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The Hydrostatic Balance and Archimedes' Principle: impacts of a didactic sequence based on primary sources from the History of Science [+*]

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Abstract

In teaching materials, regarding Archimedes' Principle, the reproduction of the pseudo-history related to the problem of the adulteration of the crown of the king of Syracuse, narrated in the work De Architectura, by Marcus Vitruvius, is still noted. This narrative attributes to Archimedes a method that is contradictory to historical evidence, imprecise and physically unfeasible. In his work La Bilancetta, Galileo suggested that Archimedes would have used a Hydrostatic Balance to solve the crown problem. From a didactic point of view, the discussion of this mechanism can favor the approach to the concept of buoyancy and Archimedes' Principle. Thus, we planned and applied, in the context of Basic Education, a didactic sequence based on the discussion of primary sources from Vitruvius and Galileo, and the reproduction of a Hydrostatic Balance as a historical experiment. In this work, we discuss the educational impact of the proposal, based on the analysis of responses from students who participated in this intervention to a research instrument.

Keywords: *Archimedes' Principle; Hydrostatic Balance; Historical Experiment; Primary Sources.*

[+] A Balança Hidrostática e o Princípio de Arquimedes: impactos de uma sequência didática pautada em fontes primárias da História da Ciência

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I. Introduction

A key aspect when considering high-quality basic education is that students develop critical and creative thinking. In this regard, historical-philosophical approaches have been identified as a potential means to teach science content and contextualize discussions related to the nature of scientific work (Martins, 2006; Forato; Martins; Pietrocola, 2012; Moura; Guerra, 2016; Ortega; Moura, 2020; Mendonça, 2020; Peduzzi; Raicik, 2020).

Regarding the Nature of Science (NoS), Peduzzi and Raicik (2020) present a series of commented assertions aimed at promoting a contextualized reflection on science. Among these propositions, we highlight, as one of the aspects underpinning the present work, that scientific observation is selective, meaning it occurs considering specific interests and problems; discovery is more complex than merely observing and having an "insight"; and scientific laws and theories are intellectual constructions, meaning they are not immediate conclusions drawn from experiments.

History of Science is one possible approach for addressing this type of proposition in the classroom, which can be facilitated by using primary sources. These are understood as cultural productions that express the intentionality of historical figures and are conceived as objects to be interpreted considering the context in which they were produced. Thus, they can be introduced into the school context from a dialogical and interpretive perspective, enhancing the understanding of characteristics of the scientific production process: the motivations of those involved in historical episodes of science, conflicts of interest, the role of criticism in scientific development, and the credibility of the methods used (Sasseron; Nascimento; Carvalho, 2009; Wineburg, 2010; Batista; Drummond; Freitas, 2015).

Despite the evident relevance of these aspects of the historical construction of scientific concepts, textbooks largely continue to neglect them. As a result, academic research has paradoxically highlighted the textbooks themselves as "pedagogical obstacles that pose challenges to the implementation of historical approaches in the classroom." (Moura; Guerra, 2016, p. 797)[2].

Equally concerning as these gaps is the reproduction of romanticized pseudo-histories about scientific development in educational contexts. Such narratives convey simplistic views of science, such as empiricism-inductivism, where a gradual and collective "discovery" is attributed to a single individual who supposedly made it through a sudden insight (Allchin, 2004; Forato, Martins, Pietrocola, 2012). A relevant example of this issue pertains to Archimedes' Principle. The most widespread pseudo-history regarding this physical concept is

[2] In Brazil, only two collections approved in the 2018 call for submissions of the National Textbook Program included robust historical-philosophical approaches to Archimedes' Principle, drawing from historical sources and historiographical works on Archimedes (Hidalgo; Queiroz; Oliveira, 2021). None of the collections approved in the 2021 call for submissions of the National Textbook Program featured this type of approach to Archimedes' Principle (Santos Júnior, 2023).9 9

even cited in textbooks approved in the 2018 and 2021 editions call for submissions of the National Textbook Program[3].

This pseudo-history, which harms students' critical development by perpetuating the notion that science is done by chance through sudden insights, traces back to the Roman author Marcus Vitruvius (80 BC – 15 BC), in a passage from his work *De Architectura*. According to Vitruvius, it was suspected in the past that part of the gold entrusted by Hieron II of Syracuse to a craftsman to make his crown had been replaced with silver. Wanting to investigate the matter without damaging the crown, Hieron sought the help of Archimedes. Enthusiastically, Archimedes is said to have run naked through the streets after suddenly solving the problem. Upon entering a bathtub filled with water, Archimedes supposedly noticed that the submerged volume of his body displaced an equivalent volume of water. He then compared the volumes of water displaced from a full container when a silver object, a gold object, and the crown – all the same mass – were submerged. The crown displaced more water than the gold object, proving the craftsman's guilt (Vitruvius, 1958).

Although Vitruvius' narrative is intriguing, it is unreliable both from a physical and historical perspective. Regarding physical coherence, the water's surface tension would make it impossible to draw reliable conclusions using this method. The immersion of a 1 kg crown, with a density of 15 g/cm³ (an intermediate value between the densities of gold and silver) and a diameter of 20 cm, in a cylindrical container with a 15 cm radius filled with water, would cause a water level change of less than 1 mm, making it imperceptible. From a historical credibility standpoint, Vitruvius lived about two centuries after Archimedes and did not cite the sources of his account (Martins, 2000; Santos Júnior, 2023).

Given the improbability of this version of the crown's case, how might Archimedes have solved the problem? Galileo Galilei (1564-1642), a great admirer of Archimedes, criticized the inaccuracy of Vitruvius' method. Drawing on Archimedes' own work on levers and equilibrium, Galileo proposed a different, physically viable method, which he believed Archimedes had employed. In his work *La Bilancetta*, or *The little balance*, Galileo stated that Archimedes had used a Hydrostatic Balance (Galilei, 1986). In the 19th century, ancient historical evidence was found that reinforced Galileo's hypothesis by attributing the use of the Hydrostatic Balance directly to Archimedes in solving the crown's problem (Martins, 2000).

The Hydrostatic Balance allows for the comparison of the apparent weight reductions of the crown and objects made of gold and silver, of the same mass, when submerged in water. Based on modern concepts, we can say that objects with the same mass, but different densities experience different buoyant forces when submerged, as they have different volumes. This

[3] Two collections selected in the 2021 call for submissions of the National Textbook Program recommend that students read texts that reference the pseudo-historical narrative related to the discovery of buoyancy (Santos Júnior, 2023). A similar situation had already been observed by Hidalgo, Queiroz, and Oliveira (2021) in three collections approved in the 2018 call for submissions of the National Textbook Program.

highlights the educational potential of the Hydrostatic Balance mechanism (Fig. 1) as a resource for contextualizing the concept of buoyancy and Archimedes' Principle itself.

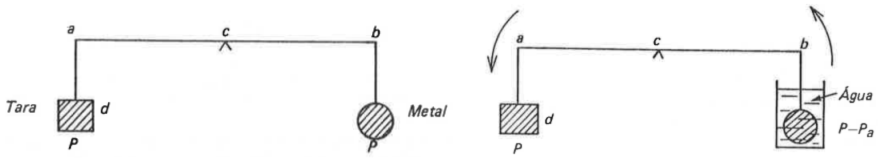

Fig. 1 – Representation of the Hydrostatic Balance. Source: LUCIE, 1986, p. 97.

We explored these potentialities in a historical-philosophical approach that starts from the crown episode, contrasting Vitruvian and Galilean versions of this event. We developed and implemented a didactic sequence that uses a low-cost Hydrostatic Balance and excerpts from primary sources of Archimedes and Galileo. This sequence also promotes, through an investigative experimental demonstration (Carvalho, 2010), a discussion on the limits of validity of Archimedes' Principle, as the statement usually found in textbooks is only valid when the dimensions of the container holding the fluid are much larger than the dimensions of the object to be submerged (Silveira; Medeiros, 2009)[4].

The extracurricular mini-course was applied to 24 high school students at the Federal Institute of Education, Science, and Technology of Piauí (IFPI) – Corrente campus, Brazil. Of these, 22 were in the 3rd year of the integrated technical course in informatics, and 2 were in the 1st year of the integrated technical course in environmental studies. The study aims to analyze the students' responses to a research instrument applied after the intervention, which sought to evaluate the educational impact of this approach.

II. The didactic sequence implemented[5]

The didactic sequence developed comprised two stages: the first, with an estimated duration of 90 minutes, and the second, approximately 210 minutes (Tables 1 and 2). The first part of the implementation took place on August 23, 2023. The second part, which began on the same day, was concluded two days later. The objectives of this intervention were: to foster more complex views on NoS through the discussion of primary sources related to Archimedes' Principle; to explore the historical episode of Archimedes and the king's crown, highlighting

[4] Only when these conditions are met will the buoyant force be equal to the weight of the volume of liquid displaced by the object. This becomes clear when we observe that objects immersed in a container with dimensions comparable to their own can float even while displacing a smaller volume of liquid than predicted by the most widely disseminated statement of Archimedes' Principle (Silveira; Medeiros, 2009).

