was removed because it has no listed companies.

The portfolios were separated according to three characteristics, trading volume, presence on exchanges, and equal weighting, with the same weight of all the assets. The portfolios were thus designated as GDP-volume, GDP-presence and GDP-number.

3.3 Formation of the GDP-volume portfolio

To compose the first portfolio, called GDP-volume, we selected 68 companies of the segments present in composing GDP, where each company's weighting percentage was determined by dividing the number of firms in the sector by the percentage of each in the composition of GDP. Therefore, the return of the market portfolio was calculated according to the modern portfolio theory of Markowitz (1952), as denoted by Equation 15.

$$R_m = \sum_{i=1}^{N} X_{sector} R_{sector} \tag{15}$$

Where: R_{sector} = return of the sector; X_{sector} = weight of the sector in the portfolio; $X_{i,sector}$ = weight of the firm within its sector of participation; and R_i = return of the firm.

The criteria for selection of the firms of each sector were similar to those for composing the ibovespa, but these could not applied strictly because the portfolio presented a high number of firms and sectors that often did not figure in the group of firms with highest trading volume. We chose firms with the largest trading volume, with each group having at least one firm, as identified in Table 4.

Table 4:
Composition of the GDP-volume market portfolio

Classification		Number of assets	Composition of the portfolio
Agribusiness		4	BRFS3 / JBSS3 / MRFG3 / BEEF3
Industry	Mineral ext.	5	PETR4 / VALE3 / PETR3 / OGXP3 / MMXM3
Industry	Manufacturing	12	GGBR4 /ABEV3 / USIM5 / CSNA3 / GOAU4 / EMBR3 / CSAN3 / WEGE3 / KLBN4 / RAPT4 / MYPK3 / MDIA3
Industry	Construction	4	CYRE3 / MRVE3 / EVEN3 / EZTC3
Industry	Utilities	12	CCRO3 / CMIG4 / VIVT4 / BRKM5/ SBSP3 / ENBR3 / EQTL3 / CESP6 / CPFE3 / CPLE6 / LIGT3 / TRPL4
Services	Commerce	8	LREN3 / LAME4 / PCAR4 / NATU3 / RENT3 / RADL3 / HGTX3 / BTOW3
Services	Trans., storage and delivery	2	GOLL4 / POMO4
Services	Information	4	CIEL3 / ESTC3 / TOTS3 / VLID3
Services	Finance and insurance	8	ITUB4 / BBDC4 / BBAS3 / ITSA4 / BBDC3 / BRSR6 / SULA11 / PSSA3
Services	Others	3	BRAP4 / OIBR4 / ODPV3
Services	Real Estates	6	BRML3 / PDGR3 / GFSA3 / MULT3 / RSID3 / IGTA3

Source: research data

3.4 Formation of the GDP-presence portfolio

The formation of the second proposed portfolio, called GDP-presence, was based on the same criteria for formation of the GDP-volume portfolio, except that here we chose firms with greatest level of presence in the trading sessions of the [B][3], within the sectors and the period studied, as shown in Table 4.

Table 5:
Composition of the GDP-presence market portfolio

Classification		Number of assets	Composition of the portfolio
Agribusiness		5	BRFS3 / JBSS3 / MRFG3 / BEEF3 / SLCE3
Industry	Mineral ext.	4	PETR4 / VALE3 / PETR3 / OGXP3
Industry	Manufacturing	11	GGBR4 /ABEV3 / USIM5 / CSNA3 / GOAU4 / EMBR3 / CSAN3 / WEGE3 / KLBN4 / MYPK3 / PMAM3
Industry	Construction	4	CYRE3 / MRVE3 / EVEN3 / EZTC3
Industry	Utilities	12	CCRO3 / CMIG4 / VIVT4 / BRKM5/ SBSP3 / ENBR3 / EQTL3 / CESP6 / CPFE3 / CPLE6 / LIGT3 / TRPL4
Services	Commerce	8	LREN3 / LAME4 / PCAR4 / NATU3 / RENT3 / LAME3 / GRND3 / BTOW3
Services	Trans., storage and delivery	3	GOLL4 / POMO4 / TGMA3
Services	Information	2	TOTS3 / VLID3
Services	Finance and insurance	9	ITUB3 / BBDC4 / BBAS3 / ITSA4 / BBDC3 / BRSR6 / SULA11 / PSSA3 / ABCB3
Services	Others	3	BRAP4 / OIBR4 / ODPV3
Services	Real estate	7	BRML3 / PDGR3 / GFSA3 / MULT3 / RSID3 / IGTA3 / TCSA3

