Original Paper

Performance of the Fama-French five-factor model in the pricing of anomalies in the Brazilian market

Performance do modelo de cinco fatores de Fama e French na precificação de anomalias no mercado brasileiro

Desempeño del modelo de cinco factores de Fama y French en la valoración de anomalías en el mercado brasileño

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Abstract
The purpose of this paper was to analyze the performance of the Fama-French five-factor model in the Brazilian stock market, in comparison to the three-and four-factor models, as well as to verify whether there are risk premiums associated to the anomalies size, book-to-market index, momentum, profitability and investment. Thus, we sought to analyze the efficiency of these models as a tool to support financial decision making in risk conditions. For this, the Fama-MacBeth methodology was used. The results of the first step regressions suggested that the Fama-French five-factor model presents the best performance in explaining the stock returns when compared to the others. However, in the results of the second step regressions, there were not found risk premiums associated to the profitability and investment factors, added to the three-factor model. Only the risks related to market, size, and book-to-market index consistently explained the cross-section of stock returns.
Keywords: Asset pricing; Market anomalies; Fama-French three-and five-factor models; Carhart four factor model

Resumo
O objetivo deste artigo foi analisar a performance do modelo de cinco fatores de Fama e French no mercado acionário brasileiro, em comparação às dos modelos de três e quatro fatores, bem como verificar se existem prêmios de risco associados às anomalias tamanho, índice *book-to-market*, momento, lucratividade e investimento. Buscou-se, assim, analisar a eficiência desses modelos como ferramentas de apoio à tomada de decisão financeira em condições de risco. Para isso, utilizou-se a metodologia de Fama e MacBeth. Os resultados das regressões do primeiro passo sugeriram que o modelo de cinco fatores apresenta o melhor desempenho na explicação dos retornos das ações quando comparado aos demais. No entanto, nas regressões do segundo passo, não se verificaram prêmios de risco associados à lucratividade e ao investimento, fatores adicionados ao modelo três fatores. Apenas os riscos relacionados ao mercado, tamanho e índice *book-to-market* explicaram consistentemente os retornos *cross-section* das ações.
Palavras-chave: Precificação de ativos; Anomalias de mercado; Modelos de três e cinco fatores de Fama e French; Modelo de quatro fatores de Carhart

Resumen
El objetivo de este artículo fue analizar el desempeño del modelo de cinco factores de Fama y French en el mercado brasileño, en comparación con los modelos de tres y cuatro factores, así como verificar si existen primas de riesgo por las anomalías tamaño, ratio book-to-market, momentum, rentabilidad y inversión. Así, buscamos analizar su eficiencia como herramienta de apoyo a la toma de decisiones en condiciones de riesgo. Para eso se utilizó la metodología de Fama y MacBeth. Los resultados del primer paso sugirieron que el modelo de cinco factores presenta el mejor desempeño para explicar la rentabilidad de las acciones en comparación con los demás. Sin embargo, en los resultados del segundo paso, no se encontraron primas de riesgo asociadas a la rentabilidad e inversión, adicionados al modelo de tres factores. Solo los riesgos relacionados con el mercado, tamaño y ratio book-to-market explicaron consistentemente los rendimientos de las acciones.
Palabras clave: Valoración de activos; Anomalías de mercado; Modelos de tres y cinco factores de Fama y French; Modelo de cuatro factores de Carhart

1 Introduction

In capital markets, various securities issued by listed companies are traded. These securities carry information about the characteristics of such companies or the macroeconomic environment in their prices, which, in turn, imply various levels of exposure to sources of systemic risk. Such factors lead investors to demand a premium when acquiring these assets; in other words, to price the risks.

Particularly in the Brazilian market, an increase in the volume of securities has been observed in the last five years, considering the different types traded (fixed and variable income securities). From 2015 to 2019, funding in this segment of the financial market soared from R$ 122 to R$ 435 billion BRL (in approximate values). In 2020, however, this growth rate dropped, and approximately 300 billion BRL was raised, given the scenario of an economic downturn (Anbima, 2020). Such information is relevant insofar as it reveals the pertinence of developing asset pricing models that effectively support investors in decision-making about the best options available to allocate their resources in portfolios of securities traded on capital markets (Paliienko, Naumenkova & Mishchenko, 2020).

The financial literature has been prolific in proposing models that seek to explain how assets are priced according to their systemic risk levels. Based on *Capital Asset Pricing Model* (CAPM), by Sharpe (1964), Lintner (1965) and Mossin (1966) – elaborated from the mean-variance theory by Markowitz (1952) – , *Intertemporal Capital Asset Pricing Model* (ICAPM) by Merton (1973) and *Arbitrage Pricing Theory* (APT) by Ross (1976) – developed in the context of multifactor models that link returns to different sources of systemic risk –, several empirical models have been proposed and tested over the years, typically incorporating the effects associated with macroeconomic aspects or firm characteristics. As suggested by Paliienko et al. (2020), the ability of a model to price assets accurately lies in its power to explain the anomalies identified in empirical studies, considering that they result either from a market failure or from the incorrect specification of previous models, particularly the CAPM. For Fama and French (2018), this scenario encourages debate on which factors can be incorporated into the models.

Building on the APT and ICAPM approaches – and the empirical evidence related to CAPM anomalies documented throughout the 1980s –, Fama and French (1993) proposed the three-factor model, in which asset returns are explained not only by the market premium but also by size and book-to-market ratio – anomalies what were documented by Banz (1981) and Lakonishok and Shapiro (1986), respectively. Following the identification of the momentum effect by Jegadeesh and Titman (1993), Carhart (1997) proposed the incorporation of a factor to the Fama-French three-factor model to account for that anomaly, which resulted in the four-factor model. In the years that followed, several anomalies were identified. As a result, their pricing was tested by Fama and French (1996, 2008), for example.

From the evidence of the effects of profitability and investment of firms (asset growth) on stock portfolios returns – documented in studies such as Titman, Wei and Xie (2004) and Novy-Marx (2013) –, Fama and French (2015) added two other factors to their previous model to capture these anomalies, thus proposing a five-factor model. The test results for the North American market showed that the five-factor model explains returns better than the three and four-factor models.

Indeed, the relevance of the investment and profitability anomalies prompted empirical studies on the five-factor model in different markets. In the Brazilian context, these include Siqueira, Amaral and Correia (2017), Vieira, Maia, Klotzle, and Figueiredo (2017) and Leite, Klotzle, Pinto, and Silva (2018), among others. Some of these tests showed that, in the Brazilian and North American markets, this model had had a superior performance in explaining stock returns, as the coefficient of determination proved to be higher through the addition of risk factors built from these two accounting variables.