[5] This work is part of a master's research in Science Teaching. Further details about the didactic sequence and the application of theoretical frameworks can be found in the dissertation developed by one of the authors (Santos Júnior, 2023).

factors that render Vitruvius' account historically and conceptually inconsistent from a physical standpoint; to present Galileo's method for solving the crown problem attributed to Archimedes as both physically plausible and supported by historical evidence; to reproduce the Hydrostatic Balance described by Galileo and use it to discuss the concept of buoyancy in the context of analyzing a sample composed of different materials; and to address Archimedes' Principle in a historically contextualized manner, discussing its limits of validity and the possibility of an updated statement of this principle in light of the Hydrostatic Paradox.

As a theoretical framework, we adopted the "Three Pedagogical Moments" – initial questioning, organization, and application of knowledge (Muenchen; Delizoicov, 2014). Additionally, we drew on the perspective of investigative experimentation developed by Carvalho (2010). In this type of activity, students construct scientific knowledge by responding to intermediate questions, which help them to appropriate the characteristic language of science. This process is important because it requires students to think critically about justifications for their ideas and express them verbally. Such experimental activities should go beyond mere observation of phenomena, providing opportunities for students to construct, through dialogue with the teacher, the physical concept to explain the observed phenomena. Investigative demonstrations should be infused with questioning, encouraging students to think about the problem and propose possible answers (Carvalho, 2010).

Table 1 – Summary of Part I of the didactic sequence.

Questioning	**Step 01:** Presentation of images related to Archimedes' Eureka episode (see Santos Júnior, 2023). Students are asked if they are already familiar with the story.
	Step 02: Presentation of biographical information about Archimedes (287 BCE – 212 BCE)[6] and his alleged involvement in the king's crown case, emphasizing that there are no records from Archimedes himself about the episode. Comments on *De Architectura*, in which Marcus Vitruvius (80 BCE – 15 BCE) narrates the event. The teacher notes on the board the time periods during which Archimedes and Vitruvius lived.
	Step 03: Collective reading of a translated excerpt from *De Architectura*, in which Vitruvius narrates his version of the episode (Appendix I). The teacher illustrates the method narrated by Vitruvius with the help of an illustrative figure (see Santos Júnior, 2023). A collective discussion follows, based on questions that address the reliability of the narrative from both physical and historical perspectives: "Is it clear how Vitruvius became aware of the episode? What was his source?"; "If Archimedes held a prestigious position, who might have prepared his bath?"; "If you were to prepare a bath in a bathtub, would you fill it to the brim?"; "Why would a silver object, with the same mass as a gold object, cause more water to overflow when placed in a container filled with water?"; "How would this have helped to solve the crown problem?"; "By how much would the water level change if we inserted a crown into a partially filled cylindrical container? For

[6] We recommend the introductory chapter of the work *Archimedes, the Center of Gravity, and the Law of the Lever* (Assis, 2008) as a resource for teachers, available at: https://www.ifi.unicamp.br/~assis/Arquimedes.pdf.

	example, if we inserted a crown with a mass of 1 kg, a density of 15 g/cm³ (intermediate between gold and silver), and a radius of 10 cm into a cylindrical container with an internal radius of 15 cm, would the rise in the water level be noticeable?”[7]
	Step 04: Questions about the reliability of Vitruvius' narrative related to the surface tension of the liquid: “What happens when we gently place a coin into a glass of water filled to the brim?”
	Step 05: Investigative demonstration with dialogue (Carvalho, 2010) to address the subject of surface tension. In this context, we conducted an investigative demonstration in which coins were progressively inserted into a glass filled with water. During the experimental activity, questions were posed for collective reflection: “What will happen if we slowly insert a coin into this full glass?”; “How many coins do you think we can insert before the liquid overflows?”[8]; “Does the liquid spill immediately after surpassing the rim of the glass? Why doesn’t the water overflow right away?”; “When the liquid overflows, do you think the volume of water spilled is equal to the volume of the coins inserted?” In this way, students were invited to reflect on the situation. We went beyond the simple observation that the liquid would not immediately overflow, providing an opportunity for students to actively construct knowledge through interaction.
Organization of Knowledge	**Step 01:** Discussion on the surface tension of liquids. Presentation of examples.
	Step 02: Mathematical demonstration, involving student participation, of the change in water level expected when inserting a crown into a cylindrical container, as described at the end of Step 3 of the Questioning stage. The prediction is a change of 0.94 mm, indicating a practically imperceptible change in level (see Santos Júnior, 2023).
Application of Knowledge	**Step 01:** Discussion: “How would surface tension influence the method described by Vitruvius?”; “Considering surface tension and the result of the change in water level, what can we infer about this narrative?”; “From a historical perspective, how does this affect its reliability?”; “Given the impracticality of the method described by Vitruvius, how might Archimedes have solved the problem?”[9]

Source: Own authorship.

Table 2 – Summary of Part II of the Didactic Sequence.

Questioning	**Step 01:** Collective reading of a translated excerpt from *La Bilancetta* (Appendix II/A-B), with the authorship initially undisclosed to the students. In this excerpt, Galileo critiques Vitruvius' narrative, which is based on the measurement of displaced volumes of water. Several questions are posed to the students: “What does the author think about the method that Vitruvius attributes to Archimedes? Does the author believe that Archimedes used

[7] In this stage, the issue is merely raised. The calculation will take place in a subsequent stage.

[8] A total of 14 coins could be added before overflow occurred. A similar demonstration, conducted by the authors of this work, can be viewed at: https://youtu.be/9LZE8hIv4M8

[9] The goal of this stage of the didactic sequence is not for students to have an answer to the last question, but rather to reflect on it as a prior preparation for the subsequent stage.

	this method? Why?"; "In the author's opinion, why might this false narrative have arisen, and how might Vitruvius have concluded that Archimedes used that method?"
	Step 02: Inquiry about the possible authorship of the text read: "Who is the author of the excerpt we read?" The teacher attributes the text to Galileo Galilei (1564-1642) and asks: "What period did Galileo live in? Was he a contemporary of Archimedes? What do you know about him?" After listening to the students, the teacher provides a chronological context for Archimedes, Vitruvius, and Galileo, and presents biographical information about Galileo[10]. He indicates the scholar's interest in and admiration for Archimedes' work, which led Galileo to recognize that the method attributed by Vitruvius to Archimedes was inconsistent.
	Step 03: Question that will lead to an alternative hypothesis for the method used by Archimedes: "What method involving water might Archimedes have employed to resolve the issue of the crown's falsification?" To foster the formulation of a hypothesis, the teacher revisits Archimedes' biography, emphasizing his study of levers. Finally, a prototype of a Hydrostatic Balance (Appendix III) is presented to the students, which operates based on the idea of a lever and is related to water: "How could the Hydrostatic Balance have been used by Archimedes to determine if there was fraud in the crown?" The students manipulate the instrument, and their possible suggestions are recorded.
Organization of Knowledge	**Step 01:** Dialogical investigative demonstration (Carvalho, 2010) using the prototype of the Hydrostatic Balance, in which we gradually present questions for reflection, inviting students to construct knowledge through dialogue with the teacher. Two samples of the same mass but different densities, along with a counterweight of the same mass, are used. Questions stimulate the formulation of hypotheses, which are then tested: "If we place one of the samples 10 cm from the axis of rotation (fixed at the center of gravity), where will we need to position the counterweight for the balance to remain level?" (in this case, we have a regular balance in air); "One of the samples and the counterweight of the same mass are in equilibrium in the air when positioned equidistant from the axis of rotation of the balance. What happens if we insert the sample into the water?" (this procedure is carried out after students have practiced their hypotheses); "In this situation, what do we need to do with the counterweight to restore equilibrium?" By replacing the first sample with another of lower density but the same mass, the previous procedure is repeated: "This time, do we need to move the counterweight closer to or further away from the axis of rotation?"; "If a sample with even lower density were used, where would the counterweight need to be placed to restore equilibrium after inserting the sample into the water?"; "Returning to the situation where the object is placed in water, what causes that imbalance? What makes the sample 'weigh' less in the water?"; "What conclusions can you draw from what we observed and discussed?"

[10] For information on Galileo, we suggest the following resources for teachers: Lucie's (1986) text titled Galileo and the Archimedean Tradition, available at: https://www.cle.unicamp.br/eprints/index.php/cadernos/article/view/1218; and the post Galileo: Beyond the Myths on the Universo Racionalista platform, available at https://universoracionalista.org/galileo-para-alem-dos-mitos/.