Source: research data

3.5 Formation of the GDP-number portfolio

This portfolio had as criterion an equal number of firms among the sectors. Hence, this portfolio contained 55 assets, with 5 companies per segment of GDP, as presented in Table 6

Table 6:
Composition of the GDP-number market portfolio

Classification		Number of assets	Composition of the portfolio
Agribusiness		5	BRFS3 / JBSS3 / MRFG3 / BEEF3 / SLCE3
Industry	Mineral ext.	5	PETR4 / VALE3 / PETR3 / OGXP3 / MMXM3
Industry	Manufacturing	5	GGBR4 /ABEV3 / USIM5 / CSNA3 / EMBR3
Industry	Construction	5	CYRE3 / MRVE3 / EVEN3 / EZTC3 / HBOR3
Industry	Utilities	5	CCRO3 / CMIG4 / VIVT4 / BRKM5/ SBSP3
Services	Commerce	5	LREN3 / LAME4 / PCAR4 / NATU3 / RENT3
Services	Trans., storage and delivery	5	GOLL4 / POMO4 / TGMA3 / KEPL3 / LOGN3
Services	Information	5	TOTS3 / VLID3 / CARD3 / ESTC3 / CIEL3
Services	Finance and insurance	5	ITUB3 / BBDC4 / BBAS3 / ITSA4 / BBDC3
Services	Others	5	BRAP4 / OIBR4 / ODPV3 / UNIP6 / BBRK3
Services	Real estate	5	BRML3 / PDGR3 / GFSA3 / MULT3 / RSID3

Source: research data

4 Results

Figure 3 depicts the evolution of the returns over time of the market portfolios tested. In general, the portfolios had similar movement. However, the movement of the market portfolios based on the composition of GDP was smoother. This behavior can be explained by the diversification of the assets; the high concentration of the ibovespa caused it to be more volatile.

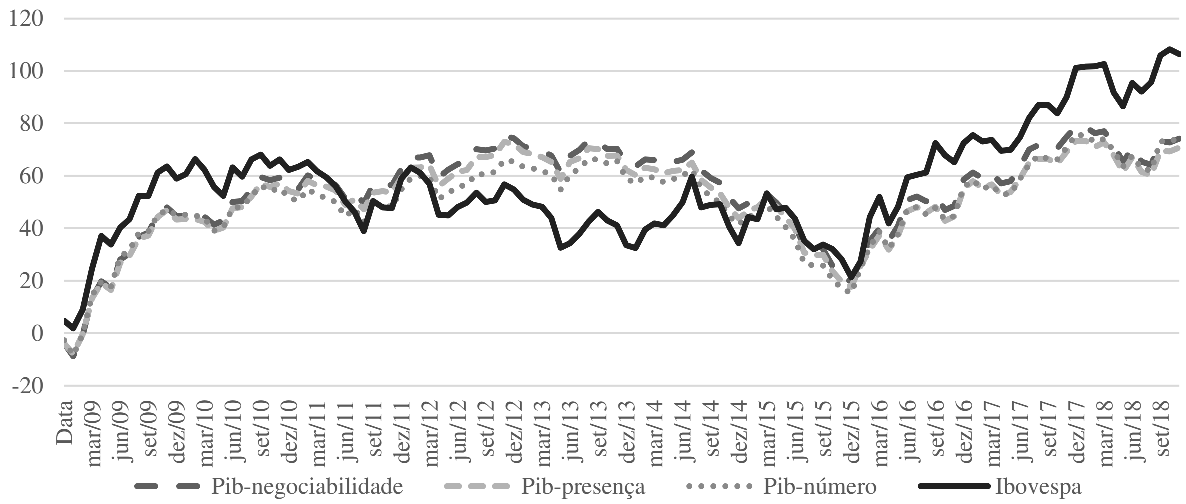

Figure 3: Percentage return of the market portfolios over time
Source: research data

In quantitative terms, Table 7 reports the descriptive statistics of the portfolios. Besides this, the correlation of the portfolios clearly indicates the tradeoff between their risk and return. This is in line with the theory of Markowitz (1952).

