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COEXISTENCE OF PROFIT-SEEKING AND RENT-SEEKING BEHAVIORS AS AN EVOLUTIONARY STABLE STRATEGY IN A LARGE-POPULATION PLAYING THE FIELD MODEL

COEXISTÊNCIA DE PRODUTORES E NÃO-PRODUTORES COMO UMA ESTRATÉGIA EVOLUCIONARIAMENTE ESTÁVEL EM UM MODELO PLAYING THE FIELD

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ABSTRACT

Objetive: This paper explores the coexistence of productive, profit-seeking behavior and unproductive, rent-seeking behavior using an evolutionary game approach. It identifies a unique Evolutionarily Stable Strategy (ESS) and shows how the prevalence of unproductive agents directly influences the incentives to either engage in productive activities, modeled on Cournot competition, or participate in rent-seeking contests, similar to the Tullock framework, for a share of the productive sector's output. The paper further explores how institutional factors—such as the marginal return to contest participation and government taxation of the productive sector—affect the spread and persistence of rent-seeking behavior. In line with empirical evidences, the analysis highlights how unproductive behaviors can become entrenched and pervasive within a population, influencing the long-term evolution of economic systems.

KEYWORDS: Rent-seeking contest. Evolutionary games. Evolutionary Stable Strategy. Large-population playing the field model.

RESUMO

Objetivo: Este artigo explora a coexistência de comportamentos produtivos, orientados ao lucro, e comportamentos improdutivos, de busca por renda econômica (*rent*), usando uma abordagem de jogos evolucionários. Identifica-se uma Estratégia Evolucionariamente Estável caracterizada pela presença duradoura de agentes improdutivos influencia diretamente os incentivos para se engajar em atividades produtivas, formalizadas como um oligopólio de Cournot, ou participar de atividades improdutivas, formalizadas como competições por rendas econômicas (*rent-seeking contests*), semelhantes ao torneio de Tullock. O artigo também explora como fatores institucionais – como o retorno marginal da participação em competições por rendas econômicas oriundas da tributação governamental sobre o setor produtivo – afetam a propagação e a persistência do comportamento de busca por rendas econômicas. Em linha com evidências empíricas, a análise destaca como comportamentos improdutivos podem se consolidar e se tornar perenes em uma população, influenciando a evolução de sistemas econômicos a longo prazo.

PALAVRAS-CHAVE: Torneios do tipo “rent-seeking”. Jogos evolucionários. Estratégia evolucionariamente estável. Modelo “playing the field” com grandes populações.

Classificação JEL: C73, D43, D72

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1 INTRODUCTION

Rent-seeking, a concept first popularized by economist Gordon Tullock in the 1960s, refers to the pursuit of wealth through manipulation or exploitation of the political and economic environment rather than through productive economic activity. This behavior results from the allocation of resources towards activities that do not generate wealth, leading to inefficiencies in the economy.

Traditional economic models have largely approached rent-seeking through static analyses, emphasizing individual rationality and equilibrium states. However, these models fail to capture the dynamic and adaptive nature of rent-seeking behavior. As agents interact within an environment, their behavior evolve over time, influenced by competition, learning, and the relative success of different strategies. Evolutionary models, on the other hand, put emphasis on explaining the diffusion of certain behaviors in a population, instead of explaining the agents' actions.

Evolutionary approaches to rent-seeking have two main streams in the literature. One is a historical and institutional perspective which aims to explain how the institutions, where understood as the "rules of the game", create the incentives to agents engage in either productive or unproductive (rent-seeking) behavior, and how rent-seekers adapt to the changes and evolution of those institutions, such that, rent-seeking exists from ancient Roman society until our days (Baumol, 1990; Davidson; Ekelund, 1994; Henrekson; Sanandaji, 2011). The second aims to explain, through evolutionary models, how rent-seeking behavior can diffuse and persist in a population even though being an inefficient allocation of resources. For instance, Hehenkamp et al. (2004) and Leininger (2003) have used the concept of an Evolutionary Stable Strategy (ESS) to analyze the amount of effort and rent-dissipation in a Tullock contest.

This paper follows the second stream from the two above mentioned. Differently from early evolutionary rent-seeking contest games, such as Hehenkamp et al. (2004) and Leininger (2003), our model explores the coexistence of a productive *profit-seeking* sector – described as a Cournot game – and an unproductive *rent-seeking* sector - described as a rent-seeking contest – whose rent is captured from government taxes, and shows how institutional features such as the marginal return of the entry cost on the rent-seeking contest or the government's taxation on the productive sector affect the pervasiveness and persistence of unproductive behavior in the economy.

The remainder of this paper is structured as follows. The next section shows some motivating evidence about rent seeking and briefly discusses some related literature. The third section presents the structure of the evolutionary game model set forth in this paper and demonstrates the existence and uniqueness of an evolutionary stable strategy featuring behavioral heterogeneity (i.e., coexistence of both productive and unproductive behaviors). A final section with concluding remarks closes this paper.

