RCC
**Revista Contemporânea
de Contabilidade**

On the relationship between implied volatility index and stock return index

Relação entre índice de volatilidade implícita e índice de retorno de ações

Relación entre el índice de volatilidad implícita y el índice de rentabilidad de las acciones

Aparecida de Fátima Ferreira Martins*
Mestre em Ciências Contábeis (UFES)
Operadora de Mesa na Gerência Nacional de Renda
Variável da Caixa Econômica Federal, São Paulo/SP, Brasil
aparecida.martins@caixa.gov.br
https://orcid.org/0000-0002-2636-1539

Patricia Maria Bortolon
Doutora em Administração (COPPEAD/UFRJ)
Professora do Programa de Pós-Graduação em Ciências Contábeis
(UFES), Vitória/ES, Brasil
patricia.bortolon@ufes.br
https://orcid.org/0000-0001-8087-3837

Vinicius Mothe Maia
Doutor em Administração (PUC-RJ)
Professor do Programa de Pós-Graduação em Ciências Contábeis
(UFRJ), Rio de janeiro/RJ, Brasil
vmaia@facc.ufrj.br
https://orcid.org/0000-0001-6156-3104

Primary contact address for correspondence*
Av. Paulista, 2300, 11º andar, CEP: 01310-300, São Paulo/SP, Brasil

Abstract
This study investigated the relationship between the Ibovespa returns and the implied volatility index in Brazil (IVol-BR). We analyzed whether the level of the IVol-BR is related to the present and future Ibovespa returns. OLS and quantile regressions were used to investigate possible differences in the relationships along the distribution of the IVol-BR. To test the robustness of the model, we applied an alternative proxy for volatility, estimated from a GARCH (1,1) model. An asymmetrical relationship was found in Brazilian investors' response to different moments of market volatility, suggesting that negative Ibovespa returns have a stronger relationship with the IVol-BR than positive returns. The coefficients were higher at the extremes of the IVol-BR distribution. These results suggest that the Brazilian market reacts more strongly to bad news than to good news, in line with the ideas developed in the behavioral finance field.
Keywords: Volatility index; IVol-BR; Ibovespa; Behavioral finance

Resumo
O estudo investigou a relação entre os retornos do Ibovespa e do índice de volatilidade implícita IVol-BR. Foi analisado se o nível do IVol-BR tem relação com os retornos contemporâneos e futuros do Ibovespa. Foram utilizadas regressões por MQO e regressão quantílica para investigar eventuais diferenças nas relações ao longo da distribuição da medida IVol-BR. Para testar a robustez do modelo foi adotada uma *proxy* alternativa para a volatilidade, estimada a partir do modelo GARCH (1,1). Foi encontrada uma relação assimétrica na resposta do investidor brasileiro aos diferentes momentos de volatilidade no mercado, sugerindo que retornos negativos do Ibovespa possuem relação mais intensa com o IVol-BR que retornos positivos. Os coeficientes são maiores nos extremos da distribuição do IVol-BR. Tais resultados sugerem que o mercado brasileiro reage de forma mais exacerbada a notícias ruins do que a notícias boas, indo ao encontro das ideias desenvolvidas pelo campo das finanças comportamentais.
Palavras-chave: Índice de volatilidade; IVol-BR; Ibovespa; Finanças comportamentais

Resumen
Este estudio investigó una relación entre los retornos de Ibovespa y el índice de volatilidad implícita IVol-BR. Foi analisado se o nível do IVol-BR tem relação com os retornos contemporâneos e futuros do Ibovespa. Foram utilizados regressões por MQO e regressão quantílica para investigar eventuais diferenças nas relações ao longo da distribuição da medida IVol-BR. Para testar a robustez do modelo foi adotada uma proxy alternativa para a volatilidade, estimada a partir de modelo GARCH (1,1). Foi encontrada uma relação

assimétrica na resposta do investidor brasileiro aos diferentes momentos de volatilidade no mercado, sugerindo que retornos negativos do Ibovespa possuem relação mais intensamente com o IVol-BR que retornos positivos. Os coeficientes são maiores nos extremos da distribuição do IVol-BR. Tais resultados sugerem que o mercado brasileiro reage de forma mais exacerbada a notícias ruins do que a notícias boas, indo ao encontro das ideias desenvolvidas pelo campo das finanças comportamentais.

Palabras clave: Índice de volatilidad; IVol-BR; Ibovespa; Finanzas conductuales

1 Introduction

Volatility attracts the attention of researchers and market participants because it is a variable that cannot be directly observed, so it needs to be estimated (Pinho, Camargos, & Figueiredo, 2017). The models used for this purpose have evolved from estimation via the standard deviation, a classical measure of statistical dispersion, to multivariate models such as those of the general autoregressive conditional heteroscedasticity (GARCH) family, where volatility is considered a measure of the uncertainty about the performance of an asset.

Volatility is also used to calculate performance metrics, such as the Sharpe ratio, which estimates the tradeoff between return (the risk premium) and risk (Bodie, Kane, & Marcus, 2015). Market players also employ volatility to estimate and price derivatives, such as options (Mello, 2009). The correct estimation of volatility thus is highly relevant for pricing assets and formulation of investment strategies (Pinho et al., 2017).

Starting with the groundbreaking work of Engle (1982), the study of volatility evolved to conditional heteroscedasticity models, where the variance is not constant over time, but rather is conditional on past behavior. These are statistical models of the ARCH family.

The approaches described so far are based on past data, obtained from observing the variations of returns over a relevant interval. However, since past behavior will not necessarily be repeated in the future, their use to estimate future volatility may not be the ideal solution.

Due to this observation, the idea of implied volatility was developed. This volatility measure is based on the future expectation of risk implicit in the premiums of the options traded in the market. The most common approach to calculate implied volatility is the application of the inverse of the Black-Scholes model, i.e., the implied volatility is estimated from the price of the option traded. Since the inverse function does not have an explicit analytic solution, numerical methods are used for its calculation (Mello, 2009).

Measures of implied volatility have been gaining space in the financial literature, mainly since the development of the implied volatility index of the American market, the VIX. This index initially measured the market's risk expectation based on at-the-money options of the S&P 100. In 2003, the VIX was updated, whereby the "new VIX" is based on the S&P 500 index, the main American stock market index. It reflects the dynamics of two variables: the risk level, estimated by the expected future variance; and the price of that risk, which is an investor risk aversion measure (Whaley, 2000).

This new way of measuring volatility, with the particular advantage of allowing estimation of future volatility, affects the financial literature in three aspects: (i) a new risk measure can affect portfolio decisions according to the portfolio theory of Markowitz (1952); (ii) the efficient market hypothesis of Fama (1970) is challenged since future volatility can be utilized to predict returns and winning investment strategies; and (iii) the approximation of the fields of finance and psychology and the resulting development of behavioral finance, and more recently the adaptive market hypothesis, can be interpreted as a measure of investor fear (Kahnemann & Tversky, 1979; Lo, 2004).

The analysis of the current relationship of the VIX with the returns introduces the index as a proxy for investor fear. Whaley (2000) showed that the market reacts more negatively to an increase in the VIX than positively when the VIX declines. This asymmetry, i.e., the stronger response of the market to the increase in volatility, justifies using the VIX as a proxy for investor fear. The findings of behavioral finance explain this movement by showing that investors react more strongly to bad news than to good news. Therefore, when the expected volatility of the market increases, investors believe this portends bad news and demand a higher return, causing stock prices to decline. This movement does not have the same intensity when the volatility decreases (good news). Smales (2016) showed that the relationship between changes in the level of investor fear and financial market returns suggests a consistent reaction of flight to quality when fear increases[i].