Table 7:
Descriptive statistics of the market portfolios

	$\bar{X}$	*σ*	*M*	*Min*	*Max*	*Assim*	*Curt.*	*ρ GDP-vol*	*ρ GDP-pres*	*ρ GDP-num*	*ρ Ibov*
GDP-vol	0.6175	43.509	0.2466	-84.211	139.549	0.3628	-0.1003	10.000	0.9963	0.9854	0.8339
GDP-pres	0.5889	42.963	0.2189	-89.626	136.958	0.3398	-0.1248	0.9963	10.000	0.9851	0.8324
GDP-num	0.6232	44.404	-0.0462	-90.502	149.397	0.4883	0.2672	0.9854	0.9851	10.000	0.8396
Ibov.	0.8864	59.957	0.5650	-118.600	169.700	0.2485	-0.3314	0.8339	0.8324	0.8396	10.000

Source: research data

In parallel, we analyzed beta portfolios, designated as such because their selection of assets occurred according to the size of the beta of each asset in the portfolio. For this purpose, we separated 70 stocks into 10 groups, in decreasing order of the betas. Tables 9 and 10 reveal the versatility of the portfolios, demonstrated by their descriptive statistics and pairwise correlations. Their standard deviations range from 3.98 to 8.12, the returns vary by 41.49 percentage points, and the correlation coefficients are decreasing. That matrix arrangement generated a large number of possibilities for testing, creating a proportionally greater number and higher volatility of results, bringing many benefits for theoretical analysis due to the closer reflection of the real world.

Table 8:
Descriptive statistics of the beta portfolios

	Beta 1	*Beta 2*	*Beta 3*	*Beta 4*	*Beta 5*	*Beta 6*	*Beta 7*	*Beta 8*	*Beta 9*	*Beta 10*
$\bar{X}$	1.15	1.55	1.38	1.43	1.25	1.77	1.54	0.82	1.47	1.25
σ	8.08	8.12	6.73	6.79	6.31	5.94	5.03	5.07	4.94	3.98
M	0.90	1.13	1.45	1.40	1.64	2.01	1.22	0.28	1.04	0.75
Min	-14.33	-14.53	-13.04	-14.88	-14.45	-12.27	-10.58	-12.47	-11.54	-8.19
Max	24.09	26.61	17.08	21.70	16.54	16.66	14.57	17.94	17.78	11.32
Assim	0.51	0.44	-0.11	0.33	0.01	0.07	0.15	0.34	0.28	0.23
Kurt.	0.03	0.31	-0.30	0.35	-0.32	-0.64	-0.21	0.35	0.36	0.06