On the one hand, the various empirical studies on factor models in the Brazilian market (and other emerging economies) have shared the common purpose of testing the adequacy of the models in explaining asset returns in contexts different from those for which they were developed. As argued by Fama and French (1998) and Atilgan, Demirtas and Gunaydin (2020), this can be explained by (i) the potential of emerging markets to allow the out-of-sample testing of these models, since they are segmented from developed markets due to their specificities; or, as suggested by Gregory, Tharyan and Christidis (2013), based on the arguments and results of Griffin (2002), by (ii) evidence that the Fama and French-style models have superior performance within countries when compared to world models (or their international versions). Evidence such as that presented in Matos and Rocha (2009), Santos, Famá and Mussa (2012), and Leite et al. (2018) revealed the efficiency of the models (more specifically of the four-factor model). On the other hand, studies on the predictive power of models in the Brazilian market have shown divergent results. For instance, empirical evidence from Mussa, Rogers and Securato (2009) and Rizzi (2012) suggests that none of the factor models they tested was able to explain the variations in returns. Therefore, empirical evidence does not point to a model i.e. consistently robust in the Brazilian market. This disagreement points to the existence of an opportunity to further the debate on the stock return generation process in the Brazilian market. As Griffin (2002) suggests, using an incorrect model can lead to errors in decision-making processes, such as those involving capital budgeting, portfolio assessment, and risk analysis.

The motivation of this study emerged in this context, i.e., to test the potential superiority of the five-factor model by Fama and French (2015) in explaining stock returns in the Brazilian market (first hypothesis). Thus, the performances of (i) the Fama-French three-factor model (1993), (ii) the Carhart four-factor model (1997), and (iii) the Fama-French five-factor model (2015) were compared. Then, we proceeded to test the hypothesis of the existence of risk premiums (the pricing of risk) associated with the factors of the models based on the following anomalies: size, book-to-market ratio, momentum, profitability, and investment (second hypothesis).

The difference between this study and other research conducted in Brazil was the adoption of the Fama and MacBeth (1973) methodology to estimate the models, which consists of a two-step regression. The two steps allowed (i) to evaluate the performance of the five-factor model (compared to the other models) and (ii) to verify the existence of risk premiums associated with the factors of the tested models. The five-factor model has been less frequently investigated in the Brazilian market when compared to the three and four-factor ones. Therefore, it seems pertinent to use it as an out-of-sample test. Furthermore, studies assessing the pricing of risk factors in the three, four, and five-factor models have remained scarce, i.e., those seeking to verify whether risks help explain the cross-section returns of stock portfolios based on anomalies. Most focus exclusively on analyzing the performance of models (first-step regressions). Therefore, given its objective, i.e., to analyze asset pricing models that capture the most commonly documented anomalies, this study is relevant in the context of decision-making on investments in Brazil.

2 Literature Review

2.1 The Fama-French and Carhart Asset Pricing Models

Based on empirical evidence that stock returns are associated with the anomalies size and book-to-market ratio, in addition to market premium (CAPM), Fama and French (1993) proposed the three-factor model, which is expressed by the econometric equation (1). The SMB factor (size) results from the difference between the returns of small-cap stock portfolios (small) and large-cap stock portfolios (big). The HML factor (book-to-market ratio) results from the difference between the stock portfolio returns of firms with a high book-to-market ratio (high) and those with a low book-to-market ratio (low). The market factor (R_m-R_f) is the difference between the returns on the market portfolio and the returns on risk-free assets. To test this model, Fama and French (1993) used data from Fama and French (1992), although relying on a different approach, namely the time-series test proposed by Black, Jensen and Scholes (1972).

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + s_i SMB + h_i HML + \varepsilon_i \quad (1)$$

Lather, based on the evidence presented by Jegadeesh and Titman (1993) that the strategy of buying stocks with high past returns and selling stocks with low past returns (momentum) produced abnormal positive returns during the subsequent year, the asset pricing studies, among them Fama and French (1996), tested this anomaly as well. Fama and French (1996) note that although the three-factor model explains many of the anomalies identified in empirical studies, it fails to capture the momentum effect. Indeed, Carhart (1997) added a factor representing this anomaly to the three-factor model, aiming to assess the performance of mutual funds. The four-factor model (econometric equation 2) incorporates momentum, which results from the difference between the returns of winning portfolios (*winners* – stocks with the highest accumulated returns in the recent past, and *losers* – stocks with the lowest accumulated returns in the recent past).

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + s_i SMB + h_i HML + \mathrm{w}_i \mathrm{WML} + \varepsilon_i \quad (2)$$

Motivated by previous evidence that the anomalies profitability and investment also contribute to describing stock returns (Titman et al., 2004; Novy-Marx, 2013; Fama & French, 2008; Fama & French, 2012), Fama and French (2015) incorporated two factors into the Fama-French model (1993), aiming to capture these sources of systematic risk and therefore proposed the five-factor model (econometric equation 3). The RMW (profitability) factor results from the difference between returns on the stock portfolios with high operating performance (robust) and those with low operating performance (weak). The CMA (investment) factor results from the difference in the average returns of firms with low investments (conservative) and those with high investments (aggressive).

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + s_i SMB + h_i HML + r_i RMW + c_i CMA + \varepsilon_i \quad (3)$$

In the empirical tests of this model, Fama and French (2015) used a sample of 606 observations of NYSE, AMEX, and NASDAQ stocks in the period 1963-2013. They initially built 25 portfolios based on combinations of the following anomalies: (i) size and book-to-market ratio; (ii) size and profitability; and (iii) size and investment. Then, the authors built 32 portfolios by combining (i) size, book-to-market ratio, and profitability; (ii) size, book-to-market, and investment ratio; and (iii) size, profitability, and investment. The results suggested an average return pattern associated with size, book-to-market ratio, profitability, and investment. Although the Gibbons, Ross and Shanken (1989) test (GRS) rejects the five-factor model (the intercepts were jointly different from zero), it showed an explanatory power ranging from 71% to 94%.

Furthermore, Fama and French (2015) found that the explanatory power of the HML factor is largely captured by RMW and CMA. To solve the redundancy problem, this factor was orthogonalized, i.e., obtained through the regression between HML and the other factors. In conclusion, Fama and French (2015) suggest that the model's main problem lies in its inability to fully capture the low returns of very small stocks (microcaps).

The five-factor model was subsequently tested in international markets, namely North America, Europe, Japan, and Asia-Pacific. Fama and French (2017) observed that stock returns for the North America, Asia-Pacific, and Europe regions increase with book-to-market ratio and profitability and decrease with investment. For Japan, average returns were found to have a weak relationship with profitability or investment and a strong relationship with book-to-market ratio. The results also revealed that the main problem of the model is associated with very small stocks (microcaps), whose returns behave like those of companies that invest aggressively, even though they have low profitability – similarly to what is documented in Fama and French (2015; 2016).