Giot (2005) pointed out the lack of focus of researchers on the possible relationship between implied volatility and future stock returns, which he attributed to the belief that markets are efficient. Banerjee, Doran and Peterson (2007) found evidence that the VIX has a strong capacity to predict the future returns of portfolios. Smales (2016) also identified this predictive capacity, but only for extreme levels of investor fear. Giot (2005) analyzed the present and future relations of the VIX and VXN indexes, based respectively on the S&P500 and Nasdaq100 indexes. He concluded that a strong negative relationship exists between changes in the implied volatility indexes and the stock return indexes investigated.

In Brazil, researchers from the University of São Paulo (USP) developed an implied volatility index based on the VIX for the Brazilian market, called the "IVol-BR". This index is based on the VIX, and applies adjustments that take into account the low liquidity of the Brazilian market. The IVol-BR is based on daily prices of options on the Ibovespa (the country's main stock index), with 2 months maturity.

Vicente and Guedes (2010) studied the informativeness of implied volatility on future volatility using data on the performance of Petrobras shares from January 2006 to December 2008. They concluded that the implied volatility of in-the-money (ITM) options is not relevant to explain realized volatility, while the implied volatility of at-the-money (ATM) options contains information about future volatility, although with bias. The authors estimated that the difference between implied volatility and realized volatility can be attributed to the risk premium required by investors, showing that options are used for hedging in risky settings.

Mello (2009) studied the forecasting capacity regarding future volatility by comparing moving average and GARCH models with implied volatility. They found evidence that implied volatility contains information from realized volatility, but both estimators tested were biased. The results suggested that the market overestimates future volatility, attributed to the fact that options to a certain extent act as a safe harbor, and agents naturally are willing to pay a premium for this "fair value" due to the existence of events they are unable to predict to assign the price of options, known as the "Black Swan" effect[ii] (Taleb, 2010).

Given the findings of studies carried out regarding volatility and behavioral finance, relating implied volatility and market return, besides volatility and investor behavior, here we investigate the following question: Is there a relationship between implied volatility, measured by the IVol-BR, and the returns of the Ibovespa?

By responding to the research question, two objectives are attained. The first is to analyze the contemporary relationship between the IVol-BR and the Ibovespa returns, to verify the utility of the IVol-BR as a "fear index", since previous research has shown that the American market declines in face of implied volatility, and the downward reaction is not proportional. The second goal is to verify whether implied volatility has a relationship with future market returns. According to Giot (2005), in periods of high implied volatility, investors exaggerate by selling assets indiscriminately to obtain money or limit losses, leaving the assets cheap, hence the study of the possible relationship between implied volatility and future market returns is of interest.

This article is similar to that of Cainelli, Pinto and Klotzle (2021), who also investigated the relationship between the IVol-BR and Ibovespa returns. However, there are relevant methodological differences. First, those authors' focus is on the relationship between the IVol-BR and future returns, without exploring the relationship between the IVol-BR and current returns of the Ibovespa, as suggested by Giot (2005) and adopted by us. In this study, we use the IVol-BR as the dependent variable, while Giot (2005) and Cainelli et al. (2021) used it as an explanatory variable. Second, they focus on the extremes of the IVol-BR distribution, while we investigate possible different relations along the entire distribution of the index, in two distinct ways – with predefined percentiles and through quantile regression, possible since here the IVol-BR is a dependent variable. Third, they use interpolation for dates without an IVol-BR value, a procedure we avoid since we believe it alters the average and other statistics of the real series observed. Finally, the period studied here is longer, ending in April 2019. Hence, the present study complements the findings of that article in aspects we present in the section on analysis of the results.

As an academic contribution, this study relates the IVol-BR with Ibovespa returns, helping to validate the index developed for the Brazilian market, which does not have an official metric for implied volatility. Methodologically, this article contributes in two ways: (i) by testing different volatility metrics, using the IVol-BR to estimate implied volatility and the GARCH model as an alternative; and (ii) by employing different estimation methods, including quantile regression.

As a practical contribution, the article presents a new way to understand the relationship between risk and return in the Brazilian market, helping portfolio managers to formulate their investment strategies. By identifying a possible asymmetric behavior of the market, managers can improve their method of structuring their portfolios, anticipating their movements and obtaining better than average returns through better timing of assuming and abandoning positions.

2 Literature Review and Development of Hypotheses

According to the portfolio theory of Markowitz (1952), the choices of assets and portfolios depend on the beliefs about the future performance of those assets (expected returns) and the expected variance of this return. Investors seek high returns with low risk and consider the covariance between assets as a way to minimize the portfolio risk. Estimating future returns is a challenge, and more so future variances and covariances. Studies of volatility and ways to understand its behavior form a relevant part of the financial literature.

All efforts to analyze these variables to obtain abnormal gains were questioned by Fama (1970) in his efficient market hypothesis (EMH). In an efficient market, prices completely reflect all available information, so there is no possibility of obtaining abnormal gains. Market efficiency can be classified in three

forms: weak, semi-strong and strong. In the weak form, prices incorporate all past information, while the semi-strong form also incorporates all publicly available information, and the strong form means that prices reflect all available information, public or not.

The theoretical support for analysis and decision-making under uncertain conditions, based on the theory of utility and rational decisions, developed by Friedman and Savage (1948), started to be challenged in studies combining behavioral psychology and finance in the 1970s. The paper of Kahneman and Tversky (1979) is a milestone in this literature, presenting the so-called prospect theory. They presented evidence that decision-makers have cognitive biases that are incompatible with rational choices and the correct assessment of probabilities and information. According to the prospect theory, the value is attributed to the gains and losses, not the final asset prices. The loss function indicates that risk aversion is greater in the field of losses than gains.

Decision-making against a backdrop of sinking asset prices or crises has been widely investigated in the behavioral finance literature. Imas (2016) described the contradictory results in this literature, where some studies indicate greater risk-taking after losses and others indicate a lower propensity to take risks. He reconciled the results by showing that after a loss is realized, investors tend to avoid risk, but when the loss is not realized, they are willing to take more risk. In turn, Guiso et al. (2018) showed that after crises, risk aversion increases significantly. They also showed that individuals who experience a large increase in risk aversion after crises are four times more likely to sell shares during the worst moment of the crisis than individuals who do not have this increase in risk aversion.

While psychology associated with finance gave rise to a new field of study, Lo (2004) also brought concepts of sociology and evolutionary biology to investigate the disputes over the EMH, by presenting the adaptive market hypothesis (AMH). According to him, market efficiency cannot be evaluated in a vacuum, since it depends on a dynamic context. This dynamic context has features similar to the increase or decrease of a population of insects, for example, which is a function of years' season, presence of predators and the insects' ability for adaptation. Prices reflect a combination of environmental conditions and the nature of the "species" in the economy. The species are the distinct participants in the market, each with characteristic behaviors. All of them compete for scarce resources. According to the author, from this standpoint, it is possible to understand the apparent contradictions between the EMH and the presence and persistence of behavioral biases.

According to the AMH, innovation is the key to survival. The AMH suggests that to achieve consistent returns, it is necessary to adapt investment strategies to changes in the market conditions, something that involves multiple skills. Portfolio managers, for instance, are a species at less risk of extinction than retail investors. This brings a clear implication for all financial market participants: survival is the only objective that matters (Lo, 2004).

Studies about risk and volatility with their varied measures fit in this context of evolution of financial investigations. The volatility metrics range from measures of dispersion of data around the arithmetic mean, such as the standard deviation, to moving average models, which allow a better evaluation of the movement of data over time and the existence or not of a long-term trend in the series (Levine et al., 2016). The moving average approach can be fine-tuned by weighting the moving averages, where different weights are attributed to different lags, and the exponential smoothing model, where the forecast is made through the exponentially weighted average of prior observations.