	Returning to the problem of the crown, "If the goldsmith replaces part of the mass of gold with silver, what happens to the density of the crown?"; "How could the Hydrostatic Balance be applied to resolve this issue?"[11]
	Step 02 - Presentation of Archimedes' Principle: The principle is introduced, accompanied by its mathematical development as outlined in textbooks. We based our approach on propositions taken from *On Floating Bodies*[12], by Archimedes, to highlight the distinction between how we understand its validity today (for fluids) and what Archimedes established in his work (for liquids). It was emphasized that the concept of buoyancy, the formal statement of the principle, and the mathematical notation were not included in Archimedes' original work but are rather the result of a collective historical construction. This step involves an implicit approach to the "provisional nature of scientific knowledge," as this characteristic of science can be inferred from the discussion.
	Step 03: Using an excerpt from *La Bilancetta* (Appendix II/C), we explain how Galileo suggested that the Hydrostatic Balance could have been used by Archimedes to solve the crown problem. Figures representing the balance in different situations are presented[13]. Discussion questions include: "Why does the balance become unbalanced when gold is placed in the water?"; "What does the buoyant force acting on the gold depend on?"; "How is this compensated by moving the counterweight closer to the axis of rotation (c)?"; "Why does the balance become unbalanced when silver is placed in the water?"; "Is the volume occupied by silver greater or lesser than the volume occupied by the gold sample of the same mass?"; "Is the buoyant force acting on the silver greater or lesser than the buoyant force acting on the gold object?"; "How does this influence the position the counterweight needs to take to restore balance?"; "In the situation where an alloy of gold and silver is used, how does the volume of this object relate to the volumes of the bodies made of pure gold and silver? And how does the buoyant force exerted on this alloy relate to the buoyant forces experienced by the other two bodies?"; "We could have a certain mass of gold on one side balancing the crown on the other side, with the balance in the air, giving the impression that the goldsmith used all the gold received. Knowing the volume of the crown would solve the problem because Archimedes could compare the densities and thus identify any fraud. However, he also did not know the volume of the crown, which is an irregular object. So, what would happen in the case of fraud if the crown were inserted into the water...?" Through dialogue, we collectively explore the possibility of using the Hydrostatic Balance to resolve the problem.
	Step 04: It is explained that Galileo concluded that the Hydrostatic Balance was used by Archimedes after studying the Greek author's work on levers and considering that it was known that the employed method involved water. A passage from *La Bilancetta*, in which Galileo explains the functioning of the Hydrostatic Balance (Appendix II/C), is read collectively.

[11] Students are encouraged to begin reflecting on this question, which will be revisited in a subsequent stage.

[12] As presented in the annotated translation by Assis (1996).

[13] Figures from the introduction by Lucie (1986) of the work *La Bilancetta* were used.

	Discussions using figures (Santos Júnior, 2023) follow: "Suppose the crown contained much more silver than gold; when inserted into the water, would balance be achieved with the counterweight closer to (e) or (f)?"; "Now imagine that a small percentage of gold was replaced by silver; would the counterweight need to be moved closer to (e) or (f)?"; "And if the goldsmith were innocent? To which point would the counterweight need to be moved to restore equilibrium in the water?" This approach seeks to foster an understanding of the use of the Hydrostatic Balance as a solution to the problem. At the end of this step, it is noted that Galileo's version of the episode is physically coherent and supported by historical evidence found[14].
	Step 05: Establishing a validity limit for Archimedes' Principle: it is valid when the dimensions of the container holding the fluid are much larger than the dimensions of the object submerged in the fluid (Silveira; Medeiros, 2009). A dialogical investigative demonstration is conducted to illustrate Galileo's Hydrostatic Paradox (Carvalho, 2010). A 350 ml soda can is placed in a container that holds 280 ml of water, and it floats (see Santos Júnior, 2023). Questions are posed: "According to Archimedes' Principle, what volume of liquid would be displaced for the can to achieve equilibrium? Is there enough liquid in the container for the can to float based on this principle?"; "How is it possible for what we are observing to occur?" This leads to the presentation of a revised statement of the principle, which addresses the Hydrostatic Paradox[15].
Application of Knowledge	**Step 1:** Students use the Hydrostatic Balance to determine the proportion of marbles and coins in a sample ("set"). For this, two other sets with the same mass as the one being studied are used, one containing only coins and the other only marbles. First, using a counterweight of the same mass as the three sets, the set composed only of marbles is manipulated: "Since the counterweight and the set of marbles have the same mass, what should be their distance from the axis of rotation for them to be in equilibrium in the air?"; "If we insert the marbles into the water, what will happen to the system?"; "By performing this procedure, why is the equilibrium disturbed? What force starts to act on the marbles?"; "By moving the counterweight, how can we make the balance return to equilibrium?"
	Step 2: The same procedure is repeated for the set containing only coins and for the set with both coins and marbles. Students analyze the position to which the counterweight was moved in the three situations: "Does the position of the counterweight that restores equilibrium when using the set with coins and marbles resemble more the position when using only marbles or that when using only coins?"; "In this way, do you think we have a greater mass of coins or marbles in the mixed set? How can we estimate the relationship between these masses?" The teacher records possible suggestions. Through dialogue, the

[14] For this information, the teacher can refer to Martins (2000).

[15] The revised statement of Archimedes' Principle is presented by Silveira and Medeiros, who demonstrate that, to avoid the Hydrostatic Paradox, it must be stated that "any body immersed in a liquid experiences an upward buoyant force equal to the weight of the fluid contained in a volume identical to the submerged volume of the body in the fluid" (2009, p. 289).

	teacher demonstrates mathematically how to obtain the masses of each distinct portion in the mixed set. The relationship obtained is used (see Santos Júnior, 2023).
	Step 03 - Extracurricular Activity: Students complete an investigative questionnaire[16].

Source: Own authorship.

III. Research Methodology

Upon concluding the didactic sequence, the students responded to a research instrument consisting of 11 open-ended questions, through which we aimed to investigate the impact of the implemented proposal. The questions were organized into the thematic blocks presented in Table 3, as follows[17].

In total, 16 of the 24 students who were engaged in both stages of the didactic sequence responded to the questionnaire. Of this total, 15 were enrolled in the 3rd year of the integrated technical course in computer science, and 1 was enrolled in the 1st year of the integrated technical course in environmental studies. The nomenclature used to identify the participants will be Pn, with n varying from 1 to 16[18].

Table 3 – Post-intervention questionnaire organized by blocks.

Block	**Questions**
Block 1 - Physical factors related to the methods described by Vitruvius and Galileo.	1, 2, 4, 5
Block 2 - Historical factors related to the different versions of the crown theft episode.	3, 6
Block 3 - Understanding of Archimedes' Principle and its limits of validity.	7, 8, 11
Block 4 - Conceptions about the Nature of Science (NoS).	9, 10

The data obtained, which are descriptive in nature, will be presented in tables, aimed at capturing the participants' conceptions while simultaneously allowing for comparison. The responses to the questions were analyzed both individually and by thematic block. Significant elements in the discussions traverse various questions. For example, to optimize the organization of the results, we chose to incorporate the discussion of the validity of the

[16] This post-intervention questionnaire did not have a summative evaluative character. There was no discussion with the students following the administration of the questionnaire, as it served solely as a research instrument.

[17] The statements of the questions will be presented in Section IV.

[18] The use of responses in this research was authorized through the signature of an informed consent form by the respective guardians or by the students themselves in the case of those of legal age. This form adheres to the general guidelines established by Federal University of Rio Grande do Norte for its preparation (see Santos Júnior, 2023).

Archimedes' Principle into Block 3, which relates to the discussion of this physical content, rather than into Block 4, which pertains to NoS, as it would also fit there. This approach aims to encourage students to understand that scientific knowledge does not possess unrestricted validity.

It is important to emphasize that, although the analysis blocks were established *a priori* based on the guiding objectives of the didactic sequence itself, the analysis categories were constructed from the central ideas expressed in the participants' responses, according to content analysis criteria (Bardin, 2004). It is also worth noting that, in some instances, participants' responses could be classified into more than one category.

IV. Research Results

We present tables with the results for each block of questions, categorized according to the proposed analysis categories, followed by discussions of these results.

Block 1 - Physical Factors Related to the Methods Narrated by Vitruvius and Galileo

The questions 1 2, 4, and 5 from the post-intervention questionnaire are related to the theme *"physical factors regarding the methods narrated by Vitruvius and Galileo"*. This analysis was based on the framework provided by Martins (2000).

The first proposed question concerned the method that Vitruvius attributed to Archimedes: *According to the method described by Vitruvius, how would Archimedes have determined whether the crown had been falsified?*

Table 4 – Summary of responses to the first question.