2.2 Empirical Evidence of Anomalies in the Brazilian Market

The Fama and French (1993; 2015) and Carhart (1997) pricing models have been adopted in most studies on asset pricing in the Brazilian market. In these analyses, the performance of the factor models is usually compared to that of CAPM. From this perspective, Rodrigues (2000) and Málaga and Securato (2004) evaluated the impact of market, size, and book-to-market factors on the returns of stocks listed on the São Paulo Stock Exchange. The performance of the three-factor model was superior to that of CAPM in both studies.

The performance of the Carhart (1997) model was also tested by several Brazilian studies in comparison to CAPM and the three-factor model. On the one hand, there is evidence of the superior performance of this model in relation to CAPM and the three-factor model. Matos and Rocha (2009) noted that: (i) CAPM was unable to capture the common sources of risk for the sample of 18 investment funds in the period 1997-2006; (ii) the predictive power of the four-factor model is superior to that of CAPM and the three-factor model. Similarly, Santos et al. (2012) observed, for a sample of stocks traded on the São Paulo Stock Exchange in the period 1995-2006, that (i) the three-factor model performed better than CAPM, and (ii) the four-factor model was superior to the three-factor model in explaining the variations in returns of the portfolios in question. On the other hand, there is no consistent evidence of risk premiums associated with the factors in the Carhart model (1997). Although Mussa et al. (2009) and Rizzi (2012) – who applied the Fama and MacBeth (1973) methodology – found similar results regarding the superior performance of the four-factor model, none of the models tested by them were able to explain the cross-section variations in returns, given the statistical significance of regressions intercepts. Furthermore, Mussa et al. (2009) found risk premiums associated exclusively with the market factor and the book-to-market ratio, which were consistently significant. No risk premiums associated with size and momentum were observed. Rizzi (2012), in turn, only documented the significance of market premium.

Following the proposition of the Fama and French (2015) model, its applicability to the Brazilian market was also tested. Martins and Eid Jr. (2015) showed that market, size, and book-to-market ratio seem to capture most of the variation in the returns of stock portfolios traded on the São Paulo Stock Exchange between 2000 and 2015. The five-factor model explained almost all variations in returns; i.e., it performed better than the Fama-French model (1993). However, the profitability and investment factors impacted returns less significantly when compared to the factors present in the Fama and French (1993) model.

Later, Siqueira et al. (2017) used the three, four and five-factor models to verify whether the informational asymmetry – measured by the Volume-Synchronized Probability of informed trading – represents an explanatory factor for the variations in the returns of stock portfolios traded in Brazil, Bolsa, Balcão (B3). Their results showed that market, size, profitability, investment, and asymmetric information factors form the combination that best explains portfolio returns. As in Fama and French (2015), the HML factor is redundant in the Brazilian market.

The performance of three, four, and five-factor models and CAPM in emerging markets – Brazil, Chile, Mexico, Argentina, India, China, Thailand, Malaysia, Turkey, Poland, Romania, and Russia – were analyzed by Leite et al. (2018). These authors observed that the four and five-factor models performed better than the three-factor model in most tests performed over the 2007-2017 period. However, as in Fama and French (2015), the HML factor has become somewhat redundant in the presence of profitability and investment factors. I.e., in the tests to assess whether the influences of this factor would be captured by the other factors in the model, its effect was practically fully explained by the profitability and investment factors.

With a different focus, Vieira et al. (2017) tested the risk factor pricing capacity of the Fama and French (2015) five-factor model for sector portfolios. The results pointed to the greater relevance of the investment factor, which was statistically significant in three of the five sectors analyzed.

In view of the above, the studies seem to disagree not only with the performance of asset pricing models tested in the Brazilian market but also the relevant risk premiums, which constitute the gap explored in this article.

3 Methodology

3.1 Analyzed sample

The population analyzed herein consists of all stocks listed on B3 between June 2000 and June 2017. At the individual level, data were collected from June 1999 to July 2018, which were necessary to construct the factors of the Fama and French (1993; 2015) and Carhart (1997) models. The criteria used for sample selection were the availability of the following firm's data (i) market value on June 30, with tolerance for the previous 30 days; (ii) market value and positive equity value on December 31 of the previous year – with a tolerance of 30 days prior to market capitalization; (iii) total assets on December 31 corresponding to the two years prior to the formation of the portfolios; (iv) operating profit on December 31 of the year prior to the year the portfolio was formed; and (v) consecutive monthly stock prices for the 12-month period before and after the portfolio formation (prices from the previous period were necessary for the construction of the momentum factor, and those from the subsequent period for the calculation of stock returns). Stocks of financial firms were excluded from the sample due to their high degree of leverage – as in Fama and French (1992; 1993). The information was treated at the asset level due to the low number of stocks listed on B3. I.e., companies that owned preferred and common stocks were treated as two different assets. This implies that two stocks of the same firm may have been used to construct different risk factors.

As a result, data were collected for 388 stocks for the entire sampling period. Each portfolio consisted of at least three stocks – a procedure similar to the one adopted by Málaga and Securato (2004) and Mussa et al. (2009). Python was used for data analysis and processing.

The closing prices of the stocks were adjusted to their dividends, the returns on individual assets were transformed into a Neperian logarithm, and the return on the portfolios was obtained using equation 4.

$$R_{p,t} = \sum_{i=1}^{n} \frac{VM_{i,t}}{VM_{p,t}} \left(R_{i,t}\right) \tag{4}$$

Where: $R_{p,t}$ represents the return on the portfolio *p* in *t*; $VM_{i,t}$ represents the market value of the stock *i* in *t*; $VM_{p,t}$ represents the market value of portfolio *p* in *t*; and $R_{i,t}$ represents the rate of return on the asset in *t*.

Table 1 shows how to operationalize the variables associated with the characteristics underlying the formation of portfolios.

3.2 Independent Variables – right-hand-side factors (RHS)

To outline the risk factors adopted in this study (independent variables of the first-step regressions), we resorted to the procedures described in Fama and French (1993, 2015) and Carhart (1997), totaling 388 stocks traded on B3. The factors of the three and four-factor models were built from similar criteria, consisting of the classification of stocks according to size, book-to-market ratio, and, for the momentum factor, their performance in the previous year (corresponding to the previous eleven months). For the five-factor model, stocks were ranked according to their size, book-to-market ratio, profitability, and investment. For the construction of portfolios, assets were classified increasingly by their market value in June of each year *t*, between 2000 and 2017. Once the market values for each asset were estimated, they were divided by their median in groups (i) big (B) and (ii) small (S). The first group consisted of assets with a high market value (with a market capitalization above the median), and the second by those with a low market value (with a market capitalization below the median).