2 PERSISTENCE AND PERVASIVENESS OF RENT-SEEKING: SOME MOTIVATING EVIDENCE AND RELATED LITERATURE

Rent-seeking refers to a behavior where agents' (individuals, firms, interest groups) effort to maximize value generates social cost instead of social surplus (Buchanan, 1980). In other words, rent-seeking designates the expenditures made to change the institutional setting in order to capture *rent* – here understood as the excess of opportunity cost – instead of allocating those resources to productive activities. The social cost arises because the resources used for rent-seeking have a positive opportunity cost elsewhere in the economy. Rent-seeking is at best a zero-sum activity when it simply reallocates endowments among people and groups and is probably negative-sum if traditional deadweight costs result as a by-product of such activities (Tollison, 2012).

The concept of rent-seeking dates back to Tullock (1967), although the term only appears later in the literature. Tullock argues that protective taxes or tariffs not only create social costs due to the deadweight loss but also due to the allocation of productive resources in inefficient activities. Agents who might benefit from these protective measures have incentives to invest resources in lobbying for the taxes or tariffs. Those lobbying expenditures, which might compensate each other, are ultimately unproductive for society since they are an attempt to transfer or resist the transfer of wealth rather than a contribution to wealth creation.

The term rent-seeking was first introduced by Krueger (1974) and, even though she was unaware of Tullock's work, both presented very similar ideas. In her paper, Krueger shows that the welfare costs of rent-seeking exceed those of government intervention by analyzing the competition for import licenses. In a competition for import licenses where the allocation of those licenses is proportional to the firm's physical plant, some firms may expand their productive capacity beyond the efficient level to secure more import licenses. Another example is when the import licenses are allocated *pro rata*. In this case, each importer receives fewer licenses than they would buy in a free-market scenario, and competition for rents occurs through entry into the industry with smaller-than-optimally sized firms.

Posner (1975) stated the first version of a rent-seeking contest, where participants bid (lobby) to obtain a monopoly. His model describes a constant-cost game where the probability of winning corresponds to the level of investment made, with the total available rents being fully dissipated. Posner assumes that if the government or a regulatory agency assures a monopoly, there will be economic agents willing to bid for the control of that monopoly up to the point where, at the margin, the cost of obtaining the monopoly equals the expected profit from being a monopolist.

Besides these three classical examples mentioned above, rent-seeking has been used to study other cases such as price control on consumption goods (televisions and cars) in Poland (Tarr, 1994); agricultural promotion in Austria (Salhofer, 2000); and export quotas on the coffee sector in Brazil (Jarvis, 2005}, among others.[1] A common feature of all is that rent is created through the regulatory power of the government. However, it is possible to have rent-seeking strictly in a private economy. For instance, rent-seeking activities may arise among siblings for the dispute of inheritance within the family; or in certain labor markets where exists competition for higher corporate positions or a limited number of openings (admission to a graduate program or a spot on a professional sports team roster) can lead to costly rent-seeking activities by participants (Tollison, 2012).

In this paper, rent-seeking is understood as an unproductive activity (Baumol, 1990; Bhagwati, 1982). According to Bhagwati (1982), an unproductive activity represents a form of income generation by engaging in directly unproductive activities, i.e., they yield pecuniary returns but do not produce goods or services that enter a social function through increased production or availability to the economy. Moreover, we are interested in cases of rent-seeking in large populations – as opposing to industries or specific sectors. Some of the examples below help to illustrate those cases.

1 See Del Rosal (2011) for a literature review on empirical measurements of rent-seeking costs.

In Ancient Rome, the chances of making money through politics were huge. Wealth flowed in from war pillage, fines, provincial taxes, loans, and various other sources like never before in Greco-Roman history, and it increased rapidly. While the public treasury gained from this, a lot of it likely ended up in the hands of the nobles. Although the Roman system allowed people to earn money through business and commerce, it also came with a loss of status. So, economic activity was not seen as a way to get rich (Baumol, 1990, p. 899). In China, families invested a lot of time and resources into preparing their children for the imperial examinations. During the Sung dynasty, these exams took place every three years, and only a few hundred people across the country managed to pass each time. Wealth was in prospect for those who passed the examination and who were subsequently appointed to government positions (Baumol, 1990, p. 901). During the Middle Ages in England, prestigious and wealth-generating activities were religious or warfare, instead of commerce or entrepreneurship. Due to the primogeniture rule, younger sons who chose not to enter the clergy life had no socially accepted option to build a fortune but warfare. Clearly, all that money came either from loots or the government revenue, i.e. taxes (Baumol, 1990, p. 903).

Although the examples are from pre-capitalist societies, they help us to illustrate cases where part of the population engages in unproductive activities in order to capture some of the wealth created by the remaining. Some contemporary examples are companies that try to tackle their competitors through the judicial system (litigation) instead of "fair" market competition (Parisi and Luppi, 2015); or the individual participation in political destabilization process and military coups (Mbaku and Paul, 1989; Mbaku, 1994).

A traditional form to model rent-seeking behavior is through a rent-seeking contest (Nitzan, 1994; Tullock, 2001). In those models, rent seekers enter a costly dispute for either a share of the rent (shared-prize models) or for the totality of the rent (winner-takes-it-all models). With no intention to cover the entire literature on modeling rent-seeking games, we present some previous works on rent-seeking contest which helps to illustrate our modeling choices in the following section.

Park et al. (2005) model a rent-seeking game in a general equilibrium framework where rent-seekers capture their rent directly from public coffers. The authors formalize the productive sector and assume that the government chooses the tax rate in order to maximize economic growth. Opportunistic rent-seeking behavior emerges from agents who attempt to capture the taxes collected by the government. One of the main results of the paper is that a higher tax rate leads to a lower fraction of labor effort allocated to productive (work) relative to unproductive (rent-seeking) activities.