Category	Archimedes' method according to Vitruvius	Participants
A	He submerged the crown and objects of equal mass made of silver and gold. He observed that the crown caused more water to overflow than the gold object and compared their densities.	P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13}, P_{14} e P_{16}.
B	He discovered how to resolve the case while taking a bath.	P_6 e P_{10}.
C	He used the Hydrostatic Balance.	P_3 e P_{15}.
D	He applied the principle of Buoyancy.	P_{11}.

Source: Own authorship.

Fourteen participants were classified as Category A (Table 4). They indicated that Vitruvius stated that Archimedes compared the volume displaced by the submersion of the crown to the volumes displaced by the submersion of objects of equal mass made of silver and gold. Responses of this nature were anticipated and desired (according to the reference by

Martins, 2000), given the discussion held regarding the excerpt from *De Architectura*, in which Vitruvius describes the supposed method used by Archimedes. As P1 noted, “he would have made two pieces weighing the same as the crown, one made of gold and the other of silver. Upon submerging both pieces in water, Archimedes would have recorded the volume of water displaced by each.”

Another point to highlight is the mention of the differences in the densities of gold, silver, and the crown by participants included in this category. As P9 commented, “because they are materials of different densities, the blocks did not have the same size.” P6 and P10 were classified into two categories (A and B), as they not only indicated the method of comparing the displaced volumes but also recalled Vitruvius' narrative about Archimedes' supposed insight in the bathtub, which led him to conduct the experiment of submerging the crown in water. As P9 noted, “He had gone to take a bath and realized that the more he submerged his body, the more water spilled out.”

Such statements indicate that the students mentioned in these categories understood the method narrated by Vitruvius. Furthermore, the fact that Categories A and B together have such a significant incidence is an initial indication that the application of the didactic sequence was successful, considering the goal of providing the group with an understanding of the physical aspects underlying the method described by Vitruvius (according to the reference by Martins, 2000).

Student P11 had his response classified in Category A and simultaneously in Category D, as he stated, “Archimedes used the principle of buoyancy, in which he submerged the crown and an object of equal weight to pure gold in a container of water.” At no point during the intervention the teacher asserted that Vitruvius stated that Archimedes used the “principle of buoyancy”. However, it is common for popular science websites and textbooks to refer to this episode in the form of the pseudo-historical Vitruvian narrative, using this expression, even though it is an anachronism (Hidalgo; Queiroz; Oliveira, 2021; Santos Júnior, 2023). Thus, there is an indication that, when responding to the questionnaire, the participant may have sought available sources online that explain the case, overlooking the anachronisms present in those explanations.

Despite the clear distinction between the methods narrated by Vitruvius and Galileo in the didactic sequence, P3 and P15 seem to have been confused. These students, situated in Category C, indicated that the version of Vitruvius suggested that Archimedes used the Hydrostatic Balance. According to P15, “he would have determined if the crown was fraudulent through the method of the Hydrostatic Balance, comparing the weight of the crown with the weight of an equivalent quantity of pure gold”. The students' misunderstanding may stem from the emphasis we placed on the use of the Hydrostatic Balance, so they may have only recalled that method. Alternatively, it may indicate inattention when responding to the questionnaire or even some kind of difficulty during the intervention. We observed that these participants, although they mentioned the Hydrostatic Balance in their responses, did not correctly describe its functioning in any item of the questionnaire, which may signal a lack of comprehension regarding this mechanism.

The second question posed concerned evaluating the Vitruvian narrative from a physical standpoint: *What physical factors discredit the version described by Vitruvius?* (Table 5). Again, some participants were included in more than one category. P9, for example, indicated: “Several physical factors question the version described by Vitruvius. First, it was unlikely that Archimedes had access to accurate measuring equipment at the time, and the surface tension of the water could have affected the amount of water spilled. Additionally, the amount of liquid spilled would be so small that making a comparison would be nearly impossible”. The mention of various factors (imprecise measuring instruments, surface tension, minimal amount of liquid spilled, difficulty comparing displaced volumes) qualified this participant for inclusion in four categories (A, B, C, and F).

The most frequently present aspect in the responses was the effect of surface tension, explicitly cited by 11 of the 16 students, who were included in Category A. This was expected given the content covered in the intervention, as we emphasized the relationship between this phenomenon and other aspects, such as the difference between the volume of liquid displaced and the submerged volume of the objects (according to the reference by Martins, 2000). To exemplify the students’ perception, we can cite P2’s response: “the surface tension of the water could have affected the amount of water spilled, and it was unlikely that Archimedes had access to accurate measuring equipment at that time”. It can be noted that the participant established a parallel between the surface tension of the water and the fact that the volume displaced by this liquid is distinct from the volume of the objects inserted into it. The established relationship is appropriate, as discussed during the investigative demonstration of inserting coins into a glass of water on the verge of spilling, where the action of surface tension caused the volume of water spilled to be much smaller than the volume of the coins submerged in the fluid. A similar response was provided by P10, who stated, “The surface tension would make it difficult or even prevent the water from spilling”.

P2's response also exemplifies another aspect addressed in the intervention. Thus, in Category B, which includes 9 out of 16 responses, there is mention of the absence of sufficiently precise instruments to measure the variation in the liquid level when the objects were submerged (Martins, 2000). In addition to P2, student P15 stated, “Some physical factors that discredit the version described by Vitruvius are the difficulty of making precise measurements at that time, the lack of appropriate instruments to determine the volume of the crown, and the possibility of variations in the densities of the gold and silver used in the crown”.

This response not only indicates the lack of precision in the instruments but also suggests that there could be variations in the densities of the gold and silver used in the crown. This latter aspect, not discussed during the intervention, is an interesting question raised by the student. The density of the gold object used for comparison would correspond to the density of the gold used in the crown, if, for example, the materials had the same origin.

Table 5 – Summary of responses to the second question.

Category	Physical factors that discredit the Vitruvian version	Participants
A	Surface tension of water.	P_1, P_2, P_4, P_6, P_7, P_8, P_9 P_{10}, P_{12}, P_{13} e P_{16}.
B	Imprecise measuring instruments in Archimedes' time.	P_1, P_2, P_3, P_8, P_9, P_{11}, P_{12}, P_{13} e P_{15}.
C	The volume of displaced liquid would differ from the submerged volume of the object.	P_2, P_6, P_7, P_9, P_{10} e P_{16}.
D	The small variation in water level.	P_1, P_6, P_7 e P_{16}.
E	Others (weight, density, and volume of the elements).	P_4, P_5 e P_{14}.
F	Difficulty in comparing displaced volumes.	P_2 e P_9.

Source: Own authorship.

In Category C, 6 participants were allocated. They cited that the volume of liquid displaced would not correspond to the volume of the objects inserted into the container. This aspect, connected to the surface tension of water, had been highlighted in an investigative experimental activity when coins were inserted into a glass filled with water. At that time, participants noted that the volume of liquid displaced was much smaller than the volume of the coins. As an example of a response classified in this category, P6 indicated, “Even if there were spillage, the volume of liquid spilled would not correspond to the volume of the crown”. Thus, the participant implicitly demonstrated that spillage might not even occur, and that even if the liquid spilled, the volume extruded would not be identical to that of the crown (Martins, 2000).

In Category D, 4 students were listed who pointed out that when inserting the crown into a container with water, the variation in the liquid level would be minimal. Among the responses presented, the one from student P16 stands out, relating the effect to surface tension: “the variation in the liquid level would be negligible, contributing to the lack of spillage, due to surface tension”. This kind of observation was likely based on the mathematical demonstration conducted during the intervention, indicating that there would be an increase of about 1 mm in the water level in the considered situation.

Category F contains two responses that highlight how difficult it would be to compare the displaced volumes from the container with water during the immersion of the crown, silver, and gold. This argument, which we emphasized several times during the intervention, curiously appeared only in two responses. As an example, P9 stated that “the amount of liquid spilled would be so little that it would be almost impossible to make a comparison”.

Finally, in Category E, there are three participants who provided identical responses[19], composed basically of the following phrase: “The weight, density, and volume of the elements”. Unfortunately, there does not seem to be a more sophisticated reasoning to justify such a response. Student P4 further noted: *“*because it is not possible to compare all this just with a basin full of water and spilled water”. This statement demonstrates disbelief on the part of the student in the efficiency of the method narrated by Vitruvius but lacks grounding in their argumentation.

In the fourth question, the students were asked about Galileo's assessment of the Vitruvian narrative: *What does Galileo point out about Vitruvius's version?* (Table 6).

Table 6 – Summary of responses to the fourth question.

Category	Galileo's remarks on the Vitruvian version	Participants
A	Vitruvius wrote about a rumor/information/legend/myth that was disseminated.	P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{12}, P_{13}, P_{14} e P_{15}.
B	The method described by Vitruvius is inaccurate/without scientific foundation.	P_4, P_5, P_{10}, P_{14}, P_{15} e P_{16}.
C	The version was invalid according to the principles of the time.	P_3.
D	Galileo does not mention anything regarding this.	P_{11}.