Table 1
Operationalization of the variables (characteristics) underlying the portfolios

Variables	Formula	Description
Size (Market Value)	$VM_{i,t} = P_{i,t} \times N_{i,t}$	$VM_{i,t}$ represents the market value of stock *i* in *t*; $P_{i,t}$ represents the price of stock *i* in *t*; $N_{i,t,}$ represents the number of shares outstanding *i* in *t*.
Book-to-Market Ratio	$B/M_{i,t} = \frac{PL_{i,dez(t-1)}}{VM_{i,dez(t-1)}}$	$B/M_{i,}$ represents the book-to-market ratio of firm *i*, in *t*; $PL_{i,dez(t-1)}$ represents the shareholder's equity of firm *i* on December 31 of year *t-1*, in relation to the year of formation of the portfolio; $VM_{i,dez\ t(-1)}$ represents the market value of firm *i* on December 31 of year *t-1*, in relation to the year of formation of the portfolio.
Profitability	$Lucr_{i,t} = \frac{Lop_{i,dez(t-1)}}{PL_{i,dez(t-1)}}$	$Lucr_{i,t}$ represents the profitability of firm *i*, in year *t*; $Lop_{i,dez(t-1)}$ represents the operating profit of firm *i* on December 31 of year *t-1*; $PL_{i,dez(t-1)}$ represents the shareholders' equity of the firm *i* on December 31 of year t-1.
Investment	$Inv_{i,t} = \frac{(AT_{i,dez(t-1)} - AT_{i,dez(t-2)})}{AT_{i,dez(t-1)}}$	$Inv_{i,t}$ represents the investment value of firm *i*, in *t*; $AT_{i,dez(t-1)}$ represents the total assets of firm *i* on December 31 of year *t-1*; $AT_{i,dez(t-2)}$ represents the total assets of firm *i* on December 31 of year *t-2*, in which *t* refers to the year of formation of the portfolio.

Source: Adapted from Fama and French (1993, 2015) and Carhart (1997).

After the previous classification, also for the same month *t*, the assets were reclassified based on their book-to-market ratio, momentum, investment, and profitability values, hence forming the low, medium, and high groups. In other words, the factors were obtained by keeping size as the main variable and changing the second (classifying) variable in order to build the portfolios with the combination of two firm characteristics, according to a 2x3 criterion.

For book-to-market ratio, the stocks were ordered and separated into three groups according to their percentiles, so that they sequentially contained the following components: (i) 30% of stocks with the lowest book-to-market ratio (low); (ii) 40% of stocks with medium book-to-market ratio (medium) values; and (iii) 30% of stocks with the highest book-to-market ratio (high) values.

As for momentum, a similar procedure was carried out, in which the stocks were increasingly ranked according to their accumulated returns in the previous 11 months. The stocks were separated into three groups so that they contained: (i) 30% of stocks with the lowest returns; (i) 40% of stocks with medium returns; and, finally, (iii) 30% of stocks with the highest returns. The returns corresponding to the last month were ignored, according to Jegadeesh and Titman (1993) and Carhart (1997).

For profitability, the stocks were sorted and divided into three groups, also according to the 30-40-30 percentiles. The first group contains the stocks with the lowest profitability (weak). The second group contains stocks with medium profitability (medium). Finally, the third group contains the stocks with the highest profitability (robust).

For the investment factor, assets were sorted in ascending order and separated into three groups, according to the same procedure described above. The first group contains stocks with the lowest investments (conservative); the second group contains stocks with medium investments; and, finally, the third group contains the stocks with the highest investments (aggressive).

The procedure adopted for building the portfolios was repeated in June of each year of the sample period so that the newly published information available could be incorporated into the composition of the portfolios. Therefore, for each year of the sampling period, from July of the year *t* to June of the year *t+1*, the monthly returns of each portfolio were calculated, considering the weight on the market value of each asset in relation to the total market value of the portfolio.

These procedures led to the creation of 30 RHS portfolios (Appendix 1), which were used to build the risk factors (independent variables) of the pricing models estimated in the first step of the methodology proposed by Fama and MacBeth (1973). Table 2 shows the operationalization of the RHS factors of the three, four, and five-factor models.

Table 2
Operationalization of the RHS factors of the three, four, and five-factor models

RHS Factors	Formulas	Models
SMB Factor. Difference between the average returns of small and big portfolios.	$SMB = \bar{R}_S - \bar{R}_B$ $\bar{R}_S = \frac{SL_t+SM_t+SH_t}{3}$; $\bar{R}_B = \frac{BL_t+BM_t+BH_t}{3}$	Three-factor Four-factor
	$SMB = \bar{R}_S - \bar{R}_B$ $\bar{R}_S = \frac{SL_t+SN_{BMt}+ SH_t+SW_t+SN_{LUCt}+ SR_t+SC_t+ SN_{INVt}+SA_t}{9}$ $\bar{R}_B = \frac{BL_t+BN_{BMt}+BH_t+BW_t+BN_{LUCt}+BR_t+BC_t +BN_{INVt}+BA_t}{9}$	Five-factor
HML Factor. Difference between the average returns of the portfolios with high and low book-to-market ratios.	$HML_t = \bar{R}_H - \bar{R}_L$ $\bar{R}_H = \frac{SH_t+BH_t}{2}$; $\bar{R}_L = \frac{SL_t+BL_t}{2}$	Three, four, and five-factor
MKT Factor – Market Risk Premium. Difference between the average monthly returns, weighted by the monthly market value of all stocks in the sample analyzed in each year of the sample period, and the monthly return of the Interbank Deposit Certificate (CDI).	$MKT = R_{market\ portfolio,t} - R_{CDI,t}$ $R_{\text{market portfolio, t}}$ = proxy for market portfolio. $R_{\text{CDI,t}}$ = proxy for risk-free return on assets (CDI).	Three, four, and five-factor
WML Factor – Momentum. Difference between the average returns of the winning stock portfolios ("winners") and the average returns of the losing stock portfolios ("losers"). Winners are those with the highest cumulative returns over the previous 11 months; and losers are those with the lowest returns.	$WML_t = \bar{R}_W - \bar{R}_L$ $\bar{R}_W = \frac{SW_t+BW_t}{2}$; $\bar{R}_L = \frac{SL+BL_t}{2}$	Four-factor
RMW Factor – Profitability. Difference between the average returns of stock portfolios with robust operating performance and portfolios with weak operating performance.	$RMW_t = \bar{R}_R - \bar{R}_W$ $\bar{R}_R = \frac{SR_t+BR_t}{2}$; $\bar{R}_W = \frac{SW_t+BW_t}{2}$	Five-factor
CMA Factor – Investment. Difference between the average returns of conservative investment portfolios and aggressive investment portfolios.	$CMA_t = \bar{R}_C - \bar{R}_A$ $\bar{R}_C = \frac{SC_t+BC_t}{2}$; $\bar{R}_A = \frac{SA_t+BA_t}{2}$	Five-factor

Source: Prepared by the authors based on Fama and French (1993, 2015) and Carhart (1997).