Source: research data

Table 9:
Correlation matrix of the beta portfolios

	Beta 1	*Beta 2*	*Beta 3*	*Beta 4*	*Beta 5*	*Beta 6*	*Beta 7*	*Beta 8*	*Beta 9*	*Beta 10*
Beta 1	1.0000	0.8287	0.8062	0.8030	0.7910	0.7147	0.7633	0.6455	0.4885	0.4370
Beta 2	0.8287	1.0000	0.7816	0.7123	0.7559	0.6422	0.7416	0.6106	0.4750	0.4865
Beta 3	0.8062	0.7816	1.0000	0.7076	0.8092	0.6510	0.7562	0.5860	0.5482	0.5715
Beta 4	0.8030	0.7123	0.7076	1.0000	0.7262	0.7025	0.6984	0.6278	0.3963	0.4484
Beta 5	0.7910	0.7559	0.8092	0.7262	1.0000	0.7195	0.8021	0.6269	0.6608	0.5420
Beta 6	0.7147	0.6422	0.6510	0.7025	0.7195	1.0000	0.6501	0.6731	0.4534	0.5182
Beta 7	0.7633	0.7416	0.7562	0.6984	0.8021	0.6501	1.0000	0.6053	0.5590	0.5714
Beta 8	0.6455	0.6106	0.5860	0.6278	0.6269	0.6731	0.6053	1.0000	0.4345	0.4343
Beta 9	0.4885	0.4750	0.5482	0.3963	0.6608	0.4534	0.5590	0.4345	1.0000	0.4393
Beta 10	0.4370	0.4865	0.5715	0.4484	0.5420	0.5182	0.5714	0.4343	0.4393	1.0000

Source: research data

Therefore, we start by analyzing the interactions between the portfolios presented, i.e., those based on GDP and the “beta” portfolios. Table 10 shows the results of the estimators using the GDP-volume portfolio as a proxy for the market portfolio.

As expected, the best results were obtained where the portfolios had p-values of the variable α greater than the significance level of 5% (0.05), so that the null hypothesis associated with Equation 4 was not rejected. In turn, Table 11 shows that the best results were obtained over the shorter time frames, with highlight on the window from 07/2016 to 12/2018. In this interval, only portfolio β_6 had a p-value less than 0.05.

A relevant observation is the long-term result, in the period from 01/2009 to 12/2018, in which only β_8 and β_{10} obtained p-values greater than the stipulated significance level. Thus, curious cases can be noted, such as portfolios β_1 and β_4, for which H_0 was only rejected over the long run. An exceptional case was portfolio β_{10}, for which the null hypothesis was not rejected in any of the windows tested. This can be related to its low causal correlation with the GDP-volume portfolio. Also, portfolio β_8 stands, out, with a p-value very near 0.05 in the window from 01/2014 to 06/2016

Table 10:
Result of the GDP-volume portfolio as a proxy for the market portfolio

Portfolio	Intercept of the regression lines				
	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
Beta 1	0.6656	0.8691	-0.1309	0.8407	10645
p-valor	0.0335	0.1952	0.8062	0.2326	0.1386
Beta 2	10500	22601	0.1801	-0.1963	0.5453
p-valor	0.0079	0.0023	0.7701	0.8568	0.4051
Beta 3	0.8416	0.4663	0.9895	18179	0.5186
p-valor	0.0025	0.3801	0.0297	0.0109	0.4050
Beta 4	0.8630	0.9644	0.5162	14493	0.7548
p-valor	0.0202	0.1626	0.3977	0.1688	0.3248
Beta 5	0.6748	0.4603	0.3834	18683	0.6993
p-valor	0.0288	0.4132	0.5410	0.0044	0.2302
Beta 6	11653	0.1861	12116	0.6395	25894
p-valor	0.0013	0.7888	0.0766	0.3882	0.0029
Beta 7	0.9118	15749	11770	0.9531	0.4210
p-valor	0.0012	0.0173	0.0255	0.1133	0.4494
Beta 8	0.1714	-0.0103	0.9262	-13762	0.2614
p-valor	0.5886	0.9872	0.1238	0.0489	0.6894
Beta 9	0.7828	0.7324	0.1524	23834	0.3453
p-valor	0.0373	0.3682	0.8506	0.0120	0.4969
Beta 10	0.5372	0.6803	0.7969	-0.3369	0.2639
p-valor	0.0712	0.2342	0.1705	0.6449	0.6479

Source: research data

According to Table 11, the J_0 ratios are all below the significance level of 0.05. Therefore, it is possible to reject the null hypothesis in multivariate form. However, the values J_1, J_2 and J_3 are above that significance level, so the null hypothesis that the intercept is equal to zero cannot be rejected. Since the statistics J_1 and J_3 take into account that the sample is finite, there is a chance of the portfolios being efficient in the short run. This is mainly the case for J_2 and J_3, which stipulate the test statistic for the models where the restriction of $\alpha = 0$ was applied, as per Equation 4. Thus, all the p-values in the sub-intervals were higher, or very near, 0.05, with the exception of the statistic J_1 for the window between 01/2014 and 06/2016, which in the individual test was the interval with the most intercepts rejected.