Source: Own authorship.

In Category A, 12 out of 16 participants were grouped. They indicated Galilei's conviction that Vitruvius must have relied on some form of unreliable or fanciful information circulating in his time (Martins, 2000). A high incidence in this category was expected since students P6, P8, P15, P9, P17, P11, P12, and P16 had already verbally expressed similar perceptions regarding the Vitruvian narrative during the class. The terms used in this reference were varied. For instance, P1 stated, “Some writer must have recorded the event, adding something to the limited understanding he had based on the rumors that circulated”. P15, on the other hand, employed the term “legend” to refer to the supposed sources of Vitruvius: “Galileo points out that Vitruvius’s version is based on legends”. Similarly, P13 asserted that Galilei believed Vitruvius’s version was based on “a myth that gained notoriety*”*. These statements align with Galilei's criticism of the historical credibility of the Vitruvian narrative (Galilei, 1986; Martins, 2000), indicating that the students understood this aspect evidenced by the interpretation of Galilei's historical text during the intervention.

Complementing this factor, P4, P5, P14, and P15 added Galilei’s points regarding the lack of precision in the method described by Vitruvius (Galilei, 1986; Martins, 2000). It is important to highlight that P4, P5, and P14 quoted a passage from *La Bilancetta* where Galilei

[19] P4, P5, and P14 responded to almost all questions identically. Thus, throughout most of the analysis, they will be categorized similarly.

criticized the method narrated by Vitruvius. In doing so, they relied on the historical document interpreted in the intervention to illustrate the imprecision noted by Galilei. P15 added that, for Galilei, Vitruvius's version "lacks solid scientific foundations".

These responses were included in Category B, along with responses from P10 and P16, who mentioned the same aspects but separately, without reference to the points understood in Category A. P16 stated, "Besides being imprecise, it could have been done in another way that would be more accurate". Thus, the student recognized that Galilei not only criticized the Vitruvian version for its imprecision but also proposed an adequate solution from the standpoint of accuracy (Martins, 2000). P10 asserted, "It was not precise, as there was nothing that proved the fact," echoing his peers' responses regarding Galilei's criticism of the method described by Vitruvius. Furthermore, in this response, he seemed to signal the absence of historical evidence supporting the Vitruvian version (Martins, 2000).

In Category C, inadequately representing the discussion conducted during the intervention, P3 stated, without explaining his argument, that Galilei deemed Vitruvius's narrative as "invalid according to the principles of the time". P3's response was not structured enough to convey what the student intended to express, although it is evident that he recognized Galilei's rejection of the Vitruvian narrative.

In Category D, we placed P11's response, who declared, "There is no mention whatsoever of Galilei regarding Vitruvius's version". This assertion may have resulted from inattention during the intervention. Galilei does not explicitly name Vitruvius, but he refuses the most widely circulated version of the historical episode attributed to the Roman architect.

In the fifth question, participants were also prompted to explain Galilei's narrative regarding the use of the Hydrostatic Balance by Archimedes: *According to Galilei, how would Archimedes have used a Hydrostatic Balance to analyze the case of the crown? What would Archimedes have done to determine if there was fraud?* (Table 7).

In Category A, 9 out of 16 participants were noted, whose responses referred to comparing the weight of the crown, immersed in water, to the weights of samples of gold and silver of the same mass (as the crown). In fact, this kind of response aligns with the method described by Galilei, based on weight comparisons, differentiating it from the method narrated by Vitruvius, which is based on comparisons of displaced volumes (Martins, 2000). This distinction had been extensively discussed during the intervention using the historical sources written by Galilei, coupled with the experimental discussion of this procedure. The incidence in this category indicates that students understood the essence of how the method described by Galilei operates.

Table 7 – Summary of responses to the fifth question.

Category	How Archimedes used the Hydrostatic Balance to analyze the case of the crown, according to Galileo	Participants
A	He compared the difference between the weights of the crown and the samples of gold and silver of the same mass when submerged in water.	P_1, P_2, P_6, P_7, P_8, P_9, P_{11}, P_{12} e P_{13}.
B	He analyzed the buoyant force acting on each object.	P_{10}, P_{15} e P_{16}.
C	He compared the volumes displaced after the immersion of the crown and the samples of gold and silver in water.	P_4, P_5 e P_{14}.
D	He compared how much each object sank in the water.	P_3.

Source: Own authorship.

It is important to highlight, however, that the students' responses appear to differ from the method presented in the intervention based on *La Bilancetta*. In the intervention, we indicated that each one of the three samples, made of different materials, was gradually inserted into the water, causing an imbalance in the balance that could only be corrected by adjusting the counterweight held in the air, to three distinct positions, each one related to the immersion of a different material (Galilei, 1986). In that context, during the investigative activity with the Hydrostatic Balance, the students expressed an understanding that, the greater the volume of the object, the lesser its weight in water, and that the counterweight would need to be moved closer to the axis of rotation. In contrast, the responses to the questionnaire seemed to suggest the simultaneous immersion of the crown and the other objects in water, gradually evaluating the difference between the weights of the investigated crown and those of the submerged samples.

Among the responses included in this category, it is noteworthy the statement from P12, who mentioned that, according to Galilei, Archimedes "would have placed the three masses of equal weight (gold, silver, and the crown) in the water, comparing the 'weight' that they would now have in the water, due to the difference in density between gold and silver". It is evident that the student understood that the samples had the same weight since they possessed the same mass and were subjected to the same gravitational acceleration. To refer to the apparent weight in water, which differs for each object, the student employed quotation marks as a rhetorical device. It can also be observed that he recognized a distinction between these apparent weights, given the difference in densities between gold and silver. A similar response was provided by P9: "Archimedes placed the crown and the same weight of silver and gold in the water and then analyzed the weight in water due to the difference in density between silver and gold".

We differentiate Category A from Category B based on the use of the term "buoyancy" in the explanations provided by participants classified in the latter category. In contemporary language, the Hydrostatic Balance allows for the investigation of the differences between the buoyant forces acting on each object. In Category B, we identified participants P10, P15, and P16, who mentioned the buoyant force to explain the functioning of the Hydrostatic Balance. For instance, participant P15 stated that “Archimedes would have used a Hydrostatic Balance to analyze the case of the crown. He would have submerged the crown and a sample of gold of the same mass in water and compared the buoyancy generated by each object”. The participant did not explain which factors would cause different magnitudes of buoyant forces on each submerged object.

The distinction between the responses of P15 and P12, assigned to Categories B and A respectively, lies primarily in the substitution of P12’s explanation – more consistent with Galilei's discourse – with the term “buoyancy,” which the researcher did not employ.

Another interesting response was presented by P10: “He would have used the Hydrostatic Balance, placing the crown on one side and a mass of gold of the same weight on the other side in a container of water, to see the buoyant force; if the crown had been fraudulently made, the buoyant force would be less than that of the gold mass”. The participant indicated that the buoyancy experienced by the crown was different from that experienced by an object of the same mass of gold. However, he confusedly indicated that the crown would experience a lesser buoyant force due to its composition. This misunderstanding does not appear in the same student's responses to questions 7 and 8 of the questionnaire, which we will still analyze in this section. Thus, it seems that the student adequately understood the functioning of the Hydrostatic Balance.

In Category C, we placed participants who incorrectly attributed to Galilei the description of the method outlined by Vitruvius. In this regard, P4, P5, and P14 stated that Galilei indicated that Archimedes “immersed an object of the same weight as gold and another of the same weight as silver, and then the crown, comparing the amount of water displaced to determine whether there was more gold or more silver in the crown”. These individuals provided nearly identical responses throughout the questionnaire, suggesting joint execution of the activity (contrary to the instructions given) or possibly copying responses from one individual.

In Category D, we assigned a single participant, P3, who stated that the functioning of the Hydrostatic Balance, a method attributed by Galilei to Archimedes, was related to comparing how much each body sank when immersed in water. This assertion is likely connected to the experimental activity conducted during the intervention, as the samples subjected to greater buoyancy remained suspended without completely sinking when the Hydrostatic Balance was unbalanced (Santos Júnior, 2023).

Block 2 – Historical factors related to the versions of the episode of the crown theft

Thematic coverage includes questions 3 and 6 from the post-intervention questionnaire. This analysis was based on the framework by Martins (2000).

In question 3, participants were encouraged to recall historical aspects related to the questioning of Vitruvius' version: *What historical factors discredit the version described by Vitruvius? Table 8 summarizes the arguments presented.*

Allocated to Category A, the most frequently cited factor by students, mentioned in 14 out of 16 responses, was the absence of historical records from Archimedes regarding the episode of the crown's forgery. Nearly all participants, except for P3 and P11, emphasized that this aspect made Vitruvius' narrative less reliable. For instance, P2 stated: “There is no historical evidence supporting the crown story because Archimedes left no record of what happened”. This view was reinforced by the other students.