3.3 Dependent Variables – left-hand-side (LHS) portfolio returns

The dependent variables used in both stages of this analysis correspond to the excess returns on LHS portfolios. For each year of the sample period, stocks were divided into five quintiles and classified by size. Subsequently, each quintile was reclassified according to a second variable (second ranking) and divided into quintiles. The portfolios were formed according to the following combinations: (i) size and book-to-market ratio; (ii) size and profitability; and (iii) size and investment. Therefore, for each variable considered in the second ranking, 25 portfolios were formed. The dependent variables of the three models analyzed here were calculated as the excess returns of these portfolios in relation to the return on the risk-free interest rate (whose proxy was the CDI), weighted by the market value of the assets with which the portfolios were formed.

3.4 Model estimation

The three (equation 1), four (equation 2), and five-factor (equation 3) models were estimated according to the methodology of Fama and MacBeth (1973), which consists of two-step regressions. In the first step, the excess returns on the 75 LHS portfolios (dependent variables) formed based on the anomalies size, book-to-market ratio, profitability, and investment, while keeping the size factor constant and varying the second characteristic, were regressed over the RHS factors (risk factors) of the estimated models – MKT, SMB, HML, WML, RMW and CMA (independent variables), according to each model. In other words, this step involved the estimation of time-series regressions to measure the sensitivities of the returns on LHS portfolios to variations in the returns of the RHS factors of the three, four, and five-factor models. As the objective of this study (the first hypothesis) was to verify whether the five-factor model produced improvements in explaining the average returns on the LHS portfolios, in the first step, the performance of the three pricing models analyzed was evaluated using the GRS test. The GRS test – used by Fama and French (1993, 1996, 2015, 2017) in their respective studies – is constructed to check whether the intercepts from the time-series regressions are jointly equal to zero (null-hypothesis) or different from zero (alternative hypothesis). To this end, the mean values of the descriptive statistics of the regression estimates were calculated. According to Fama and French (2015, 2017), the following statistics allow evaluating competing models: (i) GRS and its p-value; (ii) A|αi| – absolute values of intercepts α_i (represents the part of the returns on the LHS portfolios that the model does not explain); (iii) s(α) – standard deviation of intercept values; (iv) $A|\alpha_i|/A|r_i|$ – absolute mean value of intercepts α_i over the absolute value of r_i (difference between the absolute value of the average returns on the portfolio *i* and the average returns on all portfolios formed from the variables considered in the construction of the portfolio *i*) – measures the dispersion of the models intercepts in relation to the dispersion of LHS portfolio returns; and (v) adjusted R-squared. – adjusted coefficients of determination.

In the second step, the goal was to check whether there are premiums associated with the systematic risk coefficients estimated in the first step (sensitivities associated with RHS factors) in the Brazilian stock market; i.e., the second hypothesis formulated in this study. As the impact of these risk coefficients on returns is analyzed, the pricing of the risk they represent is verified. Therefore, at this stage, the cross-section regressions of excess returns on LHS portfolios (dependent variables) on the sensitivity coefficients associated with RHS factors (independent variables) were estimated. Subsequently, the estimates obtained from the regressions were analyzed using descriptive statistics. This analysis is essential because it reveals whether the pricing of systematic risks exists; i.e., whether investors demand a higher return as they perceive the presence of a specific type of risk and whether they discount it.

4 Discussion of Results

4.1 Descriptive Statistics of RHS Factors

Table 3 presents the descriptive statistics of RHS factors (independent variables in the first step regressions) of the estimated models. Among all factors, SMB showed the highest average monthly return (and the most statistically significant as well): (i) 1.06 for the three and four-factor models; and (ii) 1.03 for the five-factor model. Indeed, there is different evidence for this factor in Brazil: Machado and Medeiros (2011) and Siqueira et al. (2017) found negative mean values; and Martins and Eid Jr. (2015) and Vieira et al. (2017) found positive ones. The WML showed the second-highest average return (premium) of 0.61. However, as in Santos et al. (2012), this value was not statistically significant.

The MKT factor showed an average monthly premium of 0.25 in all models. Fama and French (1993, 2015) found a premium associated with MKT of (i) 0.43 for the three-factor model and (ii) 0.50 for the five-factor model. Relatively larger premiums were documented in: (i) Santos et al. (2012) – 1.56 in the period between 1995 and 2006; and (ii) Martins and Eid Jr. (2015) – 0.86 in the period between 2000 and 2012. Possibly, such divergence is due to differences between the samples and periods analyzed. As for the other factors, HML had a premium of 0.27; and CMA 0.54. The RMW factor had no premium (negative average return). In this case, it is possible to infer that highly profitable Brazilian companies do not seem to provide higher returns than low profitability ones, which suggests the potential irrelevance of this factor. However, none of these factors proved to be statistically significant.

Table 3
Summary statistics of RHS factors (three, four, and five-factor models)

Panel (a): Three and four-factor models

	MKT	SMB	HML	WML
Mean	0.25	**1.06**	0.27	0.61
Standard deviation	6.03	4.09	5.53	5.94
Skewness	-0.56	0.30	-0.70	-0.19
Kurtosis	1.83	0.28	4.52	1.19
t-statistic	0.62	3.81	0.72	1.51
P-value	0.54	**0.00**	0.47	0.13

Panel (b): Five-factor model

	MKT	SMB	HML	RMW	CMA
Mean	0.25	**1.03**	0.27	-0.23	0.54
Standard deviation	6.03	3.95	5.53	4.65	5.00
Skewness	-0.56	0.24	-0.70	-0.83	0.37
Kurtosis	1.83	0.07	4.52	3.76	2.80
t-statistic	0.62	3.84	0.72	-0.73	1.58
P-value	0.54	**0.00**	0.47	0.47	0.12

Source: Research Data.

As in Martins and Eid Jr. (2015), the mean value of SMB was the only significant value in this analysis, which corroborates the existence of the size effect in Brazil. Santos et al. (2012), in turn, only found significance for the HML factor. Thus, the results found here are similar to the results found in other pricing studies that analyzed the Brazilian market. However, discrepant results were also identified: Machado and Medeiros (2011) found statistical significance for the means of the factors of the three and four-factor models, except for SMB. Siqueira et al. (2017) reported that size had the lowest average returns for the three and four-factor models. These differences are possibly due to the number of portfolios and periods analyzed, which differ among studies.

4.2 Descriptive Statistics of LHS Portfolios

Table 4 shows the average excess returns on LHS portfolios (dependent variable) and the standard deviation of each one of them. Panel (a) presents descriptive statistics for the 25 portfolios formed by size and book-to-market ratio. The columns show the size effect, represented by a decrease in the average return on each portfolio as the size of the firm increases (with a few exceptions). This effect is similar to that documented by Fama and French (2015). However, contradicting that, Mussa et al. (2009) and Siqueira et al. (2017) found higher returns for stock portfolios of large companies listed on the Brazilian market. When comparing the returns of the low and high portfolios in the lines of the panel (a), the book-to-market ratio effect cannot be clearly observed. As pointed out by Fama and French (2015), the value effect was more clearly observed for small-cap stocks: (i) in the "small" line, the average return ranges from 0.46 to 1.00; in contrast, (ii) in the "big" line, a growth pattern cannot be identified.