Dari-Matticci and Parisi (2005) propose a solution to the Tullock's *over dissipation* paradox. The paradox arises in contests where the marginal return of the effort is sufficiently high such that the optimal effort becomes higher than the participation constraint (the expected value of the prize). Although the expected profit to participate in such a contest is negative, the result where no agent enters the contest (no-participation equilibrium) is not expected to be stable since all agents have the incentive to put the minimum amount of effort in the contest and take all the prize. The solution is to add an exit option as a first stage of the contest. In this setting, agents' payoffs contain both a discrete element (the decision whether or not to play), and a continuous element (their share in the prize) that depends on their efforts when playing. Total expenditures in rent seeking begin to decline after the marginal return of the effort reaches a certain value, as parties will start using the exit option, which opens the possibility that rents remain unexplored.

Finally, there are papers which also address a rent-seeking contest from an evolutionary perspective (Hehenkamp et al., 2004; Leininger, 2003). Differently from our focus in this paper – the coexistence and persistence of productive and unproductive behaviors in a large population – those previous papers aim to find the evolutionary stable strategy regarding the effort to participate in the rent-seeking contest. Both papers discuss the conditions for an ESS in a finite population, and concluded that, under some conditions, evolutionary stable behavior in Tullock contests leads to higher efforts of contestants than Nash behavior and may entail *overdissipation* of the contested rent.

3 INTERACTION BETWEEN PROFIT-SEEKING AND RENT-SEEKING BEHAVIORS AS AN EVOLUTIONARY GAME

In order to analyze the persistence and pervasiveness of rent-seeking we now develop an evolutionary game, in which each agent chooses whether or not to engage in a productive activity to earn income: she can be either productive (profit seeker) or unproductive (rent seeker). Each agent in this economy is free to switch her strategy (behavior) any time, whenever she thinks the other strategy will be more beneficial to herself.

3.1 Microeconomic Setting and Temporary Equilibrium

Consider a large economy, with n agents, where each agent's behavior has a small impact on the whole. In that economy, in each period each individual can choose between two strategies (types of behavior) mutually exclusive: to be a productive individual, becoming a producer, or not, in this case, becoming an unproductive (rent-seeking) individual. Productive agents behave as profit-maximizing producers. Meanwhile, the unproductive agents expropriate the taxes paid by the productive agents.

Let us define the *temporary equilibrium* as the time frame during which the number of individuals playing the rent-seeking strategy, denoted by u, is predetermined, so that the number of producers, $n-u$, is also predetermined.

Given the size of the productive sector, $n-u$, producers interact with each other in a Cournot game-frame, determining price and quantities produced and, consequently, a certain amount of tax. The total taxes collected becomes the rent to be contested by the u rent-seeking agents in the unproductive sector.

3.1.1 Productive sector as a Cournot game

We will formalize the interaction among producers as a Cournot game, which as well-known depends, among other things, on the number of producers, given by $n-u$.

Consider that the producers face the following inverse market demand:

(1) $$p = a - b\sum_{j\in\mathbf{P}} q_j ,$$

where $p \in \mathbb{R}_{++}$ is the market price, $q_i \in \mathbb{R}_{+}$ is the producer i's output, $\mathbf{P}$ is the set of producers and $a \in \mathbb{R}_{++}$ and $b \in \mathbb{R}_{++}$ are parametric constants.

Let the cost conditions be identical for each producer i and given by the following cost function:

(2) $$C(q_i) = cq_i ,$$

where $c \in (0,a) \subset \mathbb{R}$ is the constant marginal cost of producer i.

Taxes are collected from profits. Thus, considering (1) and (2), producer i's net profit can be written as follows:

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(3) $$\pi_i = (1-\tau)\left[a - b\left(\sum_{j\in\mathbf{P}_{-i}} q_j\right) - c\right]q_i ,$$

where $\tau \in (0,1) \subset \mathbb{R}$ is a tax rate constant and exogenously determined and $\mathbf{P}_{-i}$ is the set of producers consisting of producers other than i.

At each temporary equilibrium, quantities and prices are set as in a Cournot oligopoly game. Producers maximize their profits choosing the quantities they produce while taking into account their expectations on what other producers' chosen quantities will be. The first-order condition for an interior solution which determines the producer i's best-reply is:

(4) $$\frac{\partial \pi_i}{\partial q_i} = (1-\tau)\left[a - c - b\left(\sum_{j\in\mathbf{P}_{-i}} q_j^*\right) - 2bq_i^*\right] = 0 ,$$

where q_k^* denotes the best-reply of producer k.