Table 8 – Summary of the responses to the third question.

Category	Historical factors that discredit the Vitruvian version	Participants
A	Archimedes did not record the episode.	P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{12}, P_{13}, P_{14}, P_{15} e P_{16}.
B	Vitruvius did not indicate his source.	P_6, P_7 e P_{16}.
D	A slave would not fill the bathtub to the brim.	P_2, P_6, P_7, P_9, P_{10}, P_{11} e P_{16}.
C	Vitruvius and Archimedes were not contemporaries.	P_1, P_2, P_3, P_6, P_8, P_9, P_{10}, P_{12}, P_{13} e P_{16}.

Source: Own authorship.

In Category B, complementing the aspect of “absence of historical records by Archimedes”, P6, P7, and P16 added another important element: the absence of source indications in Vitruvius' text (Martins, 2000). P16, for example, stated that “Vitruvius [...] at no point indicated the sources on which he based himself”. Indeed, as emphasized in the intervention, Vitruvius did not explicitly cite any sources to support his narrative.

In Category C, 10 participants noted that Vitruvius did not live during the same period as Archimedes (Martins, 2000). Among the responses, P9 highlighted: “Vitruvius lived nearly two centuries after Archimedes, a period too long to have any theoretical foundation”. In fact, during the intervention, participants were asked to imagine what might happen to a narrative if it were verbally transmitted over a century and a half by different people. On that occasion, participants remarked that the narrative would weaken as it was disseminated over such an extended period.

In Category D, another frequently mentioned factor, noted in 7 responses, was the improbability that a servant would completely fill the bathtub (Martins, 2000). P11 commented, “A servant wouldn’t fill a bathtub to the brim knowing they would have to clean up afterward”. Indeed, as discussed in the intervention, it is likely that a servant, given Archimedes' status, would have prepared his bath. Thus, it would not be sensible to assume that this person would fill the bathtub to the top, as they would then be responsible for cleaning it. As P2 remarked: “Archimedes' servants wouldn’t fill the bathtub completely because they would have to clean up afterward, which would be more work”.

In the sixth question, participants were asked about historical evidence supporting the Galilean version: *What historical factors strengthen Galileo's version?*

The most recurring factor in the responses (Table 9) was related to the fact that Galileo's described method was based on Archimedes' own work. Indeed, the functioning of the Hydrostatic Balance is grounded in two themes studied by Archimedes: the mechanics of levers and hydrostatics (Martins, 2000). Responses recalling this aspect, which was covered during the intervention, were grouped in Category A. For example, P1 stated: “The narrative proposed by Galileo is more consistent, as the method of solution attributed to Archimedes aligns with principles derived from his recorded research”. Similarly, P12 responded that “Galileo’s version makes more sense because the solution method attributed to Archimedes is related to his documented studies”.

Table 9 – Summary of the responses to the sixth question.

Category	Historical factors that support the Galilean version	Participants
A	Coherence with the works of Archimedes.	P_1, P_2, P_7, P_8, P_9, P_{11}, P_{12} e P_{13}.
B	Corroborated by medieval documents and/or Latin poems.	P_4, P_5, P_6, P_{10} e P_{14}.
C	Other answers.	P_{11}, P_{15} e P_{16}.

Source: Own authorship.

It is notable that 8 out of the 16 students emphasized this aspect, which indeed supports Galileo's version over Vitruvius' narrative (Martins, 2000). However, fewer responses mentioned the documentary evidence that corroborates Galileo's narrative (only 5 students highlighted this factor). Thus, in Category B, participants who cited the existence of medieval documents and/or Latin poems that describe the use of the Hydrostatic Balance to solve problems like that of the crown are listed. For instance, P10 pointed out “the fact that there are medieval documents that mention the Hydrostatic Balance to solve problems like the crown's, and Latin poems that describe the use of the balance”. As mentioned during the intervention, these documents were reportedly discovered at the end of the 19th century by the French researcher Marcel Berthelot (1827-1907).

Category C lists students who provided responses that diverged from what was discussed during the intervention. P15, for example, mentioned among the historical factors “more detailed records of Archimedes' experiments made by later scientists and the greater understanding of physics and hydrostatics over the centuries”. It is possible that he was attempting to refer to documents postdating Archimedes that mention the Hydrostatic Balance. P11 stated that “Galileo was influenced by various historical factors, such as the Renaissance, which valued empirical observation and experimentation”. Although the information provided is true, it does not apply as a response to the question posed. P16 cited as historical factors “Archimedes' principles regarding the law of buoyancy”. It is possible that the student was attempting to refer to the fact that Galileo's version is more compatible with Archimedes' work on hydrostatics (Category A); however, this intention is not clearly conveyed in his statement.

Block 3 – Understanding Archimedes' Principle and its limits of validity

Thematic alignment includes questions 7, 8, and 11. This analysis was based on the framework by Silveira and Medeiros (2009).

The seventh question inquired about the concept of buoyant force: *What is the relationship between the magnitude of the buoyant force acting on an object and its submerged volume?*

The most common pattern in the responses (Table 10), noted in 7 out of 16 answers, was the emphasis on the relationship between the magnitude of the buoyant force and the volume of displaced fluid. These responses were categorized under Category A. These students indicated that the larger the volume of displaced fluid, the greater the buoyant force. As P1 stated, “The buoyant force exerted by a fluid on a submerged object is equal to the weight of the fluid displaced by the object”. This statement refers to Archimedes' Principle, which, as discussed earlier, has a limit of validity. It is important to note that the submerged volume of the object is not necessarily identical to the displaced volume of fluid; however, these quantities are directly proportional. Thus, even though these students did not explicitly mention “submerged volume” in their answers, indicating that the buoyant force is directly proportional to the displaced fluid volume is a satisfactory response. Student P6 used the exact wording of Archimedes' Principle presented during the intervention, stating, “An object fully or partially immersed in a fluid experiences an upward force equal to the weight of the fluid displaced by the object”. Again, there was no direct mention of the object's submerged volume, but the response is acceptable for conditions that do not approach the principle's limit of validity.

In Category B, 5 out of the 16 participants emphasized the direct proportionality between an object's submerged volume and the intensity of the buoyant force acting on it. These responses are considered satisfactory as they refer to the experimental activity with the Hydrostatic Balance and highlight an aspect related to the revised statement of Archimedes' Principle discussed in the intervention, which replaces the reference to the displaced fluid volume with the submerged volume of the object, thereby avoiding the Hydrostatic Paradox (Silveira; Medeiros, 2009).

Table 10 – Categorization of the responses to the seventh question.

Category	Relationship between the magnitude of the buoyant force acting on an object and its submerged volume	Participants
A	The greater the volume of fluid displaced, the greater the magnitude of the buoyant force.	P_1, P_2, P_6, P_7, P_8, P_9 e P_{11}.
B	The greater the submerged volume of the object, the greater the buoyant force.	P_{10}, P_{12}, P_{13}, P_{15} e P_{16}.
C	Conditions for floating	P_4, P_5 e P_{14}.
D	The buoyant force is equal to the weight of the object in water.	P_3.

Source: Own authorship.

The incidence of this response pattern demonstrates that the discussion regarding the update of Archimedes' Principle had positive effects. P15, for example, stated that "the relationship between the magnitude of the buoyant force acting on an object and its submerged volume is direct". It is believed that by expressing that the relationship between buoyant force and submerged volume "is direct," the student intended to indicate that these quantities are directly proportional. P10 made a similar statement, noting that "the relationship is that the larger the volume (submerged), the greater the buoyant force will be". The student had previously indicated in response to question 5 that the gold and silver crown would experience less buoyant force than an object of the same mass in gold, which raised doubts about their understanding of the discussion. However, the correct indication that there is a direct relationship between the submerged volume of an object and the magnitude of the buoyant force acting on it suggests that the error in question 5 may have simply been a writing mistake.

In Category C, P4, P5, and P14 provided nearly identical responses. These students did not indicate the relationship between the buoyant force magnitude and the submerged volume of an object. Instead, they discussed the conditions for flotation, a topic that was not covered in class. The students' response was as follows: "If an object submerged in a fluid sinks, it can be stated that its density is greater than that of the liquid, and its weight exceeds the buoyant force. If the densities of the body and the fluid are equal, the body will remain in equilibrium in the liquid, and it can be stated that the buoyant force is equal to the weight of the body".

While the statement is correct, it reflects a misunderstanding of the question or a lack of interest in answering the questionnaire, as the students were not asked about the flotation conditions of an object fully or partially immersed in a fluid.

Finally, in Category D, P3 indicated that "the buoyant force is equal to the weight of the object in water". He may have also been referring to a flotation condition, but his reasoning is incomplete, failing to convey what he might have been thinking.