Panel (b) presents descriptive statistics for portfolios formed by size and investment. Fama and French (2015) found that, as one moves from stocks with low investments (conservative) to those with high investments (aggressive), the average returns tend to decrease. However, this effect has not been documented in this study.

Panel (c) presents descriptive statistics for portfolios formed by size and profitability. They corroborate the profitability effect documented by Novy-Marx (2013) and Fama and French (2015): for each quintile according to size, the stock portfolios of companies with high profitability are associated with higher average returns when compared to the average returns of stocks of companies with low profitability. However, this result differs from those found by Siqueira et al. (2017) and Martins and Eid Jr. (2015), whose studies did not find patterns in the average returns on the portfolios according to size and profitability.

Table 4
Summary statistics of LHS portfolios

Panel (a): size - book-to-market ratio										
	Mean					**Standard deviation**				
	Low	2	3	4	High	Low	2	3	4	High
Small	0.45	0.53	1.45	1.58	1.00	10.45	8.91	9.28	11.20	8.61
2	0.67	1.21	0.71	0.54	0.96	9.58	8.19	7.70	7.60	8.43
3	0.09	0.67	0.68	1.16	0.86	14.66	7.63	7.99	7.17	8.14
4	0.58	0.32	0.55	0.49	0.69	7.26	7.09	6.77	7.04	7.26
Big	-0.17	-0.09	0.08	0.05	-0.05	7.18	6.52	7.49	6.96	8.93
Panel (b): size - investment										
	Mean					**Standard deviation**				
	Low	2	3	4	High	Low	2	3	4	High
Small	0.46	1.00	0.35	1.58	1.50	10.91	11.49	9.76	8.62	8.56
2	0.74	0.59	1.11	0.55	1.17	9.39	7.60	8.04	6.93	8.95
3	1.05	1.29	0.81	-0.87	0.43	6.76	7.40	10.10	16.09	8.36
4	0.71	0.41	0.45	0.56	0.28	6.67	6.61	7.12	7.60	7.67
Big	-0.13	-0.02	0.06	0.08	-0.33	6.45	8.08	8.47	7.21	8.04
Panel (c): size - profitability										
	Mean					**Standard deviation**				
	Low	2	3	4	High	Low	2	3	4	High
Small	0.63	0.58	0.56	1.39	1.68	13.00	9.76	7.48	8.81	11.34
2	0.87	0.91	0.20	0.59	1.52	10.24	8.37	7.80	6.62	7.72
3	0.72	0.00	1.07	1.38	-0.02	9.31	6.98	8.65	10.42	13.97
4	0.24	0.24	-0.34	0.93	1.18	8.15	7.36	6.32	6.29	7.25
Big	-0.36	-0.43	-0.31	0.25	0.10	8.51	7.83	7.91	7.31	6.60

Source: Research Data.

4.3 First Step: Regression Statistics (time-series models)

Table 5 presents the results of the tests performed for the first-step regressions (time-series models) for the three, four, and five-factor models – conducted separately for the portfolios formed according to size and book-to-market ratio (panel (a)), investment (panel (b)) and profitability (panel (c)). As in Fama and

French (2015, 2017), the summary statistics of the regression parameters estimated in the first step were calculated to measure the performance of the factor models. The parameters were set as follows: (i) mean of the GRS statistic and p-value; (ii) mean of the absolute values of the intercepts (A|αi|); (iii) mean value of the intercepts' standard deviations (s(α)); (iv) mean of the absolute intercepts in relation to the dispersion of the LHS portfolio returns ($A|\alpha i|/A|r_i|$); and (v) the adjusted coefficients of determination (Adjusted R-squared).

The summary statistics suggest that in panels (a) and (c), the best performance was achieved by the five-factor model (smallest GRS statistic and highest p-value), corroborating the results of Fama and French (2015), Martins and Eid Jr. (2015), and Siqueira et al. (2017); whereas in panel (b), the best performance was achieved by the four-factor model. Furthermore, it appears that the null hypothesis of the GRS test was rejected for the three and four-factor models estimated for portfolios according to size and profitability. In this case, the results show that the factors failed to explain the returns on LHS portfolios and that other ones could be incorporated into them.

The A|ai| values also indicate superior performance of the five-factor model for portfolios formed by size and book-to-market ratio and by size and profitability. For portfolios formed by size and investment, the four-factor model showed the best performance.

The adjusted R-squared of the estimated models points to a better fit of the five-factor model (followed by the four and three-factor models). When compared with the results of Fama and French (2015), the coefficients of determination found in this study are much lower.

Table 5
Summary statistics for tests of the first-step regressions of factor models

Panel (a): Portfolios formed by size – book-to-market ratio						
	GRS	**p-value**	**A|αi|**	**s(α)**	**A|αi|/A|ri|**	**Adjusted R-squared**
Three-factor	1.0545	0.3998	0.2593	0.2872	0.7312	0.525361
Four-factor	1.2042	0.2400	0.2722	0.3162	0.7676	0.533313
Five-factor	0.8737	0.6414	0.2498	0.3060	0.7044	0.541159
Panel (b): Portfolios formed by size – investment						
	GRS	**p-value**	**A|αi|**	**s(α)**	**A|αi|/A|ri|**	**Adjusted R-squared**
Three-factor	1.1657	0.2761	0.3631	0.4413	0.8293	0.480316
Four-factor	0.9819	0.4934	0.3622	0.4182	0.8272	0.487639
Five-factor	1.0996	0.3466	0.3705	0.5070	0.8461	0.509247
Panel (c): Portfolios formed by size – profitability						
	GRS	**p-value**	**A|αi|**	**s(α)**	**A|αi|/A|ri|**	**Adjusted R-squared**
Three-factor	1.8679	0.0103	0.4380	0.4720	0.8543	0.482783
Four-factor	1.7613	0.0184	0.4180	0.4516	0.8153	0.492782
Five-factor	1.3852	0.1151	0.3070	0.3599	0.5989	0.512195

Source: Research Data.

In summary, the evidence suggests that the returns on stock portfolios traded on the Brazilian capital market are better explained by the four and five-factor models when compared to the three-factor one. To analyze this hypothesis further, the first-step regressions were performed with different combinations of factors, and their results are presented and discussed in the next section.