Table 11:
J ratios - GDP-volume portfolio as a proxy for the market portfolio

$H_0: \alpha = 0$	*01/2009 - 12/2018*	*01/2009 - 06/2011*	*07/2011 - 12/2013*	*01/2014 - 06/2016*	*07/2016 - 12/2018*
J0	422.347	186.093	331.565	851.983	189.341
p-value	0.0000	0.0455	0.0003	0.0000	0.0411
J1	38.363	11.786	20.999	53.959	11.992
p-value	0.0002	0.3624	0.0787	0.0008	0.3507
J2	157.155	62.879	96.989	175.297	63.747
p-value	0.1081	0.7905	0.4673	0.0634	0.7829
J3	147.987	48.208	74.358	134.395	48.873
p-value	0.1396	0.9028	0.6838	0.2001	0.8986

Source: research data

Moreover, the tests of the J ratios in the majority of cases implied acceptance of excess returns being explained only by systematic risk in multivariate form. For the CAPM to be valid, the market portfolio must also be efficient in the mean-variance sense. Therefore, we tested the following two hypotheses: $H_0: Z_m = 0$ and $H_a: Z_m > 0$. In this test, efficiency of the portfolio meant rejecting the null hypothesis. The hypothesis $Z_m < 0$ also considered the portfolio to be inefficient, which makes sense because an investor will not be interested in a portfolio with negative return.

As shown by Table 12, the null hypothesis could not be rejected in any period. Besides this, the Sharpe ratio was negative in three of the five intervals tested (01/2019 to 12/2018, 07/2011 to 12/2013 and 01/2014 to 06/2016). Hence, the portfolio was not efficient in the mean-variance sense. Even considering that the first and last sub-intervals explained the excess asset returns through the market risk, they cannot be considered efficient with respect to the market portfolio.

Table 12:
Mean-variance efficiency test of the GDP-volume portfolio

$H_0: \mu_m = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
SR	-0.0461	0.2312	-0.0713	-0.4640	0.0801
p-value	0.5184	0.4094	0.5282	0.6769	0.4684

Source: research data

We now analyze the GDP-presence market portfolio. Table 13 shows that it had efficiency similar to that of the GDP-volume portfolio (slightly better). Here, α was not rejected in the full interval (01/2009 to 12/2018) in only two of the portfolios tested, β_8 and β_{10}. For these portfolios, H_0 was also not rejected in any of the sub-intervals. However, portfolio β_{10} exhibited a substantial reduction of its p-value, with a p-value in the period very near 0.05 (5%).

It can also be seen that for the portfolios β_1, β_4 and β_6, the null hypothesis was only rejected over the long term, with highlight on portfolio β_6, for which H_0 was rejected for the GDP-volume portfolio.

Table 13:
Result - GDP-presence portfolio as a proxy for the market portfolio

Portfolio	Intercept of the regression lines				
	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
Beta 1	0.7167	0.9590	-0.1041	0.8875	10.334
p-value	0.0250	0.1581	0.8553	0.2272	0.1440
Beta 2	10.953	23.450	0.2117	-0.2585	0.5185
p-value	0.0073	0.0015	0.7425	0.8189	0.4410
Beta 3	0.8843	0.5284	10.166	18.760	0.4959
p-value	0.0017	0.3163	0.0322	0.0100	0.4323
Beta 4	0.9044	10.198	0.5482	15.550	0.7351
p-value	0.0139	0.1373	0.3553	0.1338	0.3447
Beta 5	0.7138	0.5097	0.4032	18.981	0.6706
p-value	0.0205	0.3623	0.5196	0.0063	0.2346
Beta 6	11.986	0.2339	12.280	0.7192	25.786
p-value	0.0009	0.7322	0.0744	0.3226	0.0035
Beta 7	0.9439	15.951	11.971	10.387	0.3990
p-value	0.0006	0.0139	0.0234	0.0731	0.4596
Beta 8	0.1987	0.0248	0.9462	-13.593	0.2480
p-value	0.5301	0.9682	0.1125	0.0530	0.7110
Beta 9	0.8035	0.7738	0.1682	24.247	0.3305
p-value	0.0323	0.3454	0.8342	0.0110	0.5136
Beta 10	0.5589	0.7037	0.8144	-0.2502	0.2480
p-value	0.0557	0.2047	0.1561	0.7288	0.6609