The analysis of historical volatility evolved to the ARCH and GARCH models, proposed by Engle (1982). These models enable analyzing the existence of volatility clusters, i.e., the tendency for periods of high volatility to be conditional on other periods of high volatility. These models permit the conditional variance to change with time. In the ARCH model, the variance of the error term is related with the square of the error term of the previous period, and in the GARCH model, this relation occurs with the squared error terms in various previous periods.

All the previous measures are used to analyze past data, but the decision-making models of investors according to the financial theories discussed previously require estimates of expected values. The measures of implied volatility presented below have been developed in this sense.

The VIX is the implied volatility index of the American market and reflects the consensus opinion of a representative sample of investors regarding the expected future volatility of the options market. The index is based on the S&P500 index, the main indicator of the American stock market. The VIX reflects the dynamic between the risk level and the price of this risk, which is a measure of investors' risk aversion. The calculation starts from the valuation model of the stock options index containing all the parameters except one, which is known or can be estimated with reasonable accuracy. The unknown parameter is the implied volatility.

The general formula to calculate the VIX is as follows:

$$\sigma^2 = \frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} Q(K_i) - \frac{1}{T}\left[\frac{F}{K_0} - 1\right]^2$$

Equation 1 – Calculation of the VIX

Where: [iii]

$$\sigma = \frac{VIX}{100} \rightarrow \text{VIX} = \sigma \; x \; 100$$

T = Maturity time of the option;

F = Level of the future index derived from the option price index;

K_0 = First price below the future index F;

K_i = Exercise price of an out-of-money option, a call if $K_i > K_0$, and a put if $K_i < K_0$, or also a put and call if $K_i = K_0$;

ΔK_i = Interval between option prices – half of the difference between the prices on either side of K_i:

$$\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2}$$

R = Risk-free interest rate; and

QK_i = Midpoint of the bid-ask spread of each option with price K_i.

The implied volatility metric has been the topic of various studies focused on using it as a possible predictor of volatility and as a new measure of risk related to current and future returns.

For Giot (2005), it is necessary to compare the belief that implied volatility has no relation with stock prices, derived from the efficient market theory (EMT), with the opinions of market participants, who see high implied volatility as a warning sign to investors with long positions. According to the author, the reasoning of market players is as follows: in periods of high implied volatility, investors tend to exaggerate by selling financial assets indiscriminately to obtain cash or limit losses, reducing the prices of assets. This reaction is coherent with the findings of Imas (2016) and Guiso et al. (2018) and also finds support from the AMH, where survival is related to remaining in or leaving a market, indicating that investors that do not survive are those that overreact and wind up being eliminated from the the financial market's "ecosystem".

Giot (2005) investigated the relationship between the implied volatility index of the American market and the return of the S&P100 index, finding evidence of a strong and negative relationship between contemporaneous changes in the implied volatility and stock price indexes used in his work, besides examining the existence of a relationship between future positive returns and extremely high implied volatility. To check whether high levels of implied volatility are relevant signs of long positions, the author used an algorithm to classify the levels of implied volatility into 20 evenly spaced percentiles and then ranked the percentiles to classify the implied volatility as high or low. The high implied volatility index was near or above the 15th percentile, while the very high implied volatility index was near the 20th percentile. In other words, Giot (2005) showed evidence that future positive returns are expected from long positions in periods of high implied volatility, but that for this to become attractive, the implied volatility must be extremely high. The findings ran contrary to the EMH but could be explained in the literature on behavioral finance.

Rubbaniy et al. (2014) investigated the predictive power of three implied volatility indexes on the future returns of stock indexes. They used daily data from 1990 to 2009 from the S&P 500, NASDAQ 100 and DAX 20, and their implied volatility indexes were the VIX, VXN and VDAX. The authors also examined this relationship during the financial crisis of 2008 and the more recent periods of a predominantly bull market (July 2002 to September 2007) and mainly bear market (October 2007 to December 2009). Their findings suggested that the implied volatility indexes have predictive capacity for future returns of 20 and 60 days, but not for 1 and 5 days. Their results regarding the different levels of volatility were coherent with those of Giot (2005), showing that high levels of implied volatility coincide with positive future returns as measured by the stock indexes. They also regressed the volatility indexes with the returns of a portfolio characterized by beta, size and book-to-market, and tested whether implied volatility was still able to forecast future returns when other risk factors were involved. The VXN and VDAX presented somewhat contradictory results, making a general conclusion difficult. They concluded that various reasons could explain these differences, such as liquidity, size, and diversity of listed firms, etc.

Rubbaniy et al. (2014) argued that implied volatility indexes can be used to anticipate the direction of the market and are good analytical tools to identify opportunities to enter and exit. Besides this, they stated that other market movements exist that is not reflected in the volatility indexes, and these are insufficient to predict the future performance of the market. Other factors should be considered to forecast future returns. Investment strategies based on the predictive ability of implied volatility indexes should be combined with other market-timing indicators.

Bagchi (2012) conducted a study to fill the gap in research on implied volatility in emerging markets. He evaluated the relationship between the Indian VIX and three parameters of each firm: beta, market-to-book value of equity and market capitalization. His results indicated that the Indian VIX has a positive and significant relationship with the returns of portfolios assembled for 30 and 45 days in the future. Finally, he concluded that, in line with other studies, the Indian VIX can represent a good market forecasting tool for traders and investors.

Lee (2015) studied the relationship between the VIX and implied volatility indexes of four emerging countries: India, South Korea, South Africa and Russia. He found evidence that changes in the respective VIX indexes impact changes in the implied volatility indexes of emerging markets.

Li, Yu and Luo (2019) studied the relationship between the volatility index and returns with intraday trading data from the Chinese options market. The objective was to identify whether that market's volatility index (iVX) could be used as a proxy for market fear, like the case of the VIX in the American market. The evidence found demonstrated that the iVX can be used as a proxy for market fear when there are large changes, and also that an asymmetric relationship exists between market returns and the iVX.

In the Brazilian market, the implied volatility index is calculated by a group called Nucleus for Financial Studies (NEFIN) of the University of Sao Paulo. The IVol-BR relies on the same factors as the VIX, combining the standard international method used in markets with high liquidity with adjustments that take into account the low liquidity and low number of options exercised in the Brazilian market. The index is based on the daily prices of options on the Ibovespa, considering a two-month horizon, accompanying the maturity of the options.

According to Astorino et al. (2017), an implied volatility index should reflect the dynamics of the level of risk found by investors – the expected future volatility – and the price of this risk – the risk aversion of investors. Given this, the IVol-BR should be higher in periods of stress, corroborating the study of Whaley (2000), acting as a fear parameter of Brazilian investors regarding falling stock prices.

According to Pinho, Camargos and Figueiredo (2017), between 2000 and 2014, in periodicals classified by Qualis-CAPES as A2, B1 and B2, 51 articles were published on topics related to volatility. With respect to implied volatility, the country's literature is scarcer, with the Brazilian index having been used to calculate volatility only three times during that interval. Since that period, we found the same pattern of scant articles examining the relationship between implied volatility and returns.

Mello (2009) tested future volatility prediction models and found that implied volatility has information on realized volatility, but both estimators used were biased.

Vicente and Guedes (2010) investigated whether implied volatility has information about future volatility using a sample of Petrobras shares from 2006 to 2008. They concluded that although the implied volatility of ATM options contains information about future volatility, the estimator is also biased.

Cainelli et al. (2021) analyzed the relationship between the IVol-BR and future returns of the Ibovespa and identified the predictive capacity of the index mainly for returns 20, 60, 120 and 250 days ahead, with weak explanatory power for returns of 1 and 5 days.