4.4 Regressions with Different Combinations of RHS Factors

In order to identify which combination of factors makes up the best model for the sample analyzed here, Table 6 presents the results of the GRS test and summary statistics for various combinations of factors. We also performed tests without the HML factor to verify its pertinence in asset pricing in the Brazilian market, as this factor proved to be redundant in studies such as those by Fama and French (2015), Siqueira et al. (2017), and Leite *et al.* (2018).

In panel (a) – portfolios formed by size and book-to-market ratio –, the best performance was observed for the combination of the following factors: R_m-R_f and SMB (smallest GRS statistic and highest p-value). In panel (b) – portfolios formed by size and investment –, the combination of factors R_m-R_f, SMB, and WML produced the model with the best performance. However, the results of both groups can be considered inferior when compared to those of the same sets of portfolios presented previously in Table 5. In panel (c) – portfolios formed by size and profitability –, the null hypothesis of the GRS test was rejected for all combinations of factors, which points to the models' inability to explain the average returns on LHS portfolios.

In general, the results discussed in this section demonstrate that the different combinations of factors were not able to outperform the models estimated in their original structure (for the estimates, see Table 5).

Table 6
Summary statistics for tests of first-step regressions for different combinations of RHS factors

Factors	GRS	p-value	A|αi|	s(α)	A|αi|/A|ri|	Adjusted R-squared
Panel (a): Portfolios formed by size – book-to-market ratio						
Rm-Rf / SMB	1.0623	0.3902	0.2811	0.3282	0.7926	0.4873
Rm-Rf / SMB / WML	1.2092	0.2355	0.3049	0.3678	0.8595	0.4978
Rm-Rf / SMB / RMW	1.1464	0.2957	0.3601	0.4862	1.0152	0.5121
Rm-Rf / SMB / CMA	1.1312	0.3116	0.3046	0.3850	0.8589	0.5136
Rm-Rf / SMB / CMA / RMW	1.1342	0.3085	0.3448	0.4636	0.9720	0.5227
Panel (b): Portfolios formed by size – investment						
Factors	**GRS**	**p-value**	**A|αi|**	**s(α)**	**A|αi|/A|ri|**	**Adjusted R-squared**
Rm-Rf / SMB	1.1857	0.2568	0.3738	0.4678	0.8537	0.4655
Rm-Rf / SMB / WML	1.0377	0.4208	0.3945	0.4675	0.9008	0.4739
Rm-Rf / SMB / RMW	1.2653	0.1896	0.4092	0.5791	0.9345	0.4837
Rm-Rf / SMB / CMA	1.2920	0.1704	0.4020	0.5056	0.9179	0.4958
Rm-Rf / SMB / CMA / RMW	1.2619	0.1923	0.4064	0.5726	0.9279	0.5040
Panel (c): Portfolios formed by size – profitability						
Factors	**GRS**	**p-value**	**A|αi|**	**s(α)**	**A|αi|/A|ri|**	**Adjusted R-squared**
Rm-Rf / SMB	1.8596	0.0108	0.4323	0.4590	0.8433	0.4668
Rm-Rf / SMB / WML	1.7756	0.0170	0.3806	0.4310	0.7423	0.4780
Rm-Rf / SMB / RMW	1.6102	0.0400	0.3442	0.4063	0.6713	0.5006
Rm-Rf / SMB / CMA	1.8268	0.0129	0.4326	0.4539	0.8437	0.4852
Rm-Rf / SMB / CMA / RMW	1.5957	0.0430	0.3430	0.3980	0.6691	0.5061

Source: Research Data.

4.5 Second Step: Cross-section Regressions Results

In the cross-section regressions estimated in the second step of the Fama and MacBeth (1973) two-step methodology, the dependent variable of the models was the average excess return of each one of the 75 LHS portfolios, and the independent variables were the estimated slope coefficients of the first-step regressions (the sensitivities associated with the RHS factors).

Table 7 shows the mean values of the estimates of the cross-section regression parameters, as well as the statistic results of the testing of the hypotheses underlying the regression models. The five-factor model had the highest adjusted coefficient of determination (approximately 66%). These values were similar for the other models: (i) approximately 60% for the three-factor model; and (ii) 61% for the four-factor model. The findings by Mussa et al. (2009) corroborate the results presented here, i.e., higher adjusted R-squared for the four-factor model, compared to the three-factor model. Rizzi (2012), on the contrary, found a higher adjusted R-squared for the three-factor model compared to the four-factor model.

All slope coefficients of the three-factor model – b, s, and h – proved to be statistically significant, suggesting the pricing of market risks, size, and book-to-market ratio by market investors. For the four-factor model, the coefficient linked to the momentum – w – was the only insignificant one, which suggests the inexistence of a risk premium associated with it. These results partially corroborate those found by Mussa et al. (2009), who only identified market and book-to-market ratio premiums. For the five-factor model, the coefficients related to profitability and investment – r and c – were not significant, pointing to the inexistence of risk premiums associated with profitability and investment. In general, market, size, and book-to-market ratio seem to explain the cross-sectional variations in portfolio returns. The incorporation of momentum, profitability, and investment into asset pricing models does not seem to be pertinent for the analyzed sample. However, the findings by Mussa et al. (2009) and Rizzi (2012), who tested the three and four-factor models for the Brazilian market using the two-step methodology, differ from those reported here: for both models tested in those studies, market and book-to-market ratio were significant, whereas size and momentum were not.

Furthermore, Table 7 shows that the mean estimates of the intercepts of the estimated models were significant. This suggests that the analyzed risks (independent variables of the three models tested in the second step) failed to explain the cross-sectional excess returns on LHS portfolios. Possibly, macroeconomic events such as financial crises or those associated with the performance of companies (firm-specific characteristics) in view of business cycles could contribute to explaining returns. In other words, other factors not included in the models could help explain the returns of stock portfolios traded in the Brazilian market.

Table 7
Summary statistics of the second-step regression parameters (cross-section)

Panel (a): Three-factor model

	α	b	s	h	Adjusted R-squared
Coeff	1.1877	1.1877	0.9649	0.4523	0.6009
p-value	0.0001	0.0003	0.0000	0.0052	

Panel (b): Four-factor model

	α	b	s	h	w	Adjusted R-squared
Coeff	1.1398	-1.1527	0.9922	0.4091	0.3319	0.6162
p-value	0.0002	0.0005	0.0000	0.0113	0.2250	

Panel (c): Five-factor model

	α	b	s	h	r	c	Adjusted R-squared
Coeff	1.1254	-1.1052	0.9639	0.7352	0.2123	-0.0666	0.6596
p-value	0.0003	0.0009	0.0000	0.0000	0.1940	0.7528	

Panel (d) Durbin-Watson and Breusch-Pagan test statistics

	DW	DW (p-value)	BP	BP (p-value)
Three-factor	2.0215	0.5030	3.8138	0.2823
Four-factor	2.0360	0.5250	4.8946	0.2983
Five-factor	2.1311	0.6875	4.7524	0.4468

Panel (e) VIF Statistics of the independent variables of the factor models

	Rm-Rf	SMB	HML	WML	RMW	CMA
Three-factor	1.01	1.02	1.02			
Four-factor	1.03	1.02	1.1	1.11		
Five-factor	1.03	1.06	1.03		1.03	1.07

Source: Research Data.
Note: the coefficients b, s, h, w, r, and c refer respectively to factors R_m-R_f, SMB, HML, WML, RMW, and CMA; α is the intercept. In panel (d), the Durbin-Watson test statistics point to the absence of autocorrelation issues. The Breusch-Pagan test statistics revealed no heteroskedasticity. In panel (e), the VIF statistics suggest the absence of multicollinearity between the explanatory variables.