Source: research data

Tables 14 shows similarity between the GDP-presence and GDP-volume portfolios. According to the statistic J_0, the null hypothesis was rejected for all the windows studied. Of particular note, in the tests J_2 and J_3, with restrictive character, H_0 was not rejected in any of the periods for either of the portfolios.

Table 14:
J ratios - GDP-presence portfolio as a proxy for the market portfolio

$H_0: \alpha = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
J0	473.179	235.401	340.292	811.711	178.529
p-value	0.0000	0.0089	0.0002	0.0000	0.0575
J1	42.980	14.909	21.552	51.408	11.307
p-value	0.0000	0.2176	0.0719	0.0011	0.3907
J2	173.233	75.467	98.777	170.661	60.836
p-value	0.0675	0.6730	0.4513	0.0729	0.8082
J3	163.128	57.858	75.729	130.840	46.641
p-value	0.0910	0.8329	0.6705	0.2190	0.9125

Source: research data

The mean-variance efficiency tests indicated in Table 15 are basically the same as for the previous portfolio, with the same periods having negative Sharpe ratios, besides no rejection of the null hypothesis in any of the cases.

Table 15:
Mean variance efficiency test of the GDP-presence portfolio

$H_0: \mu_m = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
SR	-0.0533	0.2204	-0.0776	-0.4744	0.0838
p-value	0.5212	0.4135	0.5307	0.6806	0.4669

Source: research data

With respect to the third portfolio, called GDP-number, Table 16 shows a large number of rejections of the H_0. The only interval to reach a value near acceptance was that from 07/2016 to 12/2018. However, the intercept of the portfolio Beta 6 was significant for the model. Once again, the null hypothesis was not rejected in the full period for portfolios β_8 and β_{10}, and only portfolio β_{10} corresponded to the restriction $\alpha = 0$ in all the windows studied.

Table 16:
Result of the GDP-number portfolio as a proxy for the market portfolio

Portfolio	Intercept of the regression lines				
	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
Beta 1	0.6518	12.605	-0.2094	0.5640	0.8627
p-value	0.0303	0.0480	0.6606	0.3865	0.2394
Beta 2	10.301	26.817	0.0940	-0.4522	0.3349
p-value	0.0119	0.0004	0.8681	0.6811	0.6254
Beta 3	0.8240	0.7794	0.9158	14.649	0.3689
p-value	0.0052	0.1592	0.0340	0.0516	0.6052
Beta 4	0.8543	12.285	0.4529	12.788	0.5862
p-value	0.0180	0.0668	0.4508	0.1961	0.4409
Beta 5	0.6659	0.6857	0.3330	15.919	0.5125
p-value	0.0255	0.1965	0.6004	0.0108	0.3569
Beta 6	11.601	0.4061	11.808	0.4145	24.663
p-value	0.0009	0.5116	0.0658	0.5768	0.0048
Beta 7	0.9025	17.385	11.241	0.7328	0.2892
p-value	0.0014	0.0082	0.0361	0.2268	0.5896
Beta 8	0.1556	0.2279	0.8784	-15.760	0.1502
p-value	0.6402	0.7270	0.1539	0.0300	0.8310
Beta 9	0.7751	0.8389	0.1156	21.944	0.2495
p-value	0.0402	0.2908	0.8882	0.0193	0.6324
Beta 10	0.5315	0.8648	0.7565	-0.3711	0.1852
p-value	0.0746	0.1312	0.2065	0.6047	0.7382

Source: research data

However, in the multivariate tests (J ratios), the results in Table 17 show that in the last sub-interval, H_0 could not be rejected. In this period, none of the J tests rejected the null hypothesis. Furthermore, the ratios J_2 and J_3 (with restriction) were efficient in all the windows.