Based on the international and national studies reviewed, we investigate the following hypotheses:

H1: An empirical relationship exists between the IVol-BR and the return of the Ibovespa.

This hypothesis aims to verify whether, as found by Giot (2005), the Ibovespa is in some way related to the IVol-BR.

 H2: The IVol-BR is related to the future return of the Ibovespa.

In this case, our objective is to identify whether the future return of the Ibovespa is related with the IVol-BR. If this hypothesis is confirmed, it will indicate a relationship between implied volatility and market return, a relationship that should not be confirmed if the Brazilian market behaves as a strongly efficient market.

3 Methodology

The sample includes daily data of the IVol-BR and Ibovespa in the period from August 1, 2011 to April 30, 2019. The starting date is due to the availability of IVol-BR data, since the index only began being announced on that date.

The Ibovespa data were obtained from the Comdinheiro database and the IVol-BR data from the NEFIN website. The latter data were not available for all days in the sample, so those days were excluded. The final sample contained 1,622 quotations of the indexes.

We aim to verify two relationships, namely:

1. The current relationship between relative changes in implied volatility and the stock returns; and
2. The possible relationship between implied volatility and future market returns.

For the first analysis, we observed the simultaneous changes in the IVol-BR and Ibovespa, based on daily data of the indexes. Giot (2005) found a negative and statistically significant relationship between the stock index returns and the implied volatility indexes used in his work, corroborating the findings of Whaley (2000). Giot (2005) also observed that positive returns of the stock indexes were associated with declining levels of implied volatility, while negative returns were associated with rising implied volatility levels.

In the second analysis, we focused on the possible relationship between implied volatility and future stock index returns. In this case, we investigated whether high implied volatility levels can indicate an oversold market, and thus act as a signal to buy in the short and medium-term. The method used in this case was similar to that employed by Campbell and Shiller (1998), who studied the link between the observed price/earnings ratio and the future return of stock indexes.

The final sample to analyze the hypotheses contained 1,621 returns for each index, losing only one observation given the return of the indexes utilized.

For hypothesis 1, involving estimation of the contemporaneous relationship between the daily variations of the $IVol - BR_t$ and the Ibovespa, we used the following regression equation:

$$r_{IVol-BR,t} = \beta_0 D_t^+ + \beta_1 D_t^- + \beta_2 r_{Ibovespa,t} D_t^+ + \beta_3 r_{Ibovespa,t} D_t^- + \varepsilon_t \tag{1}$$

Where:

- $r_{IVol-BR,t} = \ln(IVol - BR_t) - \ln(IVol - BR_{t-1})$, which is the daily variation of the level of the implied volatility index;
- $r_{Ibovespa,t} = \ln(Ibovespa_t) - \ln(Ibovespa_{t-1})$, which is the one-day return of the Ibovespa;
- D_t^- = a dummy variable that is equal to 1 (0) when $r_{Ibovespa,t}$ is negative (positive); and
- $D_t^+ = 1 - D_t^-$.

To analyze the coefficients β_2 and β_3, we observed their absolute value. The expectation was that β_3 will be greater than β_2 when negative returns of the Ibovespa are associated with greater changes in the IVol-BR than positive returns, as found by Giot (2005).

We also performed a test with the addition of the quadratic terms to the above regression to capture the effect of the size of the returns:

$$r_{IVol-BR,t} = \beta_0 D_t^+ + \beta_1 D_t^- + \beta_2 r_{Ibovespa,t} D_t^+ + \beta_3 r_{Ibovespa,t} D_t^- + \beta_4 r^2_{Ibovespa,t} D_t^+ \\ + \beta_5 r^2_{Ibovespa,t} D_t^- + \varepsilon_t \tag{2}$$

In light of the findings of Giot (2005), the objective here was the verify whether high or low returns of the stock index move differently, considering one-day changes in the implied volatility index. Giot (2005) found weak statistical significance for the coefficients of the quadratic terms of their models, indicating that despite the existence of an asymmetric effect, there was no effect of size.

In the case of hypothesis 2, the aim is to verify whether the implied volatility index can indicate an oversold or overbought market. Giot (2005) analyzed the question by observing the relationship between the variation of the implied volatility index at time t and the future return of the stock index at 1, 5, 20 and 60 trading days ahead.

Here we adopted a method similar to that used by Giot (2005), observing the relationship between the implied volatility index (IVol-BR) and $r_1 d_t$, $r_5 d_t$, $r_{20} d_t$ and $r_{60} d_t$, where $r_1 d_t$, $r_5 d_t$, $r_{20} d_t$ and $r_{60} d_t$ are the future returns of the Ibovespa in periods of 1, 5[iv], 20 and 60 trading days. Giot (2005) sought to identify whether very high levels of implied volatility are signs of trading relevant to long positions, and for this used an algorithm based on the method of Campbell and Shiller (1998) that classifies the levels of implied volatility according to 20 equally spaced percentiles. In this way, it is possible to shift from a qualitative vision of the data to a quantitative view of the business climate, enabling ranking the observed levels of the implied volatility index.

Giot (2005) utilized a period of 2 years to establish the percentiles of the implied volatility index and then related them with the stock return index. We applied an adaptation of the method used by Giot and Campbell and Shiller, by which we analyzed the distribution of the index from a period 1 year before t and, instead of classification in 20 percentiles, we included a dummy variable to identify the observations in the lowest 30% of the distribution and those in the highest 30%, with the objective of identifying the influence of low and high levels of implied volatility on the returns of the Ibovespa.

To evaluate Hypothesis 2, we estimated regressions as follows:

$$r1d_t = \beta_0 + \beta_1 Dlower_t + \beta_2 Dhigher_t + \varepsilon_t \tag{3}$$

$$r5d_t = \beta_0 + \beta_1 Dlower_t + \beta_2 Dhigher_t + \varepsilon_t \tag{4}$$

$$r20d_t = \beta_0 + \beta_1 Dlower_t + \beta_2 Dhigher_t + \varepsilon_t \tag{5}$$

$$r60d_t = \beta_0 + \beta_1 Dlower_t + \beta_2 Dhigher_t + \varepsilon_t \tag{6}$$

Giot (2005) found that the majority of coefficients in the central area of the ranking were not significant, that those in the highest positions were strongly positive and significant, and that the coefficients in the lowest positions in the ranking were strongly negative and significant. Those results supported the hypothesis that extremely high implied volatility levels signal a good moment to assume long positions of the referred index. He found that this was consistent with the conclusions of market participants that highly volatile markets are oversold, which should benefit traders that assume long positions.

The final sample of this study consisted of 1,369 observations, given the loss of the 252 observations that were used to construct the initial ranking for aggregation in clusters (groups). Additionally, we estimated the volatility with the GARCH (1,1) model, substituting the IVol-BR in the previous analyses, and applied to the GARCH the same methods applied to the IVol-BR. This analysis permitted a comparison of the results

found based on the IVol-BR with a classic model, as was done by Duppati, Kumar, Scrimgeour and Li (2017), functioning as a robustness test.

Since the volatility estimated by GARCH (1,1) has a daily basis and the volatility given by the IVol-BR has an annual basis, the volatility calculated via GARCH was annualized to have the same basis. Besides this, another difference in estimating the metrics by the IVol-BR and GARCH (1,1) is that for the GARCH (1,1) model, the estimation is for one day ahead of the price observation on the preceding day, while for the IVol-BR, the estimation is carried out based on options that mature in 21 days.