5 Conclusion

This study had two objectives: The first was analyzing whether Fama and French's (2015) five-factor model can explain the returns of stock portfolios traded in the Brazilian capital market better when compared to other previous models, namely the Fama and French three-factor model (1993) and the Carhart four-factor model (2007). The second was to check whether the systematic risks associated with the factors in these models are priced by investors. To carry out the analysis, the procedures proposed by such authors were followed, and the portfolios were formed based on the following anomalies: size, book-to-market ratio, momentum, profitability, and investment.

Initially, summary statistics of RHS factors were analyzed. Subsequently, these factors were used as independent variables of the models estimated in the first step of the Fama and MacBeth two-step methodology. The analysis revealed that the SMB factor was the only one in which the mean value was statistically significant in all analyzed models. Then, the summary statistics of LHS portfolios, the mean, and the standard deviations of the portfolio returns used as dependent variables suggested the following effects: (i) size – for portfolios formed by size and book-to-market ratio; (ii) value (related to book-to-market ratio) – which proved to be more pronounced for small-cap stocks; and (iii) profitability. The initial results suggested that the investment would not be priced by the market.

Afterward, the analysis of the results was performed in two steps. In the first one, the time-series regressions were estimated to obtain the impacts of variations in the RHS factors (risk factors) on the variations in the returns on LHS portfolios, as well as the intercept coefficients. Then, the performance of the three-factor models was evaluated based on the statistics referring to the intercepts. In the second, the coefficients of the RHS factors estimated in the previous step were used to test the hypothesis of the existence of risk premiums associated with the factors.

In the first-step regressions of the three, four, and five-factor models, the five-factor model was the one that best explained the average returns on portfolios. This result corroborates Fama and French (2015) and contributes as an out-of-sample test of the five-factor model, as it proved to be superior to the other models for the Brazilian market sample analyzed here. This evidence supports the first hypothesis of this study, as well as the results documented by Siqueira et al. (2017) and Leite *et al.* (2018). When performing the analysis between the different groups of LHS portfolios, we observed that the models estimated from the factors built according to size and investment performed better, suggesting that the choice of anomalies to sort portfolio returns is relevant. In total, the first hypothesis of this study was corroborated: the five-factor model has superior performance when compared to the three and four-factor models.

In the second-step regressions, the results allowed us to check whether the coefficients obtained in the models estimated in the first step help to explain the returns on LHS portfolios. Although the five-factor model had the highest adjusted coefficient of determination compared to the others, the results showed that none of the models tested here was able to explain fully the cross-section returns on LHS portfolios. This occurred because a proportion of their variances were left unexplained by the factors included in the models (the intercepts were significant in all models). Size was the most critical risk factor in the performance of the models and proved to be significant in all estimated models. Momentum, investment, and profitability were not statistically significant. In the cross-section regressions (through which the pricing of risks associated with market, size, book-to-market ratio, momentum, profitability, and investment was tested), the model with the best fit was the three-factor model, given the statistical significance of all its risk factors. This was not observed for the other models tested. Therefore, the second hypothesis of this research was rejected, as the risk factors of the five-factor model are not priced by investors.

Thus, we conclude that the five-factor model performed better than the three and four-factor models in the Brazilian context. However, the risks associated with profitability and investment are not priced. The results showed that investors demand risk premiums associated with size, book-to-market ratio, and market. Perhaps this result stems from the fact that the stocks in the sample are mainly of small companies, which are less profitable and make fewer investments. Therefore, there is no premium associated with these risks.

Finally, another possible explanation for the absence of risk premiums in the five-factor model is the fact that portfolio returns are generated by other events, such as macroeconomic ones (financial crises), those specific to companies throughout business cycles, or even by aspects that influence the industries with stocks traded on the capital market differently. These could contribute to explain them but were not considered in this paper and therefore represented a limitation of this study.

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Appendix 1 – Operationalization of the RHS factors for the three, four, and five-factor models

Fama-French three-factor model (1993)			
Portfolio	Composition	Portfolio	Composition
Small and Low	Stocks with low market value and low book-to-market ratio	Big and Low	Stocks with high market value and low book-to-market ratio
Small and Medium	Stocks with low market value and medium book-to-market ratio	Big and Medium	Stocks with high market value and medium book-to-market ratio
Small and High	Stocks with low market value and high book-to-market ratio	Big and High	Stocks with high market value and high book-to-market ratio
Carhart four-factor model (1997)			
Portfolio	Composition	Portfolio	Composition
Small and Loser	Stocks with low market value and lower past returns	Big and Loser	Stocks with high market value and lower past returns
Small and Medium	Stocks with low market value and medium past returns	Big and Medium	Stocks with high market value and medium past returns
Small and Winner	Stocks with low market value and higher past returns	Big and Winner	Stocks with high market value and higher past returns
Fama-French five-factor model (2015)			
Portfolio	Composition	Portfolio	Composition
Small and Low	Stocks with low market value and low book-to-market ratio	Big and Low	Stocks with high market value and low book-to-market ratio
Small and *Neutral*	Stocks with low market value and medium book-to-market ratio	*Big* and *Neutral*	Stocks with high market value and medium book-to-market ratio
Small and High	Stocks with low market value and high book-to-market ratio	Big and High	Stocks with high market value and high book-to-market ratio
Small and Weak	Stocks with low market value and low profitability	Big and Weak	Stocks high low market value and low profitability
Small and Neutral	Stocks with low market value and medium profitability	Big and Neutral	Stocks with high market value and medium profitability

Portfolio	Composition	Portfolio	Composition
Small and Robust	Stocks with low market value and high profitability	Big and Robust	Stocks with high market value and high profitability
Small and Conservative	Stocks with low market value and low investment	Big and Conservative	Stocks with high market value and low investment
Small and Neutral	Stocks with low market value and medium investment	Big and Neutral	Stocks with high market value and medium investment
Small and Aggressive	Stocks with low market value and high investment	Big and Aggressive	Stocks with high market value and high investment

Source: Prepared by the authors, based on Fama and French (2015).

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DATASET
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