From (5) we obtain the producer i's best-reply:[2]

(5) $$q_i^* = \frac{a-c}{2b} - \frac{1}{2}\left(\sum_{j\in\mathbf{P}_{-i}}^{i-1} q_j^*\right),$$

As all producers have the same cost function and expect the same behavior from their rivals they will all supply the same quantity q^*, that is, $q_i^* = q_j^* = q^*$ for all producers i and j. Therefore, recalling that there are $n-u$ producers and letting $\mu \equiv u/n$ stands for the proportion of rent-seeking agents in the population, it follows from (1) that the output of each producer in temporary equilibrium is given by:[3]

(6) $$q^* = \frac{a-c}{b[1+(1-\mu)n]},$$

while the market quantity in this equilibrium is given by:

(7) $$Q^* = (n-u)q^* = \left(\frac{a-c}{b}\right)\frac{(1-\mu)n}{1+(1-\mu)n}.$$

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[2] Since $\frac{\partial^2 \pi_i}{\partial q_i^2} = -2b(1-\tau) < 0$ for any $q_i \in \mathbb{R}_+$, we know that the net-profit function in (3) is strictly concave, it follows that the second-order condition for profit maximization is satisfied and, therefore, the choice in (5) is in fact a maximizer.

[3] That is, the Cournot equilibrium for given number of producers, $n-u=(1-\mu)n$. Note that for a given size of population, n, the Cournot equilibrium is parameterized by the number of rent-seekers, u, which can adjust endogenously over time towards an evolutionary stable frequency distribution of types of behavior across agents, $(\mu, 1-\mu)$.

In turn, the market price in the temporary equilibrium is found by using (1) and (6)-(7):

(8) $$p^* = \frac{a+(1-\mu)nc}{1+(1-\mu)n}.$$

Finally, based on (6) and (8), it will be useful ahead to have the gross (pre-tax) profit of each producer in the temporary equilibrium:

(9) $$\Pi^* = (p^* - c)q^* = \frac{1}{b}\left[\frac{a-c}{1+(1-\mu)n}\right]^2 \equiv \Pi(\mu).$$

Given the signs of the parametric constants assumed so far, and recalling that $a > c > 0$, from (6)-(9) we have for all $\mu \in [0,1] \subset \mathbb{R}$ the following comparative static results for the temporary equilibrium:

(10) $$\frac{\partial q^*}{\partial \mu} = \frac{(a-c)n}{b[1+(1-\mu)n]^2} > 0,$$

(11) $$\frac{\partial Q^*}{\partial \mu} = -\left(\frac{a-c}{b}\right)\frac{n}{[1+(1-\mu)n]^2} < 0,$$

(12) $$\frac{\partial p^*}{\partial \mu} = \frac{(a-c)n}{[1+(1-\mu)n]^2} > 0, \text{ and}$$

(13) $$\frac{\partial \Pi^*}{\partial \mu} = \frac{2(a-c)^2 n}{b[1+(1-\mu)n]^3} > 0.$$

Therefore, for a given population of agents, an increase in the proportion of rent-seeking agents, by decreasing the numbers of producers, results both in a higher market price and output of each producer in temporary equilibrium, which leads to an increase in gross profit of each producer in this equilibrium. However, this expansion of individual outputs does not surpass the associate reduction of the numbers of producers, so that the market quantity in temporary equilibrium decreases when the number of rent-seeking agents in the economy is higher.

3.1.2 Unproductive Sector as a Rent-Seeking Contest

Now let us consider those individuals who do not engage in any productive activity, seeing in the amount of taxes collected by the government an opportunity for rent seeking. The source of that rent is the total tax revenue collected by the government from productive agents in a determined temporary equilibrium. Considering (9), we can write the rent in a given temporary equilibrium as follows:

(14) $$V(\mu) \equiv (n-u)\tau\Pi(\mu) = (1-\mu)n\tau\Pi(\mu).$$

We will formalize the interaction among unproductive agents as a rent-seeking contest game, which will be parametrized in the temporary equilibrium by the number of unproductive agents (from now on, rent-seekers), given by u. Dari-Mattiacci and Parisi (2005) developed a deterministic rent-seeking game with two players. Drawing on it, we will propose a generalization of that game for any finite number u of players.

More precisely, consider u self-interested rent seekers who engage in a contest in which each one spends costly effort to capture a net share (i.e. net of this effort) of a predetermined rent in (14) for a given frequency distribution of types of behavior across agents, $(\mu, 1-\mu)$. Using the traditional Tullock's (logit) contest outcome (or success) function,[4] the ith rent-seeker's net share in the rent can be written as:

(15) $$S_i = \frac{(x_i)^r}{\sum_{j \in U}(x_j)^r} - x_i,$$

where $x_i \in [0,1] \subset \mathbb{R}$ is the fraction of the rent that is spent on rent seeking by the ith rent seeker, $r \in \mathbb{R}_{++}$ is the marginal return of rent-seeking activity and U stands for the set of unproductive agents (rent seekers).[5]

[4] More details on this function, see Fey (2008), Hehenkamp et al. (2004); Nitzan (1994); Tullock (2001).

[5] Following Dari-Matticci and Parisi (2005), we opt to express the ith rent seeker's utility function as in (5). It is usual to express the utility function of a rent seeker i as the difference between the absolute value of rent seeking that is captured by the rent seeker, given by $\frac{(x_i)^r}{\sum_{j \in U}(x_j)^r} V(\mu)$, and the absolute value spent to capture it, given by $x_i V(\mu)$, that is, the common utility function is $\frac{(x_i)^r}{\sum_{j \in U}(x_j)^r} V(\mu) - x_i V(\mu) = S_i V(\mu)$. Note that, for a given rent $V(\mu)$, the maximization of $S_i V(\mu)$ is equivalent to maximize S_i.