Table 17:
J ratios - GDP-number portfolio as a proxy for the market portfolio

$H_0: \alpha = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
J0	413.612	334.225	310.816	724.935	156.108
p-value	0.0000	0.0002	0.0006	0.0000	0.1113
J1	37.570	21.168	19.685	45.913	0.9887
p-value	0.0002	0.0766	0.0979	0.0021	0.4850
J2	154.341	97.537	92.637	160.073	54.584
p-value	0.1170	0.4624	0.5073	0.0994	0.8585
J3	145.338	74.778	71.021	122.722	41.848
p-value	0.1500	0.6797	0.7158	0.2673	0.9386

Source: research data

The mean-variance efficiency tests (Table 18) obtained the same results for the portfolios GDP-volume and GDP-presence, and in particular indicated the inefficiency of the GDP-number portfolio as a proxy for the market portfolio in the CAPM.

Table 18:
Mean-variance efficiency test of the GDP-number portfolio

$H_0: \mu_m = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
SR	-0.0439	0.1802	-0.0555	-0.4216	0.1071
p-value	0.5175	0.4291	0.5219	0.6618	0.4577

Source: research data

As demonstrated in Table 19, we performed efficiency tests with the ibovespa as the proxy for the market portfolio, the one most often used for that purpose. We then performed tests for comparison between the proposed portfolios. The results were more efficient than that of the portfolio currently used in the Brazilian scenario. In this respect, there were individualized efficiencies in the period from 07/2016 to 12/2018 and the complete interval, from 01/2009 to 12/2018. Note also that the ibovespa had fewer cases of rejection than the portfolios tested previously. Nevertheless, this was not sufficient to classify it as efficient.

Table 19:
Ibovespa portfolio as a proxy for the market portfolio

Portfolio	Intercept of the regression lines				
	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
Beta 1	0.2537	14.735	0.6603	-18.269	0.2722
p-value	0.5662	0.1572	0.2978	0.0294	0.7943
Beta 2	0.6630	30.512	10.584	-16.846	-0.2767
p-value	0.1977	0.0198	0.1312	0.0748	0.7932
Beta 3	0.5037	0.9483	16.232	-0.6616	-0.2082
p-value	0.2009	0.2554	0.0119	0.4528	0.8083
Beta 4	0.5545	12.383	10.861	-0.7327	0.0875
p-value	0.2011	0.0764	0.1220	0.5153	0.9273
Beta 5	0.3767	0.9410	0.7593	-0.6228	0.0366
p-value	0.3589	0.2681	0.2817	0.4520	0.9680
Beta 6	0.9099	0.5461	17.446	-11.085	21.571
p-value	0.0273	0.5080	0.0072	0.1793	0.0351
Beta 7	0.6838	17.570	16.632	-0.7256	-0.0557
p-value	0.0330	0.0099	0.0029	0.2561	0.9388
Beta 8	-0.0334	0.3819	13.365	-21.918	-0.1095
p-value	0.9314	0.6420	0.0330	0.0020	0.9058
Beta 9	0.6290	10.569	0.2782	0.8735	-0.1662
p-value	0.1227	0.2419	0.7416	0.3748	0.7425
Beta 10	0.4103	10.417	0.9526	-0.8618	0.0572
p-value	0.2318	0.1631	0.1497	0.2238	0.9289

Source: research data

The J ratios (Table 20) better explained what was seen in the individual tests. Note that the ibovespa presented fewer rejections of the null hypothesis, with highlight for the periods from 01/2009 to 12/2018, 01/2009 to 06/2011 and 07/2016 to 12/2018, when no p-value was lower than 0.05 (5% level). This shows a contradictory result between the multivariate tests and individual tests in the full 10-year interval, because the null hypotheses of the unit tests were rejected several times. However, taken together, these rejections did not have an effect, either in the restricted or unrestricted model.