Giot (2005) observed that with very high or very low volatility levels, the relationship with the stock return index was different. Therefore, to evaluate the possible effects between the IVol-BR and Ibovespa at different volatility levels, we applied the quantile regression method. This estimation method is typically employed when the relationship between the dependent variable and the independent variables is not constant throughout the distribution of the dependent variable, so the average relationship captured by traditional regression (by ordinary least squares) would be unable to explain the phenomenon of interest (Angrist & Pischke, 2009). Due to the leverage effect, widely studied in the literature, the expectation is that the relationship between volatility and return is not the same for different volatility levels (Thurner, Farmer & Geanakoplos, 2012). The same method was applied to the volatility measured by the GARCH (1,1) model.

4 Results and Analyses

The table 1 contains the descriptive statistics of the data. Table 1 shows that the data have a leptokurtic distribution and that the IVol-BR returns are skewed to the left, while the Ibovespa and GARCH returns are skewed to the right.

Table 1
Descriptive statistics of the data

	Total (n=1621)
Ibovespa Return	
Mean (Standard Deviation)	0.000353 (0.0147)
Median [Min, Max]	1.51e-05 [-0.0499, 0.0639]
Kurtosis	0.6273177
Asymmetry	0.1439438
IVol-BR Return	
Mean (Standard Deviation)	5.23e-05 (0.116)
Median [Min, Max]	-0.00120 [-0.637, 0.484]
Kurtosis	2.622028
Asymmetry	-0.1267248
GARCH Return	
Mean (Standard Deviation)	0.000123 (0.0668)
Median [Min, Max]	-0.0188 [-0.218, 0.801]
Kurtosis	29.12025
Asymmetry	3.772893

Source: Authors.

We analyzed the stationarity of the series of the IVol-BR returns along with the volatility estimated by the GARCH model and the Ibovespa, by applying the following tests: augmented Dickey-Fuller; unit root of Phillips-Perron; and Kwiatkowski-Phillips-Schmidt-Shin (KPSS). For all the variables, the tests indicated the series were stationary at significance of 0.01, and there was no unit root problem.

Analysis of the correlation between the variables used serves to identify the intensity of the relationship between them, which can suggest problems of multicollinearity. By analyzing the pairwise correlations between the data, we found a negative correlation between the Ibovespa return and IVol-BR returns, suggesting an inverse relationship between the variables. Regarding the volatility measured by the GARCH model, the correlation was positive. With respect to future behavior, all the future returns of the Ibovespa were negatively correlated with the IVol-BR, while regarding the returns of the volatility measured by the GARCH model, only the returns for 1 day and 20 days ahead presented this negative correlation[v].

To test H1, we estimated the regression model according to Equations (1) and (2) using the ordinary least squares (OLS) approach. The results of the models estimated (with robust standard errors in parentheses) are presented in Table 2.

According to the results obtained for Equation (1), presented in the first column of Table 2, the coefficients of the dummy variables suggest that positive returns of the Ibovespa have a negative relationship with the IVol-BR. When observing the returns, only the negative return had a statistically significant relationship at 1% with the IVol-BR. The negative return multiplied by the also negative coefficient resulted in a positive impact on the IVol-BR, i.e., negative returns of the Ibovespa are associated with higher implied volatility.

Table 2
Results of the model of Equations (1) and (2) for the IVol-BR

	Dependent Variable	
	$r_{IVol-BR}$	
	(1)	(2)
D_t^+	-0.020***	-0.002
	(0.006)	(0.008)
D_t^-	-0.007	-0.004
	(0.006)	(0.008)
$r_{Ibovespa,t}D_t^+$	0.532	-2.808***
	(0.413)	(1.076)
$r_{Ibovespa,t}D_t^-$	-1.854***	-1.216
	(0.449)	(1.249)
$r^2{}_{Ibovespa,t}D_t^+$		19.693
		(35.990)
$r^2{}_{Ibovespa,t}D_t^-$		91.060***
		(27.123)
Observations	1,621	1,621
Adjusted R²	0.022	0.028
F-statistic	10.299*** (df = 4; 1617)	8.835*** (df = 6; 1615)
Breusch-Pagan test of heteroscedasticity	6.9902, df = 3	4.453, df = 5
Durbin-Watson test of autocorrelation	2.8761	2.8684
Maximum VIF	2.493604	4.411165

Note: White's robust standard error between parentheses $^*p<0.1;\ ^{**}p<0.05;\ ^{***}p<0.01$
Source: Authors.

The second column presents the results of Equation (2), which investigates the possible size effect of the returns. Here the positive return shows a statistically significant negative relationship with the IVol-BR, i.e., positive returns are associated with lower implied volatility when controlling for the size effects of the equation. The size effect was only significant for the negative returns, presenting upward concavity.

The relationships identified run counter to the efficient market theory but are coherent with the findings of the behavioral finance literature. In particular, the relationship between negative returns and increased implied volatility (coefficient -1.854) is in line with studies that have associated implied volatility with an investor fear index. The prospect theory of Kahneman and Tversky (1979) indicates that investors value gains and losses differently, placing more weight on perceived gains versus perceived losses when faced with ultimately equal choices. Guiso et al. (2018) identified the risk aversion increases after crises (during crises, volatility is greater). According to Giot (2005), this heightened risk aversion prompts investors to sell assets indiscriminately in moments of high volatility, putting downward pressure on stock prices and reducing the returns. The results presented here are coherent with this prediction and with the results of Giot (2005) and Li et al. (2019).

The results in Table 2 show that the estimated model is significant, i.e., it explains a relevant part of the variation of returns of the IVol-BR, although the adjusted R² value is low, which indicates that other explanatory variables can also influence the IVol-BR.

In international studies such as those of Giot (2005), Li et al. (2019) and Rubbaniy et al. (2014), and the paper of Cainelli et al. (2021) for Brazil, implied volatility and its relations have been investigated at different levels. With this objective, and also to give more robustness to the findings, we executed the models identified above through quantile regression. Table 3 reports the results for the dependent variable return of the IVol-BR.

The coefficients of the dummies (D_t^+ and D_t^-) were statistically significant throughout the distribution of the IVol-BR, with some interesting aspects. The sign was negative for both until the median of the distribution, after which it was positive. This suggests that irrespective of what occurs in the market, volatility has a tendency to remain low (high), which supports the idea of long memory of volatility, extensively studied by Moraes, Pinto and Klotzle (2015). Furthermore, despite the equal signs, the coefficients of D_t^- were larger than those of D_t^+as of the 90th percentile. In other words, the extreme implied volatilities were more strongly

associated with the occurrence of negative returns in our dataset. This is in line with the evidence about asymmetric reactions of investors reported by the behavioral finance literature (Kahneman & Tversky, 1979; Imas, 2016; Guiso et al., 2018).

The asymmetric behavior of the relations identified can also be observed in the consistent statistically significant and negative relationship of the negative returns with the IVol-BR, and the absence of a significant relationship with positive returns (except of the 50th and 75th percentiles in the model with squared returns). The quantile regression results also indicated that the coefficients of the negative returns are greater at the extremes of the distribution of the IVol-BR in comparison with the center of the distribution. This evidence suggests that the IVol-BR, like the VIX in the international arena, can be interpreted as an index of fear in the Brazilian market. The results jibe with those of Giot (2005) and Rubbaniy et al. (2014) for the American market, and of Li et al. (2019) for the Chinese market.

Regarding the quadratic effects of the positive returns indicated in Table 3, the results of Table 4 show that this relationship was present in the central portion of the IVol-BR returns, and only for the squared positive returns.

To assess H2, we tested whether the implied volatility index can indicate an oversold or overbought market, considering the relationship between the variation of the implied volatility index at time t and the future return of the Ibovespa 1, 5, 20 and 60 trading days ahead. We analyzed separately the effects of the 30% greatest and smallest returns of the IVol-BR ($Dhigher_t$ amd $Dlower_t$) and the central 40% of the distribution of the returns (reflected in the coefficient of the *Constant*). The results of the estimated models (with robust standard errors in parentheses) are presented in Table 4.