The eighth question concerned the comparison of buoyant force magnitudes: *When submerging a crown composed of gold and silver and a sample of gold of the same mass in water, which object will experience a greater buoyant force?* (Table 11).

Table 11 – Summary of the responses to the eighth question.

Categoria	**Which object will experience a greater buoyant force: a crown made of gold and silver or a sample of gold of the same mass?**	**Participantes**
A	The gold and silver crown.	P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}, P_{13} e P_{14}.
B	Both will experience the same buoyant force.	P_{15}.
C	The crown, because it has a greater mass.	P_{16}.

Source: Own authorship.

Almost all participants (14 out of 16 students) correctly responded that the crown made of gold and silver would experience a greater buoyant force than the pure gold sample. These students were placed in Category A. An example of a statement that falls within this category was provided by P2, who indicated that the buoyant force would be greater "on the crown made of gold and silver because it has a larger volume". A similar response was given by P10, who stated that the buoyant force would be greater on the object containing "silver, because its volume is larger". Once again, P10's statement confirms the hypothesis that their response to the fifth question reflected a writing error rather than a misunderstanding of Archimedes' Principle.

Regarding Category A, it is noteworthy that P3, P4, P5, and P14 had previously provided responses to the seventh question that did not clearly explain their understanding of the relationship between the submerged volume of an object and the magnitude of the buoyant force acting on it. Nevertheless, they correctly answered the eighth question, indicating that the crown made of gold and silver would experience a greater buoyant force than a gold object of the same mass. This may suggest a misunderstanding of the previous question or difficulty in articulating their thoughts in writing. The other students mentioned in this category gave responses that were consistent with their answers to the seventh item on the questionnaire.

In Categories B and C, P15 and P16 are listed, respectively. P15 stated that "both objects will experience the same buoyant force because buoyancy depends only on the volume of displaced liquid, not on the composition of the objects". It is interesting to note that this same pattern of response is obtained when the question is posed to the artificial intelligence (AI) known as ChatGPT. The students were warned during class that the AI makes an error when analyzing the crown problem. Nonetheless, P15 seems to have persisted in relying on it, demonstrating a lack of critical judgment regarding information found on the internet. In fact, the magnitude of the buoyant force only depends on the volume of displaced fluid, but the crown made of a metal alloy displaces a larger volume of liquid than a pure gold sample of the same mass, thus experiencing a more intense buoyant force.

Meanwhile, P16 correctly stated that the crown would experience a greater buoyant force but justified this by saying, “it will be the body with the greater mass”. This statement indicates a lack of attention, as the question itself specifies that both objects have the same mass.

The eleventh question asked participants about a potential limit to the validity of the principle: *Does Archimedes' Principle have a limit of validity? Explain. To answer this question, remember the experiment on the Hydrostatic Paradox.*

Table 12 – Summary of the responses to the eleventh question.

Categoria	**Does the statement of Archimedes' Principle have a limit of validity? Explain.**	**Participantes**
A	Yes, it is only valid when the object immersed in the fluid has dimensions much smaller than those of the container.	P_4, P_5, P_6, P_{10}, P_{12}, P_{13} e P_{14}.
B	Yes, the Hydrostatic Paradox is observed when an object is submerged in a fluid contained in a container with dimensions much larger than the object itself.	P_1, P_2, P_7, P_8 e P_9.
C	Yes, it only applies to certain objects.	P_3.
D	No, but it can be challenged in some situations, such as in the Hydrostatic Paradox experiment.	P_{15}.
E	Yes, it is not always entirely accurate for determining the purity of a material.	P_{11}.
F	Yes, in the case of the bathtub, it was impractical for it to be filled to the brim for a bath.	P_{16}.

Source: Own authorship.

Listed in Category A, seven out of sixteen participants responded to the question by indicating that Archimedes' Principle would only be valid when the object immersed in the fluid has dimensions significantly smaller than those of the container holding the fluid (Table 12). Indeed, as discussed during the intervention, the usual statement of Archimedes' Principle fails to predict the magnitude of the buoyant force when the object's dimensions are comparable to those of the container. Thus, it would only be valid if the container's dimensions are much larger than the object's dimensions (Silveira; Medeiros, 2009). In this context, P12 stated that “for the principle to be valid, the object must be placed in a fluid located in a container with much larger dimensions”. Similarly, P14 wrote that “it is only valid when the dimensions of the object inserted into the fluid are much smaller than the dimensions of the container holding the fluid". These statements are equivalent, demonstrating an understanding of what was discussed.

In Category B, an inversion is noted. Five participants mistakenly indicated that the Hydrostatic Paradox occurs when the container holding the fluid is much larger than the object immersed. However, this is precisely the condition that *does not generate any paradox*. The usual statement of Archimedes' Principle works exclusively in this situation, as highlighted during the intervention.

P1's statement is an example of a response classified under Category B: "The validity of Archimedes' Principle has limitations in some circumstances, as demonstrated by the Hydrostatic Paradox experiment. It manifests when an object is immersed in a liquid held in a container significantly larger than the object". The participant recognized that Archimedes' Principle has limitations but mistakenly described the conditions under which its validity fails.

Categories C, D, E, and F each have one representative. P3, in Category C, simply indicated that "as seen in class, the principle applies to some objects but not to others," without offering any explanation to clarify their understanding. In Category D, P15 indicated: "Archimedes' Principle has no limit of validity because it is a fundamental law of physics that applies to any object immersed in a fluid. Therefore, the Hydrostatic Paradox experiment shows that the principle can be challenged in specific situations, but this does not invalidate its general applicability". It is interesting to note that, although the student incorrectly asserted that Archimedes' Principle has no limit of validity, he paradoxically acknowledged that it could be "challenged in specific situations," such as the "Hydrostatic Paradox experiment". However, the student's response lacks sufficient detail to clarify his understanding of what these specific situations might be.

In Category E, P11 suggested that Archimedes' Principle is limited because "in all cases, we do not have total precision regarding the purity of the material". This response shows either a clear misunderstanding or a lack of engagement with the question, or simply a lapse of attention during the discussion on this topic.

Finally, in Category F, P16 stated that "we can recall the bathtub incident, where it would be impractical to fill the tub to the top for Archimedes to take a bath". This response appears to reflect a poor understanding of the question.

Block 4 – Conceptions of the Nature of Science (NoS)

Questions 09 and 10 of the post-intervention questionnaire are related to this topic. Question 9 prompted students to reflect on possible changes to Archimedes' Principle over time, asking: *Can we say that the principle formulated by Archimedes in the 3rd century BCE is identical to what we consider today as Archimedes' Principle? (Consider, for example, its applicability, the terms used in its formulation etc.)*

The most frequently cited distinction between the modern statement of the principle and that formulated in the 3rd century BCE concerns its current applicability to fluids (liquids and gases), rather than exclusively to water, as established by Archimedes (Table 13). Eleven out of sixteen students mentioned this modification in the statement of Archimedes' Principle, being listed in Category A. The high recurrence of this response pattern suggests that the students recognized this alteration as significant. This perception may have been reinforced by the discussion of the example of buoyant force acting on a hot air balloon.

Student P1 provided an example of a response categorized as follows: "His formulation was specific to water and did not incorporate the characteristics of other liquids or gases. The principle he conceived does not exactly resemble what we currently recognize as Archimedes' Principle, which is a broader and more refined formulation, considering various types of fluids". The student demonstrated an understanding of the mutable nature of the principle, indicating that its applicability today differs from that of two millennia ago. However, the assertion that the principle is now more "refined" is not entirely appropriate, as it reflects a value judgment arising from a decontextualized comparison of statements from very different eras.

Table 13 – Summary of the responses to the ninth question.

Category	**Is the principle formulated by Archimedes identical to the modern Archimedes' Principle?**	**Participants**
A	No, because Archimedes' statement of the principle applied only to water (or liquids), whereas today it applies to fluids in general.	P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{12}, P_{13} e P_{14}.
B	No, since Archimedes did not account for the role of surface tension in the overflow of liquids.	P_1, P_2, P_8, P_9, P_{12} e P_{13}.
C	No, the modern formulation is broader or improved.	P_1, P_2, P_3, P_7, P_8 e P_9.
D	It is identical/similar, but with some modifications.	P_{10} e P15.
E	No, because there are now more efficient methods for determining the purity of gold.	P_{11}.
F	No, because Archimedes did not leave any account of the crown episode. Therefore, it is likely that the story has been distorted.	P_{16}.

Source: Own authorship.

A similar response was given by P4, who stated that "at that time, fluids were only liquids, and today gases are also included". The participant referred to the term "fluid," present in Archimedes' work, which, in fact, only referred to water. He correctly indicated the modern applicability of the principle to both liquids and gases.