Each rent seeker wants to maximize her net share in (15) by choosing the best possible fraction of the rent that she spends on rent seeking (i.e. her effort to capture a rent). Thus, the best-reply of rent-seeking agent i has to satisfy the following first-order condition:[6]

(16) $$\frac{\partial S_i}{\partial x_i} = \frac{r(x_i)^{r-1}\left(\sum_{j\in U}(x_j)^r\right) - (x_i)^r r(x_i)^{r-1}}{\left(\sum_{j\in U}(x_j)^r\right)^2} - 1 = 0 .$$

As all rent-seeking agents have the same contest outcome function in (14) and expect the same behavior from their rivals they will all spend the same effort x^*, that is, $x_i^* = x_j^* = x^*$ for all rent seekers i and j. Therefore, recalling that there are u rent seekers and $\mu \equiv u/n$, it follows from (16) that the optimal effort of each rent-seeking agent in temporary equilibrium is given by:

(17) $$x^* = \left(\frac{u-1}{u^2}\right) r = \left[\frac{\mu n - 1}{(\mu n)^2}\right] r .$$

In other words, in the temporary equilibrium there exists a unique symmetric Nash equilibrium in the unproductive sector, in which each rent seeker spends the proportion x^* of the rent $V(\mu)$ on rent seeking.

Based on (15) and (17) we can obtain the net share of each rent seeker in temporary equilibrium:

(18) $$S^* = \frac{(x^*)^r}{\sum_{j\in U}(x^*)^r} - x^* = \frac{1}{u} - x^* = \frac{1}{\mu n} - \left[\frac{\mu n - 1}{(\mu n)^2}\right] r \equiv S(\mu, r) .$$

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[6] From the first-order condition in (16) we obtain $\frac{\partial^2 S_i}{\partial x_i^2} = \left(\sum_{j\in U}(x_j)^r\right)^{-3}\left[r(r-1)(x_i)^{r-2}\left(\sum_{j\in U}(x_j)^r\right)^2 - \left[r(3r-1)\right](x_i)^{2(r-1)}\left(\sum_{j\in U}(x_j)^r\right) + 2r^2(x_i)^{3r-2}\right]$. As all rent-seekers' optimization problems have the same objective function in (15) and they expect the same behavior from their rivals, the rent-seeker i knows that the other rent-seekers will spend the same fraction x^* as her. Thus, as $x_i^* = x_j^* = x^*$ for all rent seekers i and j, the previous second-order derivative expression simplifies to $\left.\frac{\partial^2 S_i}{\partial x_i^2}\right|_{x_i = x_j = x^*, \forall j \in U} = \frac{1}{u(x^*)^{3r}}\left\{r(r-1)(x^*)^{r-2}\left(u(x^*)^{2r}\right) - \left[r(3r-1)\right](x^*)^{2(r-1)}\left(u(x^*)^r\right) + 2r^2(x^*)^{3r-2}\right\} = \frac{-(x^*)^{3r-2}2r^2(1-u)}{u(x^*)^{3r}}$, which is strictly negative for any $u \geq 2$.

Hereafter, we assume that the marginal return of rent seeking activity is non-increasing, i.e. $r \in (0,1] \subset \mathbb{R}$. As a result, $S^* \geq 0$ for all $\mu n \geq 1$. Given that assumption about the marginal return of rent seeking activity and the signs of the remaining parametric constants assumed so far, from (17)-(18) we have the following comparative static results for the temporary equilibrium in the unproductive sector:

(19) $$\frac{\partial x^*}{\partial r} = \frac{\mu n - 1}{(\mu n)^2} > 0$$, for all $u = \mu n > 1$,

(20) $$\frac{\partial x^*}{\partial \mu} = \frac{n}{(\mu n)^2}\left(\frac{2 - \mu n}{\mu n}\right) r \leq 0$$, for all $u = \mu n \geq 2$,

(21) $$\frac{\partial S^*}{\partial r} = -\frac{\partial x^*}{\partial r} = -\left[\frac{\mu n - 1}{(\mu n)^2}\right] < 0$$, for all $u = \mu n > 1$, and

(22) $$\frac{\partial S^*}{\partial \mu} = -\frac{n}{(\mu n)^2} - \frac{\partial x^*}{\partial \mu} = -\frac{n}{(\mu n)^3}[(1-r)\mu n + 2r] < 0$$, for all $r \in (0,1] \subset \mathbb{R}$.

Therefore, as in Dari-Matticci and Parisi (2005, p. 415), the effort in the temporary equilibrium increases in the marginal return of rent seeking. However, the net-share decreases when x^* increases as a reaction to an increase in r. Moreover, an increase in the mass of agents in the unproductive sector decreases the effort the rent seekers put on the contest when there are three or more agents in the unproductive sector. The net share is, alike, strictly decreasing in the proportion of unproductive agents if the marginal return of rent-seeking activity is decreasing or constant.

3.2 Persistence of Behavioral Heterogeneity in a Large-Population Playing the Field Scenario

In this section we will show that the persistence of rent-seeking behavior in the economy previously outlined, in which its productive sector is characterized as a Cournot oligopoly game disturbed by rent-seeking behavior, can emerge as an evolutionary stable strategy (ESS). To do so, we need to specify the expected payoff of each strategy (profit-seeking and rent-seeking behaviors).