The results for the ends of the distribution of the IVol-BR were not statistically significant, and a marginally positive sign was only observed for the relationship of the 30% lowest returns and the return of the Ibovespa one day ahead. None of the coefficients analyzed of the observations in the greatest 30% cluster had a statistically significant relationship with future Ibovespa returns. It is interesting to note the statistically significant relationship of the constant (β_0) with the future returns of 20 and 60 days of the Ibovespa. In other words, variations of the IVol-BR classified in the central 40% of the distribution have a positive relationship with the returns 20 and 60 days ahead. Furthermore, the coefficient is greater in relation to the 60-day future return (0.013) than the 20-day future return (0.009). In other words, the mean volatility levels are more relevant to explain future returns than extreme volatilities.

The results presented here contrast with those of Giot (2005), Rubbaniy et al. (2014) and Cainelli et al. (2021), since those authors found significant relationships for higher implied volatilities. Nevertheless, the significant relations for longer terms (20 and 60 days) are in line with the findings of the mentioned studies.

The investigation of the relations of the IVol-BR with future returns, i.e., the identification of a possible gap between the occurrence of the IVol-BR and the response of the Ibovespa, can serve to structure market positions seeking extraordinary returns. The results, which pose a challenge to the efficient market hypothesis, suggest potential for this type of analysis for longer periods, as well as in the mentioned international studies.

Aparecida de Fátima Ferreira Martins, Patrícia Maria Bortolon, Vinicius Mothe Maia

Table 3
Results of models (1) and (2) with quantile regression using the IVol-BR

	Dependent Variable $r_{IVol-BR,t}$													
	Model (2)							Model (1)						
	(0.05)	(0.10)	(0.25)	(0.50)	(0.75)	(0.90)	(0.95)	(0.05)	(0.10)	(0.25)	(0.50)	(0.75)	(0.90)	(0.95)
D_t^+	-0.182***	-0.133***	-0.056***	0.003	0.062***	0.120***	0.168***	-0.191***	-0.141***	-0.066***	-0.010*	0.041***	0.103***	0.142***
	(0.024)	(0.016)	(0.011)	(0.007)	(0.010)	(0.020)	(0.019)	(0.018)	(0.014)	(0.008)	(0.006)	(0.008)	(0.015)	(0.015)
D_t^-	-0.177***	-0.132***	-0.059***	-0.014*	0.053***	0.130***	0.177***	-0.191***	-0.141***	-0.060***	-0.007	0.056***	0.119***	0.152***
	(0.022)	(0.016)	(0.009)	(0.008)	(0.011)	(0.020)	(0.022)	(0.018)	(0.013)	(0.008)	(0.006)	(0.007)	(0.013)	(0.016)
$r_{Ibovespa,t}D_t^+$	-2.210	-2.060	-2.583	-3.256***	-3.264**	-2.645	-1.482	-0.258	-0.683	-0.276	-0.260	0.350	1.083	3.425
	(2.828)	(2.091)	(1.587)	(0.978)	(1.334)	(3.716)	(3.618)	(0.921)	(0.793)	(0.562)	(0.382)	(0.455)	(1.182)	(2.257)
$r_{Ibovespa,t}D_t^-$	0.114	-0.212	-1.009	-3.539***	-2.460	-0.716	0.332	-2.427***	-1.981***	-1.178**	-1.995***	-1.828***	-2.946***	-3.635***
	(2.905)	(1.762)	(1.351)	(1.218)	(1.788)	(3.133)	(2.716)	(0.789)	(0.751)	(0.510)	(0.409)	(0.578)	(0.952)	(1.180)
$r^2{}_{Ibovespa,t}D_t^-$	73.100	48.334	3.166	-52.253	-21.945	87.236	88.977							
	(55.234)	(32.018)	(37.558)	(36.718)	(60.120)	(100.945)	(62.739)							
$r^2{}_{Ibovespa,t}D_t^+$	55.414	27.929	66.772	95.046***	90.881**	90.795	147.088							
	(58.557)	(52.075)	(44.237)	(26.467)	(36.824)	(127.285)	(109.306)							
Observations	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621
Breusch-Pagan test of heteroscedasticity	4.453, df = 5							6.9902, df = 3						
Durbin-Watson test of autocorrelation	2.8684							2.8761						

Note: White's robust standard error between parentheses. *p<0.1; **p<0.05; ***p<0.01
Source: Authors.

Table 4
Results of models (3), (4), (5) and (6) using the IVol-BR

Dummy for cluster of observations of the IVol-BR	Dependent Variable:			
	r1d	r5d	r20d	r60d
	(1)	(2)	(3)	(4)
$Dlower_t$	0.002* (0.001)	0.001 (0.002)	-0.002 (0.004)	0.011 (0.007)
$Dhigher_t$	0.0002 (0.001)	-0.0002 (0.002)	-0.004 (0.004)	0.007 (0.007)
Constant	-0.0001 (0.001)	0.001 (0.002)	0.009*** (0.003)	0.013** (0.005)
Observations	1,369	1,369	1,369	1,369
Adjusted R^2	0.001	-0.001	0.001	0.0004
F-statistic (df = 22; 1367)	1.879	0.295	0.365	1.307
Breusch-Pagan test of heteroscedasticity	0.18333, df = 2	3.0159, df = 2	0.073493, df = 2	0.67758, df = 2
Durbin-Watson test of autocorrelation	1.9684	0.46709	0.11591	0.044511
VIF maximum			1.438542	

Note: White's robust standard error between parentheses.v*p<0.1; **p<0.05; ***p<0.01
Source: Authors.

5 Robustness Analysis

The volatility was estimated by the GARCH (1,1) model, replacing the IVol-BR in the analysis to test the robustness of the models. To test H1, we used the volatility estimated by GARCH as a proxy for the dependent variable employed in Equation (1). The results are reported in Table 5.

Table 5
Results of the models of equations (1) and (2) for GARCH (1,1)

	Dependent Variable	
	r_{GARCH}	
	(1)	(2)
D_t^+	-0.060***	-0.048***
	(0.002)	(0.003)
D_t^-	-0.060***	-0.043***
	(0.002)	(0.003)
$r_{Ibovespa,t}D_t^+$	5.382***	3.135***
	(0.160)	(0.407)
$r_{Ibovespa,t}D_t^-$	-5.214***	-1.713***
	(0.174)	(0.472)
$r^2_{Ibovespa,t}D_t^+$		108.017***
		(13.592)
$r^2_{Ibovespa,t}D_t^-$		61.266***
		(10.244)
Observations	1,621	1,621
Adjusted R²	0.556	0.582
F-statistic	509.395*** (df = 4; 1617)	376.439*** (df = 6; 1615)
Breusch-Pagan test of heteroscedasticity	31.327, df = 3	62.58, df = 5
Durbin-Watson test of autocorrelation	1.6012	1.5512
VIF maximum	2.493604	4.196658

Note: White's robust standard error between parentheses. *p<0.1; **p<0.05; ***p<0.01
Source: Authors

As can be seen, all the coefficients are significant at 0.01, meaning that positive or negative returns of the Ibovespa influence volatility. Also, there is no large variation in the values of the coefficients, regardless of the sign. In this case, when evaluated by the GARCH (1,1), the results run contrary to those obtained by Giot (2005) and also do not agree with those obtained when using volatility measured by the IVol-BR. In relation to the quadratic effect for the return of the volatility measured by the GARCH model, upward concavity is also present.

All the coefficients are statistically significant, which can be explained by the fact that implied volatility estimated by the GARCH (1,1) utilizes historic data of volatility itself. Table 6 presents the results of Equations (1) and (2) when using the volatility estimated by the GARCH (1,1) and quantile regression models.

