AN ELEMENTARY PROOF OF THE BOWLES-GINTIS-MORISHIMA FUNDAMENTAL MARXIAN THEOREM WITH HETEROGENEOUS LABOUR

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Abstract

This note gives an elementary proof of a marxian fundamental theorem with heterogeneous labour originally due to Samuel Bowles and Herbert Gintis.

Resumo

Propomos nesta nota uma nova demonstração, elementar, de um teorema marxiano fundamental com trabalho heterogêneo devido originalmente a S. Bowles e H. Gintis.

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In a now classical paper (Bowles and Gintis, 1977), Bowles and Gintis have reformulated the Marxian theory of value with heterogeneous labour (without joint production). The novelty of their approach was to avoid any reduction of the various types of labour to a common unit. From the point of view of the theory of exploitation and the set of possible "Marxian fundamental theorems", a most important result was their theorem 2. Their proof was however unclear, and, after some comments from Morishima (1978), Bowles and Gintis provided a revised theorem (see below, our theorem) wich is theorem 2' of their reply to Morishima (Bowles and Gintis, 1978).

The aim of this note is to give an elementary and immediate proof of their Marxian fundamental theorem 2'. Considering the part played by Morishima in the revision of the original proof, we call it a "Bowles-Gintis- Morishima Fundamental Marxian Theorem with heterogeneous labour". The Bowles and Gintis demonstrations are unnecessarily lengthy and quite intricate. They use the Perron-Frobenius artillery which, we claim, is almost entirely dispensable.

We consider an economy producing n commodities by means of these commodities and m types of labour. We use the following notation:

A the n*n matrix of material input coefficients

B the n*m matrix of consumption per unit labour

L the m*n matrix of labour input coefficients

V the m*n matrix of labour values

x the n*1 vector of gross sectoral outptus

p the 1*n vector of prices

and, as Bowles and Gintis do, make the following assumptions with respect to the production conditions:

(i) A + BL , the "socio technical" matrix of inputs (a non negative matrix), is quasi-irreducible (see Bowles and Gintis, 1977; p. 188, for the original definition of such matrices, and remark 1 below) or, more generally, is a "Sraffa-matrix", concept introduced by Krause (Krause, 1981, appendix, and remark 1 below); note that every quasi-irreducible matrix is a Sraffa-matrix.

(ii) B (a non negative matrix) contains at least one positive element in each of its columns, i. e. no type of labour survives without consuming at least one commodity.

(iii) L (a non negative matrix) contains at least one positive element in each of its columns, i. e. some labour is necessary to produce each type of commodity.

In this note, vectors and scalars defined as: ≥ 0 are non-negative; vectors defined as: > 0 are semi-positive, $>> 0$ are strictly positive; scalars defined as > 0 are positive.

The matrix of labour values is determined by:

$$V = VA + L$$

and, assuming an equalized rate of profit r, wages paid in advance and no worker savings, the price vector is given by:

$$p = (1 + r)\, p(A + BL)$$

Observe that our assumption with regard to A + BL implies that there exists $p >> 0$ associated to $(1 + r)^{-1} \geq 0$ such that

$$p = (1 + r)\, p(A + BL)$$

On the basis of this value system, theorem 2' in Bowles and Gintis is concerned with the relation between the rate of profit r and the m rates of exploitation of labour. The rate of exploitation of the s'th type of labour is defined as:

$$e_s(x) = (L_s x - V_s BLx) / V_s BLx$$

where L_s and V_s are respectively the s' th row of L and V. These rates of exploitation depend on the vector x of gross sectoral outputs, and we consider states such that $x > 0$.

If $V_s BLx = 0$, we define:

$$e_s(x) = +\infty \quad \text{if} \quad L_s x - V_s BLx > 0$$

$$e_s(x) = o \quad \text{if} \quad L_s x - V_s BLx = 0$$

THEOREM. (Analogous to theorem 2' of Bowles and Gintis (1978)).

In an economy in which some type of labour appears directly in each commodity, then,

(I) if all rates of exploitation are positive, the profit rate is positive;

(II) if the profit rate is positive, at least one rate of exploitation is positive.