Table 20:
J ratios - ibovespa portfolio as a proxy for the market portfolio

$H_0: \alpha = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
J0	151.867	136.602	457.359	344.089	124.992
p-value	0.1254	0.1891	0.0000	0.0002	0.2530
J1	13.795	0.8651	28.966	21.792	0.7916
p-value	0.1992	0.5784	0.0221	0.0691	0.6377
J2	62.103	48.889	120.654	99.547	45.378
p-value	0.7973	0.8985	0.2807	0.4445	0.9198
J3	58.481	37.482	92.502	76.320	34.790
p-value	0.8279	0.9580	0.5085	0.6647	0.9678

Source: research data

Therefore, according to Table 21, although the ibovespa was shown to be efficient in explaining the excess returns of the assets in some periods, it was not mean-variance efficient, and also had negative Sharpe ratios in two cases.

Table 21:
Mean-variance efficiency test of the ibovespa

$H_0: \mu_m = 0$	01/2009 - 12/2018	01/2009 - 06/2011	07/2011 - 12/2013	01/2014 - 06/2016	07/2016 - 12/2018
SR	0.0113	0.1816	-0.2254	-0.1069	0.2146
p-value	0.4955	0.4286	0.5884	0.5422	0.4158

Source: research data

Finally, because its mean was not statistically different from zero, the return of the ibovespa presented marginally better results than the other portfolios tested, since its Sharpe ratio was greater than zero in a greater number of periods.

5 Conclusions

The objective of this study was to test the ex ante efficiency of the proposed portfolios through the composition of GDP (GDP-volume, GDP-presence and GDP-number), utilized as proxies for the market portfolio in the context of the CAPM, comparing them with the efficiency of the ibovespa for the same purpose. Thus, we tested the following hypotheses: H_1: A hypothetical market portfolio based on the composition of GDP, formed by firms in each of its sectors, would by itself explain the return of a portfolio of assets; and H_2: A hypothetical market portfolio based on the composition of GDP, formed by firms in each of its sectors, would be mean-variance efficient.

The results showed that all the proposed portfolios were inefficient in the mean-variance sense, thus not confirming H_1 and H_2. Those conclusions were also identified by Silva and Motta (2002), Araujo, Fajardo and Di Tavani (2006), Hagler and Brito (2007), Volpe (2010), and Pereira and Laurini (2020), notwithstanding the characteristics of the Brazilian context.

However, good responses to the tests were obtained for some sub-intervals. In particular, in the window between 07/2016 and 12/2018, both hypotheses were not rejected in 90% of the cases. This suggests what as found in the studies of the correlation between GDP and the ibovespa, that the window analyzed can influence the results obtained.

The theoretical conception of using gross domestic product as a proxy for the market portfolio when applying the CAPM is important, since it is a macroeconomic variable that encompasses all output of the Brazilian economy. This explains its use by Araujo, Fajardo and Di Tavani (2006), since its strong capacity to explain the Brazilian market can cover a huge range of applications. However, even though the ibovespa presented a marginally better result than the other portfolios, the results did not allow stating it can be used as a proxy for the market portfolio in the CAPM, since this index did not satisfy the premises for efficiency of the Sharpe-Lintner version. Despite this, the index is a possible alternative for practical application in the real world.

We also could conclude that despite studying a 10-year period of relative stability of the Brazilian economy, the method discussed by Campbell, Lo and MacKinlay (1997) was not able to confirm the hypotheses for the Brazilian scenario, according to the portfolios proposed in this study. Therefore, future studies can be more precise by testing subdivisions within each proxy adopted in this study as the efficient portfolio premises for the subdivisions volume, presence and number and the ibovespa, but doing so in intervals before and after the Covid-19 pandemic, since the pandemic has had a major effect on the stability of the Brazilian economy. Future studies can also add comparison with models such as those of Guermat (2014) and Pereira and Laurini (2020) to see if different results are obtained than those of this study.

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