Aparecida de Fátima Ferreira Martins, Patrícia Maria Bortolon, Vinicius Mothe Maia

Table 6
Results of models (1) and (2) with quantile regression using GARCH (1,1)

	Dependent Variable													
	r_{GARCH}													
	Model (2)							Model (1)						
	(0.05)	(0.10)	(0.25)	(0.50)	(0.75)	(0.90)	(0.95)	(0.05)	(0.10)	(0.25)	(0.50)	(0.75)	(0.90)	(0.95)
D_t^+	-0.072***	-0.064***	-0.050***	-0.039***	-0.031***	-0.025***	-0.019***	-0.080***	-0.070***	-0.056***	-0.048***	-0.044***	-0.039***	-0.033***
	(0.010)	(0.004)	(0.002)	(0.002)	(0.002)	(0.002)	(0.005)	(0.006)	(0.002)	(0.002)	(0.001)	(0.002)	(0.002)	(0.004)
D_t^-	-0.077***	-0.067***	-0.050***	-0.038***	-0.031***	-0.024***	-0.023***	-0.084***	-0.074***	-0.057***	-0.048***	-0.042***	-0.038***	-0.032***
	(0.007)	(0.004)	(0.003)	(0.002)	(0.002)	(0.002)	(0.002)	(0.004)	(0.003)	(0.002)	(0.001)	(0.001)	(0.003)	(0.004)
$r_{Ibovespa,t}D_t^+$	2.033*	2.269***	2.357***	1.894***	1.367**	0.569	0.032	3.267***	3.275***	3.542***	4.131***	5.175***	6.574***	7.507***
	(1.173)	(0.586)	(0.458)	(0.458)	(0.584)	(0.572)	(1.600)	(0.260)	(0.107)	(0.186)	(0.111)	(0.237)	(0.351)	(0.566)
$r_{Ibovespa,t}D_t^-$	-2.052**	-2.080***	-1.914***	-1.646***	-0.984*	-0.448	-1.067*	-3.411***	-3.476***	-3.542***	-3.959***	-4.914***	-6.244***	-7.201***
	(0.895)	(0.783)	(0.378)	(0.538)	(0.522)	(0.756)	(0.585)	(0.267)	(0.200)	(0.129)	(0.141)	(0.188)	(0.391)	(0.744)
$r^2_{Ibovespa,t}D_t^+$	29.709	32.759*	34.692**	80.252***	148.722***	231.541***	286.970***							
	(29.102)	(18.815)	(15.153)	(20.994)	(29.938)	(28.161)	(84.553)							
$r^2_{Ibovespa,t}D_t^-$	37.320	40.605	56.990***	90.861***	161.933***	239.261***	253.644***							
	(24.140)	(30.219)	(11.221)	(25.192)	(27.550)	(41.274)	(28.774)							
Observations	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621	1,621
Breusch-Pagan test of heteroscedasticity	62.58, df = 5							31.327, df = 3						
Durbin-Watson test of autocorrelation	1.5512							1.6012						

Note: White's robust standard error between parentheses.   *p<0.1; **p<0.05; ***p<0.01
Source: Authors.
.

For the set of regressions, the results are reasonably different: the squared effect is statistically significant as of the 25th quantile when interacted with the dummy for negative return of the Ibovespa, and as of the 10th quantile when interacted with the dummy for positive return of the Ibovespa.

Moreover, in the model of Equation (1), the impact on the returns of volatility measured by GARCH is negative in all the quantiles for all the explanatory variables except for the interaction of the dummy for positive return with the return of the Ibovespa, which is statistically significant with positive effect in all the quantiles. Also, when interacted with the dummy for return of the Ibovespa, positive or negative, the coefficients have a stronger effect on the quantiles above the 75th.

Table 7 shows the results of the regressions of Equations (3), (4), (5) and (6) against the future returns of the Ibovespa when using the cluster as the proxy for volatility estimated by the GARCH model.

Table 7
Results of models (3), (4), (5) and (6) using GARCH (1,1)

Dummy for cluster of GARCH observations	Dependent Variable			
	r1d	r5d	r20d	r60d
	(1)	(2)	(3)	(4)
$Dlower_t$	-0.0002 (0.001)	-0.002 (0.002)	0.002 (0.004)	0.014** (0.007)
$Dhigher_t$	0.001 (0.001)	0.001 (0.002)	-0.0004 (0.004)	0.004 (0.007)
Constant	0.0003 (0.001)	0.002 (0.001)	0.006** (0.003)	0.013*** (0.005)
Observations	1,369	1,369	1,369	1,369
Adjusted R^2	-0.0003	-0.0003	-0.001	0.002
F-statistic (df = 22; 1367)	0.820	0.791	0.224	2.238
Breusch-Pagan test of heteroscedasticity	2.6656, df = 2	1.2809, df = 2	1.5279, df = 2	0.11874, df = 2
Durbin-Watson test of autocorrelation	1.9607	0.46878	0.1155	0.047454
VIF maximum	1.31701			

Note: White's robust standard error between parentheses. *p<0.1; **p<0.05; ***p<0.01
Source: Authors.

Even when using the other proxy for volatility, the results are similar to those obtained when using the returns of the IVol-BR, except for β_1, which for the estimation by GARCH is statistically significant for the returns 60 days ahead, while for the analysis by the IVol-BR it is statistically significant for the returns 5 days ahead, in all cases positive.

Regarding the central 40% of the distribution of the variations of volatility estimated by the GARCH model, the results are similar to those obtained with the IVol-BR. Of particular note is the statistically significant and positive relationship with the Ibovespa returns for 20 and 60 days ahead (with the largest coefficient being for the 60-day return). This result suggests that for median volatility levels, a relationship exists between the measures of volatility and the longer future returns.

6 Final Considerations

The objective of this study was to verify the possible existence of a relationship between the implied volatility in the Brazilian equity market measured by the IVol-BR and the returns of the Ibovespa. This relationship was investigated in two ways, both with present and future returns for 1, 5, 20 and 60 trading days. The analysis of the present relationship is important to validate the IVol-BR as a measure of implied volatility for the Brazilian market. The results also can be indicative of movements into and out of positions in the Brazilian market.

The results found challenge the EMH and present evidence in line with the findings of the behavioral finance literature. In the analysis of the current relations between the returns of the Ibovespa and IVol-BR, the negative relationship is stronger when the Ibovespa return is negative. These results corroborate the observations of Giot (2005), who concluded that negative stock index returns are associated with greater relative changes in volatility. When analyzing the relationship between the Ibovespa and IVol-BR, including the squared returns of the market index, we found that the size effect was only significant for the negative returns, graphically represented by upward concavity.

The results suggest that, as demonstrated in behavioral finance studies, investors react more strongly to bad news (negative returns) than to good news (positive returns), indicating that investors are not necessarily rational (Kahneman & Tversky, 1979; Guiso et al., 2018).

The quantile regressions allowed observing that the coefficients of the negative returns were greater at the extremes of the distribution of the IVol-BR. This evidence suggests that the IVol-BR can be interpreted as a fear index for the Brazilian market (Giot, 2005; Rubbaniy et al., 2014; Li et al., 2019).

In analyzing the relationship with the future Ibovespa returns, we observed that for moderate levels of volatility, a positive relationship exists with future returns of 20 and 60 days, indicating the existence of a long memory of volatility in the Brazilian market. These results are in line with the findings in the international and Brazilian literature regarding the longer-term relationships, but contradict the relationships suggested in studies for higher volatilities (Giot, 2005; Rubbaniy et al., 2014; Cainelli et al., 2021).

To give greater robustness to the analyses, we also estimated the volatility by the GARCH (1,1) model and used it as a proxy for implied volatility instead of the IVol-BR. The coefficients found had the same signs as found for the IVol-BR.