PROOF. By definition:

$p = (1 + r)\, p(A + BL)$

Let $H = A\,(I - A)^{-1}$, then, recalling that $V = L(I - A)^{-1}$:

(1) $p = (1 + r)pBV + rpH$

Multiplying equation (1) by the vector BLx and rearranging, we have:

(2) $r(pBVBLx + pHBLx) = pB(Lx - VBLx)$

Equation (2) provides a most useful relation between the rate of profit and the rates of exploitation of the m types of labour. From (2) we conclude:

(i) if $Lx - VBLx > 0$, then $r > 0$;

(ii) if $r > 0$, then we cannot have $Lx - VBLx \leq 0$ since, by hypothesis, $pb >> 0$, $pBV >>0$, $BLx > o$.

QED.

REMARK 1: As can easily be observed, the proof breaks down if pB is not strictly positive, which could be the case if p is not strictly positive. The characterization of the A + BL matrix as a quasi-irreducible or Sraffa-matrix excludes this possibility.

Let us recall the definition of a Sraffa-matrix as introduced by Krause (1981). By means of a simultaneous permutation of columns and rows we can put every reducible semi-positive square matrix A into the normal form of Gantmacher (1959, pp. 74-80), wich consists of block submatrices A_{ij} with $1 \leq i, j \leq s$ such that:

(i) A_{ii} is square and irreducible for all i, and $A_{ij} = 0$ for $i < j$.

(ii) For some g, with $1 \leq g \leq s$: if $i \leq g$, $A_{ij} = o$ for all $1 \leq j \leq i - 1$. If $i > g$, $A_{ij} \neq 0$ for some
$1 \leq j \leq i - 1$.

A semi-positive square matrix S is then called a Sraffa-matrix if the normal form of its transpose S^T with g = 1, which consists of block submatrices S^T_{ij} with $1 \leq i, j \leq s$, is such that the dominant eigenvalue of the square and irreducible submatrix S^T_{11} is greater than the dominant eigenvalues of all the submatrices S^T_{ii} with $1 < i \leq s$.

The importance of Sraffa-matrices lies in the fact proved by Krauser (1981, p. 177, theorem 2) that S has a unique strictly positive left eigenvector (up to a scalar) associated to its non-negative dominant eigenvalue.

Finally, let us note that a quasi-irreducible matrix, as defined by Bowles and Gintis (1977, p. 188), is a peculiar Sraffa-matrix: the normal form of its transpose is such that all the submatrices S^t_{ii} with $1 < i \leq s$ are zero matrices.

REMARK 2: Theorem 3 of Bowles and Gintis (1978) states that if the profit rate is positive, then it is less than at least one of the rates of exploitation. Using equation (2), and with no additional assumptions about matrix A, one can prove a weaker theorem: if positive, the profit rate is less than or equal to at least one of the rates of exploitation. To prove this: divide both sides of equation (2) by pBVBLx (recalling that pBVBLx > 0) and let $e^*_s(x) = \max_s\{e_s(x)\}$ so that we have:

$r\,(1 + (pHBLx\,/\,pBVBLx)) \leq e^*_s\,(x)$

and conclude $r \leq \max_s \{e_s(x)\}$.

REMARK 3: Observe that our proof does not require, as in theorem 2' of Bowles and Gintis, that "each type of labour appears directly or indirectly in each non luxury good". This is an unnecessarily strong condition.

REFERENCES

BOWLES, S. and GINTIS, H. 1977. The Marxian theory of value and heterogeneous labour: a critique and reformulation, **Cambridge Journal of Economics**, 1, 173-192.

BOWLES, S. and GINTIS, H. 1978. Professor Morishima on heterogeneous labour and Marxian value theory, **Cambridge Journal of Economics**, 2, 311-314.

GANTMACHER, F. R. 1959. **The theory of matrices**. Volume 2. New-York: ChelseaPublishing Company.

KRAUSE, U. 1981. Heterogeneous labour and the fundamental Marxian theorem, **Review of Economic Studies**, 48, 173-178.

MORISHIMA, M. 1978. S. Bowles and H, Gintis on the Marxian theory of value and heterogeneous labour, **Cambridge Journal of Economics**, 2, 305-309.