We will take as the expected payoff of profit-seeking strategy the producer's net (after-tax) profit defined in (3), which is directly proportional to the gross (pre-tax) profit in temporary equilibrium given in (9), that is,

(23) $$\pi(\mu, \tau) = (1-\tau)\Pi(\mu)$$.

In turn, considering (3), (14) and (18), the expected payoff of rent-seeking strategy, that is, the expected payoff of those individuals who do not engage in any productive activity, seeing in the amount of taxes collected by the government an opportunity for rent seeking, can be taken as:

(24) $$\psi(\mu,\tau,r) \equiv S(\mu,r)V(\mu,\tau) = \left[\frac{1}{\mu n} - \left(\frac{\mu n - 1}{(\mu n)^2}\right) r\right](1-\mu)n\tau\Pi(\rho).$$

Considering the gross profit in (9), it is straightforward to see that the expected payoffs of behavioral strategies in (23) and (24) are non-linear with respect to the proportion of rent-seekers, μ. In this case, although the payoff of each agent is (as implied by its definition as an expected utility) linear in the own mixed strategy, it also depends on the strategy frequencies in the population, given by $(\mu, 1-\mu)$. As pointed out by Crowford (1992, p. 301), this scenario is known as the "playing the field" model, which allows describing simultaneous interaction of the agents in a population in more general ways than the well-known "pairwise random matching" model. We further assume that we have a *large-population* scenario, so that each agent's expected payoff can be well approximated using, in addition to her own mixed strategy, the expected payoffs in (23) and (24), in which the strategy frequencies in the population is computed without excluding the strategy choice of agent, since excluding it would have a negligible effect on the frequencies $(\mu, 1-\mu)$. In sum, we will formalize the selection mechanism between profit-seeking and rent-seeking strategies across entire population as a *large-population playing the field* model (Crawford, 1992, p. 303).

Let y the probability that an agent chooses to be unproductive in a given period, so that the probability vector $\boldsymbol{\sigma} = (y, 1-y)$ denotes her mixed strategy. Let $E(\boldsymbol{\sigma} \mid \mathbf{v})$ stands for the expected payoff of an agent playing the mixed strategy $\boldsymbol{\sigma}$ when the population strategy frequencies are given by $\mathbf{v} = (\mu, 1-\mu)$. Based on that notation, and using (23) and (24), this expected payoff can be written as follows:

(25) $$E(\boldsymbol{\sigma} \mid \mathbf{v}) = y\psi(\mu) + (1-y)\pi(\mu).$$

Drawing on Crawford (1992, p. 303), we can define a *large-population ESS* as an individual's mixed strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$, such that for each mixed strategy $\boldsymbol{\sigma} = (y, 1-y) \neq \boldsymbol{\sigma}^*$, the following condition holds

(26) $$E\left(\boldsymbol{\sigma}^* \mid (1-\varepsilon)\boldsymbol{\sigma}^* + \varepsilon\boldsymbol{\sigma}\right) > E\left(\boldsymbol{\sigma} \mid (1-\varepsilon)\boldsymbol{\sigma}^* + \varepsilon\boldsymbol{\sigma}\right),$$

for all sufficiently small fraction $\varepsilon \in \mathbb{R}_{++}$ of agents plays a mixed strategy $\boldsymbol{\sigma}$.

The large-population playing the field model has a unique ESS $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$, where y^* is defined in (27), featuring the two strategies (profit-seeking and rent-seeking behaviors) as survivors. These results regarding the existence and persistence of behavioral heterogeneity can be formally established as follows.

Proposition (*Coexistence of profit-seeking and rent-seeking behaviors*)**.** For a given vector of parameters (n, a, b, c, τ, r), with $n \geq 2$, $a \in \mathbb{R}_{++}$, $b \in \mathbb{R}_{++}$, $c \in (0,a) \subset \mathbb{R}$, $\tau \in (0,1) \subset \mathbb{R}$ and $r \in \mathbb{R}_{++}$, there exists a unique evolutionary stable strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$ in the large-population playing the field game with expected payoffs given by (23) and (24), where

(27) $$y^* = \frac{1}{2}\left[\frac{\left(n(1-r)-r\right)\tau}{n(1-r\tau)} + \sqrt{\frac{\left[\left(n(1-r)-r\right)^2 \tau + r^2\tau - 2nr\left((1+r)\tau - 2\right)\right]\tau}{\left(n(1-r\tau)\right)^2}}\right] \in (0,1) \subset \mathbb{R}$$

is the probability with an agent becomes a rent seeker, which equalizes the expected payoffs (23) and (24) and, therefore, the following condition holds

(28) $$S(y^*, r)(1-y^*)n\tau - (1-\tau) = 0.$$

Proof: see Appendix A.

The Proposition 1 clarifies that in a situation where all agents adopt the same mixed strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$, this strategy will not be invadable by any other distinct (pure or mixed) strategy. This happens because the mixed strategy $\boldsymbol{\sigma}^*$ is, as demonstrated in the Appendix A, a strict, symmetric Nash equilibrium, that is, the mixed strategy $\boldsymbol{\sigma}^*$ is the only best response to itself. Therefore, no agent who plays another mixed strategy with a probability of being rent seeker other than $\boldsymbol{\sigma}^*$ will end up realizing that her strategy $\boldsymbol{\sigma} \neq \boldsymbol{\sigma}^*$ generates an expected payoff $E\left(\boldsymbol{\sigma} \mid \boldsymbol{\sigma}^*\right)$ strictly smaller than $E\left(\boldsymbol{\sigma}^* \mid \boldsymbol{\sigma}^*\right)$.