Overall, this study presents a new way to interpret the risk-return relationship in the Brazilian market, which can help portfolio managers at the moment of structuring their investment methods, thus having real-world applications. The study also adds to the literature by using a little-used proxy, the IVol-BR, comparing it to proxies for volatility using different estimation methods.

Future studies can examine what events or news can trigger implied volatility, besides investigating the results obtained by portfolios using strategies built based on timing extracted from analyzing the behavior of implied volatility and the main stock index in the Brazilian market, the Ibovespa.

References

Angrist, J. D., & Pischke, J.-S. (2009). *Mostly Harmless Econometrics: An Empiricist's Companion*. Princeton University Press.

Astorino, E., Chague, F., Giovannetti, B. C., & Silva, M. E. (2017). Variance Premium and Implied Volatility in a Low-Liquidity Option Market. *Revista Brasileira de Economia, 71*(1), 3-28. https://doi.org/10.5935/0034-7140.20170001

Bagchi, D. (2012). Cross-sectional analysis of emerging market volatility index (India VIX) with portfolio returns. *International Journal of Emerging Markets, 7*(4), 383–396. https://doi.org/10.1108/17468801211264306

Banerjee, P. S., Doran, J. S., & Peterson, D. R. (2007). Implied volatility and future portfolio returns. *Journal of Banking & Finance, 31*(10), 3183-3199. http://dx.doi.org/10.1016/j.jbankfin.2006.12.007

Bodie, Z., Kane, A., & Marcus, A. J. (2015). *Investimentos*. AMGH editora

Cainelli, P. V., Pinto, A. C. F., Klötzle, M. C. (2021). Estudo sobre a relação entre o IVol-BR e os retornos futuros do mercado acionário brasileiro. *Revista Brasileira de Contabilidade & Finanças* - USP, São Paulo, v. 32, n. 86, p. 255-272, maio/ago. 2021. https://doi.org/10.1590/1808-057x202009890

Campbell, J. Y., & Shiller, R. J. (1998). Valuation Ratios and the Long-Run Stock Market Outlook. *The Journal of Portfolio Management, 24*(2), 11-26. https://doi.org/10.3905/jpm.24.2.11

Duppati, G., Kumar, A. S., Scrimgeour, F., & Li, L. (2017). Long memory volatility in Asian stock markets. *Pacific Accounting Review, 29*(3), 423–442. https://doi.org/10.1108/PAR-02-2016-0009

Engle, R. F. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. *Econometrica, 50*(4), 987-1007. https://doi.org/10.2307/1912773

Fama, E. F. (1970). Efficient Capital Markets: A Review of Theory and Empirical Work. *The Journal of Finance, 25*(2), 383–417. https://doi.org/10.2307/2325486

Friedman, M., & Savage, L. J. (1948). The Utility Analysis of Choices Involving Risk. *Journal of Political Economy, 56*(4), 279–304. https://doi.org/10.1086/256692

Giot, P. (2005). Relationships Between Implied Volatility Indexes and Stock Index Returns. *The Journal of Portfolio Management, 31*(3), 92-100. https://doi.org/10.3905/jpm.2005.500363

Guiso, L., Sapienza, P., & Zingales, L. (2018). Time varying risk aversion. *Journal of Financial Economics, 128*(3), 403–421. https://doi.org/10.1016/j.jfineco.2018.02.007

Gujarati, D. N., & Porter, D. C. (2011). *Econometria Básica*. AMGH editora.

Imas, A. (2016). The Realization Effect: Risk-Taking After Realized Versus Paper Losses. *American Economic Review, 106*(8), 2086–2109. https://doi.org/10.1257/aer.20140386

Kahneman, D., & Tversky, A. (1979). Prospect Theory: An Analysis of Decision under Risk. *Econometrica, 47*(2), 263–291. https://doi.org/10.2307/1914185

Lee, Y. –H. (2015). Does the US Fear Gauge Impact on the Investor Fear Gauge in the Emerging Markets? *Journal of Emerging Market Finance, 14*(3), 197-209. https://doi.org/10.1177%2F0972652715601909

Li, J., Yu, X., & Luo, X. (2019). Volatility Index and the return–volatility Relation: Intraday Evidence from Chinese Options Market. *Journal of Futures Markets, 39*(11), 1348-1359. https://doi.org/10.1002/fut.22012

Lo, A. W. (2004). The Adaptive Markets Hypothesis. *The Journal of Portfolio Management, 30*(5), 15–29. https://doi.org/10.3905/jpm.2004.442611

Markowitz, H. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91. https://doi.org/10.1111/j.1540-6261.1952.tb01525.x

Mello, A. R. A. F. (2009). *Volatilidade implícita das opções de ações: uma análise sobre a capacidade de previsão do mercado sobre a volatilidade futura*. Dissertação de Mestrado, Fundação Getúlio Vargas, São Paulo, SP, Brasil.

Moraes, A. S. M. D., Pinto, A. C. F., & Klotzle, M. C. (2015). Previsão de value-at-risk e expected shortfall para mercados emergentes usando modelos FIGARCH. *Brazilian Review of Finance, 13*(3), 394-437-394–437. https://doi.org/10.12660/rbfin.v13n3.2015.53080

Pinho, F. M., Camargos, M. A., & Figueiredo, J. M. (2017). Uma Revisão da Literatura sobre Modelos de Volatilidade em Estudos Brasileiros de 2000 a 2014. *Revista de Administração FACES Journal, 16*(1), 9-28. http://dx.doi.org/10.21714/1984-6975FACES2017V16N1ART3213

Rubbaniy, G., Asmerom, R., Rizvi, S. K. A., & Naqvi, B. (2014). Do fear indices help predict stock returns? *Quantitative Finance, 14*(5), 831–847. https://doi.org/10.1080/14697688.2014.884722

Smales, L. A. (2016). Risk-on/Risk-off: Financial market response to investor fear. *Finance Research Letters*, 17, 125-134. https://doi.org/10.1016/j.frl.2016.03.010

Taleb, N. N. (2010). *The Black Swan: the impact of the highly improbable.* Random House Trade editora.

Thurner, S., Farmer, J. D., & Geanakoplos, J. (2012). Leverage causes fat tails and clustered volatility. *Quantitative Finance, 12*(5), 695–707. https://doi.org/10.1080/14697688.2012.674301

Vicente, J. V. M., & Guedes, T. S. (2010). A Volatilidade implícita contém informações sobre a volatilidade futura? Evidências do mercado de opções de ações da Petrobras. *Brazilian Business Review, 7*(1), 48-65. http://dx.doi.org/10.15728/bbr.2010.7.1.3

Whaley, R. E. (2000). The Investor Fear Gauge. *The Journal of Portfolio Management, 26*(3), 12-17. https://doi.org/10.3905/jpm.2000.319728

NOTES

CONSENT TO USE IMAGE
Does not apply.

APPROVAL OF THE RESEARCH ETHICS COMMITTEE
Does not apply.

CONFLICT OF INTERESTS
Does not apply.

This page intentionally left blank

---

[i] A financial market phenomenon of collective movement to more secure assets, generally observed in moments of economic crisis and uncertainty.

[ii] A concept created by Nassin Taleb to explain moments of crisis or rare events with large impact that cannot be foreseen.

[iii] For a complete description of the calculation of the VIX, see Cboe: https://www.cboe.com/micro/vix/vixwhite.pdf

[iv] For example, $r_5 d_t$ is calculated as follows: $\ln(P_{t+5}) - \ln(P_t)$, and is the prospective return related to the level of the IVol-BR observed at time t.

[v] The table of correlations has been omitted to save space, but it is available from the authors on request.