It is also worth noting that in the ESS $\boldsymbol{\sigma}^*$, since all agent adopt the same mixed strategy, represents a monomorphic situation, in which the profile over pure strategies (profit-seeking and rent-seeking behaviors) induced by the common mixed strategy $\boldsymbol{\sigma}^*$ will not be concentrated in a single type of observed behavior. In other words, each agent becomes a rent-seeking individual with probability given by (27), so that the fraction $y^* \in (0,1) \subset \mathbb{R}$ of the population of agents is made up of rent-seeking agents.

Based on the Implicit Function Theorem, we can use the condition in (28) to infer some impacts of variations of the tax rate, τ, and the marginal returns to rent-seeking expenditures, r, on the pervasiveness of rent-seeking, y^*:

(29) $$\frac{\partial y^*}{\partial \tau} = \frac{-1/\tau}{\frac{\partial S(y^*,r)}{\partial y}(1-y^*) - S(y^*,r)} > 0, \text{ for all } r \in (0,1] \subset \mathbb{R} \text{ and } y^* n > 1,$$

and

(30) $$\frac{\partial y^*}{\partial r} = \frac{\left(\frac{y^* n - 1}{(y^* n)^2}\right)(1-y^*) n\tau}{\frac{\partial S(y^*,r)}{\partial y}(1-y^*) - S(y^*,r)} < 0, \text{ for all } r \in (0,1] \subset \mathbb{R} \text{ and } y^* n > 1.$$

The sign of derivative in (29) follows from (22) and the assumption that the marginal return of rent-seeking activity is non-increasing (i.e. $r \in (0,1] \subset \mathbb{R}$), which implies that $S(y^*,r) \geq 0$ for all $\mu n \geq 1$. Therefore, when the tax rate rises (falls), the pervasiveness of rent-seeking activities in the evolutionary equilibrium, measure by y^* in (27), increases (decreases), since the net profits for the individuals that remain productive is squeezed (increased) and the rent is increased (squeezed), making the unproductive activities relatively more attractive. In turn, a rise (reduction) in the marginal return of rent-seeking activity has in the long run a negative (positive) effect on the effort the rent seekers put on the contest, consequently decreasing (increasing) the individual net share of rent-seeking activity, which generates a decrease (increase) in the relative frequency of rent seekers.

4 FINAL REMARKS

This paper analyzes the existence and persistence of rent-seeking behavior based on an evolutionary game approach showing how rent-seeking behavior can coexist with productive activities (profit-seeking behavior). We built an evolutionary game in which each individual can choose between two strategies (types of behavior) mutually exclusive: to be a productive individual, becoming a producer, or not, in this case, becoming an unproductive (rent-seeking) individual. Agents in the productive sector make their production decisions as in a Cournot game, while the remaining agents engage in a rent-seeking contest *à la Tullock* to capture the tax collected from the productive sector. We show that under some reasonable conditions a unique mixed strategy where both behaviors coexist is an evolutionary stable strategy (ESS), that is, is a strategy immune to invasions.

The proportion of unproductive activity in the population plays a crucial role in explaining this stability. In a Cournot game, marginal outputs increase and the aggregate output decreases when agents exit the game. For our case, it means that producers are better off while rent-seeker will have to fight for a smaller value of rent (the taxed part of the aggregate pre-tax output). The number of players is also important in the rent-seeking contest since more contestants decrease the expect net-share of each contestant. Thus, at a point, an increase in unproductive activities creates incentives for agents to change their strategy for productive activities. More important, this strategy is evolutionary stable, meaning that another strategy trying to compete with will be pushed out.

Our model also highlights the importance of the institutional framework, interpreted as the taxes, in the pervasiveness of rent-seeking. An increase in taxes decreases the expected profit from producers, while increases the total share of production captured by taxes and shared among the rent-seekers, making the proportion of unproductive behavior to rise. Therefore, an exogenously determined parameter, tax rate, plays a key role in the determination of the prevalence of each strategy in the population, since its increase also increases incentives to engage in unproductive activities.

Compared to the previous literature on evolutionary rent-seeking contest (Hehenkamp et al., 2004; Leininger, 2003) our model looks to the decision to participate in the (un)productive sector – instead of the effort put in the rent-seeking contest – as a rent-seeking contest with an exit option (Dari-Matticci; Parisi, 2005). Rent-seekers are not obligate to engage in the rent-seeking contest since they have the exit option of choosing the productive strategy. Moreover, our framework provided a microfundation to the productive sector, while previous works simply assume the existence of a rent for which the contestants are competing. This setting is similar to present in the corruption literature in models where the unproductive sector (corrupt) exists in a parasitic way regarding the productive sector (Griebeler; Hillbreacht, 2015; Mishra, 2006).

Our result regarding the positive effect of taxes in the rent-seeking behavior is also in line with existing literature. Park et al. (2005) propose a general equilibrium model where rent-seekers extract their rent from public coffers. The government, in turn, chooses the optimal value for the taxes in order to maximize economic growth. Likewise in our framework, a rise in taxes leads to an increase in the incentives to engage in rent-seeking activities.

Appendix A. Coexistence of profit-seeking and rent-seeking behaviors

Taking into account the Theorem 1 established and demonstrated by Crawford (1992, p. 304), we know that the mixed strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$ will be a large-population ESS, defined by the condition (26), in the large-population playing the field game proposed in this paper, if $\boldsymbol{\sigma}^*$ is a strict, symmetric Nash equilibrium in that game, and there exists a neighborhood of $\mathbf{v} = \boldsymbol{\sigma}^*$ throughout which $E(\boldsymbol{\sigma} \mid \mathbf{v})$ in (25) is continuous in $\mathbf{v} = (\mu, 1-\mu)$ for all $\boldsymbol{\sigma} = (y, 1-y)$.

Considering that the functions in (9), (23) and (24) are continuous for all $\mu \in (0,1] \subset \mathbb{R}$, it is straightforward to see that the expected payoff $E(\boldsymbol{\sigma} \mid \mathbf{v})$ in (25) is indeed continuous in $\mathbf{v} = (\mu, 1-\mu) \neq (0,1)$ for all $\boldsymbol{\sigma} = (y, 1-y) \neq (0,1)$.

Given the continuity of the expected payoff in (25), in order to determine whether there exists a large-population ESS we just need to find the strict, symmetric Nash equilibrium, that is, the mixed strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$ such that $E\left(\boldsymbol{\sigma}^* \mid \boldsymbol{\sigma}^*\right) > E\left(\boldsymbol{\sigma} \mid \boldsymbol{\sigma}^*\right)$ for any $\boldsymbol{\sigma} \neq \boldsymbol{\sigma}^*$. In other words, we need to search for the mixed strategy $\boldsymbol{\sigma}^* = (y^*, 1-y^*)$ which is not invadable by any other mixed strategy $\boldsymbol{\sigma} = (y, 1-y) \neq \boldsymbol{\sigma}^*$.

Since the functions in (9), (23) and (24) are well defined for any $y^* \in (0,1] \subset \mathbb{R}$, we know by (25) that

(A.1) $$E(\boldsymbol{\sigma} \mid \boldsymbol{\sigma}^*) = y\psi(y^*) + (1-y)\pi(y^*)$$

is differentiable at any $y \in (0,1] \subset \mathbb{R}$.

If there exists a maximizer of the expected payoff in (A.1) in the open interval $(0,1) \subset \mathbb{R}$, the following first-order (necessary) condition has to be satisfied for any $y^* \in (0,1) \subset \mathbb{R}$:

(A.2) $$\frac{\partial E(\boldsymbol{\sigma} \mid \boldsymbol{\sigma}^*)}{\partial y} = \psi(y^*) - \pi(y^*) = \left[S(y^*, r)(1-y^*)n\tau - (1-\tau)\right]\Pi(y^*) = 0,$$

where the second equality was obtained based on (9), (23) and (24). Since $\lim_{\mu \to 0^+}[\psi(\mu) - \pi(\mu)] = +\infty$ and $\psi(1) - \pi(1) = -(1-\tau)(a-c)^2/b < 0$, we can infer that a solution y^* for the equation in (A.2), if it exists, in fact is in the open interval $(0,1) \subset \mathbb{R}$.

As $\Pi(\mu)$ in (9) is strictly positive for all $\mu \in [0,1] \subset \mathbb{R}$, we can infer that (A.2) is satisfied if, and only if, the expression between brackets in (A.2), which is the condition in (28), is null. Let $\phi(y) \equiv S(y,r)(1-y)n\tau - (1-\tau)$. The condition in (28) is satisfied if, and only if, $\phi(y^*) = 0$ for any $y^* \in (0,1] \subset \mathbb{R}$. Since $\lim_{y \to 0^+} \phi(y) = +\infty$, $\phi(1) = -(1-\tau) < 0$ and ϕ is continuous along the domain $(0,1] \subset \mathbb{R}$, we can then apply the intermediate value theorem to readily conclude that there is some $y^* \in (0,1) \subset \mathbb{R}$ such that $\phi(y^*) = 0$. Actually, we can solve directly the equation in (28) to obtain the unique solution $y^* \in (0,1) \subset \mathbb{R}$ given by (27) such that $0 < y^* < 1$.

Given the existence and uniqueness of $y^* \in (0,1) \subset \mathbb{R}$, and considering that $\lim_{y \to 0^+} [\psi(y) - \pi(y)] = +\infty$, $\psi(1) - \pi(1) = -(1-\tau)(a-c)^2 / b < 0$, $\psi(y^*) - \pi(y^*) = 0$ and the differential of expected payoffs $\delta(y) \equiv \psi(y) - \pi(y) = 0$ is a continuous function of y along the interval $(0,1] \subset \mathbb{R}$, we can then apply the intermediate value theorem to readily conclude that $\delta(y) > 0$ for all $y \in (0, y^*) \subset \mathbb{R}$ and $\delta(y) < 0$ for all $y \in (y^*, 1] \subset \mathbb{R}$. As a result, given (A.2), we are able to infer that the probability $y^* \in (0,1) \subset \mathbb{R}$ is, in fact, the maximizer of the expected payoff in (A.1). Therefore, based on the Theorem 1 established and demonstrated by Crawford (1992, p. 304), we can conclude that there exists a unique ESS in our large-population playing the field game.

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