In Category B, students who mentioned that surface tension was not considered by Archimedes in the formulation of the principle are listed. P8, for instance, noted that "Archimedes developed his principle based on observations with water as a fluid, and he was not aware of the generalization of the principle for different fluids, or the limitations imposed by surface tension". Indeed, such a phenomenon was not considered, and the concept of surface tension itself did not exist in the 3rd century BCE. Although surface tension has no direct implication for the statement of the principle, it influences the fact that, even if an object is submerged in a liquid about to overflow from a container, the displaced volume may differ from the submerged volume of the object (which invalidates Vitruvius' method). The participants' reference to this effect likely stems from the investigative experimental activity conducted during the intervention, which involved placing coins into a glass of water filled to the brim.

In Category C, participants who claimed that the current statement of Archimedes' Principle is more comprehensive than that of the 3rd century BCE are listed. P1, P2, P7, P8, and P9 made this assertion while noting that the principle now applies to both liquids and gases. P3, on the other hand, simply stated that "the principle has changed over time, and as more studies were conducted, it was improved".

In Category D, students P10 and P15 indicated that the current statement of Archimedes' Principle remains identical or similar to what was written in antiquity. P10 stated that "today the principle is more elaborate, but it remains the same". The student's conclusion is somewhat paradoxical. If the principle is more elaborate today, how could it still be the same? P15 asserted that the current principle is similar to the one from antiquity, "but there may be differences in wording and applicability depending on the context". Like P10's statement, P15's response appears somewhat contradictory. How can a principle that has had its wording and applicability altered remain similar to what it was before? It was expected that students would recognize that the changes the principle has undergone over centuries would be enough to assert its considerable difference from the 3rd century BCE statement.

Finally, in Categories E and F, we find responses that diverged from the question at hand. P11 (Category E) suggested that Archimedes' Principle had not remained unchanged because more effective methods for determining the purity of gold now exist. Participant P16 (Category F) noted that "over the course of history, this story may have changed because [Archimedes] did not write about the subject". This statement refers to the fact that the episode of the crown was not reported by Archimedes. However, the question did not concern this aspect.

We now move on to the discussion of the results obtained in response to the tenth question of the questionnaire. As is well known, specialized literature has long recommended a preferably explicit and contextualized approach to the NoS through historical episodes. There are references indicating that implicit teaching does not contribute as much to the learning of NoS when compared to explicit teaching of this theme (Abd-El-Khalick; Lederman, 2000; Clough; Olson, 2008; Mccomas, 2008; El-Hani; Freire Junior; Teixeira, 2009; Forato, 2009).

As previously indicated in the synthesis of Part II of the didactic sequence, the intervention employed an implicit approach to the “provisionality of knowledge.” Thus, we sought to determine whether this type of approach, even if not explicit, yielded satisfactory results. The tenth question aimed to investigate *whether the participants were able to perceive* the provisional nature of scientific knowledge through the discussions: *Considering the previous question, what does this suggest about the nature of scientific knowledge?*

The students' responses were organized into categories, which can be consulted in Table 14.

Table 14 – Summary of the responses to the tenth question.

Category	**Consider the previous question to reflect on the nature of scientific knowledge.**	**Participants**
A	Scientific knowledge can be altered.	P_3, P_{12} e P_{16}.
B	Scientific knowledge evolves/is refined over time.	P_1, P_2, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{13}, P_{14} e P_{15}.

Source: Own authorship.

Promoting more complex views on NoS through the discussion of primary sources was one of the objectives set for the intervention. The responses to the tenth question indicate that the participants realized that the contextually addressed episode signaled the provisional nature of knowledge. For all participants who responded to the questionnaire, scientific knowledge was seen as mutable. Thus, although the approach did not explicitly focus on this aspect, there were satisfactory results in contributing to the teaching of the theme “provisionality of knowledge,” which, to some extent, aligns with perspectives highlighted in the literature.

On the other hand, there were variations in the students' responses, with more naïve views also being observed. In Category A, the answers of three participants who acknowledged the possibility of knowledge modification are listed. Included in this category is the peculiar response from P3, who stated that knowledge is provisional until definitively proven. For P3, science could undergo modifications when “the theory is not 100% proven”. In this case, we observe a more simplistic view of science, as the participant considers scientific knowledge to be mutable but believes it becomes definitive once experimentally proven. Although this was not one of the intervention's objectives, such a response highlights the need to discuss the role of experimentation and the meaning of empirical corroboration. P16, for example, suggested that in science, “changes can occur over time, along with influences”. However, he did not elaborate on what these “influences” might be, leaving us without specific indications to further explore this interesting aspect of his response.

Regarding this topic in the questionnaire, 13 out of the 16 participants explicitly stated that scientific knowledge evolves or improves over time (Category B). P15 indicated that “scientific knowledge is built over time, with new discoveries and refinements, and that scientific theories can be revised and updated based on new evidence". P9 stated that "the example shows how scientific knowledge evolves and transforms over time". Participants thus demonstrated an understanding of scientific knowledge as a construction that can be updated and revised over time. Some cited the example of Archimedes' Principle to contextualize this view. P11, for instance, responded that "scientific knowledge advances as centuries pass, which has, in a way, improved Archimedes' Principle". Given the responses grouped in Category B, we believe it would be useful to address, in such interventions, how these transformations occur, to avoid potential perceptions of a linear, continuous evolution of science or decontextualized comparisons of knowledge from different eras, which may lead to anachronistic evaluations. Deepening these discussions, possibly in an explicit manner, as suggested by the literature, could offer additional benefits to the participants.

In this section, we have analyzed the responses to the research instrument. Regarding the analysis of the physical and historical aspects of the versions of the crown episode, the students demonstrated a good understanding of the narrated methods and the reasons why Galileo's version inspires greater credibility compared to Vitruvius'. Regarding Archimedes' Principle, satisfactory responses were noted, correctly indicating the direct proportionality between the submerged volume of objects and the buoyant force acting upon them. However, some participants struggled with the Hydrostatic Paradox. As for their conceptions of NoS, interesting results were observed. A significant portion of the participants highlighted the provisional nature of scientific knowledge. It is important to note, however, that traces of value judgments stemming from decontextualized comparisons between knowledge from different periods were observed in the responses. This aspect would deserve to be addressed at an appropriate time.

V. Final comments

The impact of the approach proposed in this work can be evaluated from four aspects: 1. the understanding of the physical and historical factors that make Galileo's narrative more coherent than Vitruvius'; 2. the comprehension of the Hydrostatic Balance method and the arguments supporting its probable use; 3. the understanding of the statement of Archimedes' Principle and its limits of validity; and 4. the conceptions about the Nature of Science (NoS).

Regarding the first aspect, both during the implementation of the didactic sequence and in the responses to the investigative questionnaire, most participants correctly expressed that Vitruvius' version was based on measurements of displaced volumes and pointed out that surface tension and the minimal variation in water level would hinder the use of this method. Moreover, the interaction throughout the intervention and the responses observed in the questionnaire demonstrated an understanding of the historical aspects discussed, such as the lack of historical records left by Archimedes, Vitruvius' failure to cite sources, and the two-century gap between these two figures. The questioning stages, carried out according to the

Three Pedagogical Moments framework, were essential for developing these perceptions, highlighting the importance of this framework for intervention.

In terms of understanding the Hydrostatic Balance method, most participants correctly identified that Galileo's method was based on the comparison of apparent weights or, in modern terms, on the comparison of the buoyant forces acting on objects of the same mass but different densities. Regarding the historical arguments supporting this version, caution is necessary. Few participants were able to point out the material historical evidence corroborating Galileo's narrative and the use of the Hydrostatic Balance. This indicates the need for greater attention to this aspect in future applications, reinforcing with greater care the evidence found in the late 19th century.

As for the historically contextualized understanding of Archimedes' Principle, the perception was positive. We observed that the participants demonstrated a correct understanding of the relationship between the magnitude of the buoyant force and the submerged volume of an object in a fluid. In the questionnaire responses, most students stated that larger volume objects are subjected to a greater buoyant force when submerged. There was thus a well-founded conclusion that the fraudulent crown experienced a more significant buoyant force than a gold object of the same mass.

The participants succeeded in constructing knowledge about the Hydrostatic Balance. The fact that we relied on the investigative experimentation framework, proposing the didactic use of the Hydrostatic Balance in a dialogic manner, based on questions, contributed to these positive results. In the knowledge application stage, the participants successfully used the Hydrostatic Balance, appropriately determining the proportion between the masses of coins and marbles in a sample containing these materials. Thus, the Three Pedagogical Moments framework also demonstrated its effectiveness as a contribution to the developed work, integrating and articulating the various stages.

Regarding the limits of validity of Archimedes' Principle, most students correctly responded that the usual statement applies only to objects with dimensions much smaller than the container holding the fluid. However, it was noted during the intervention that students had difficulty understanding the distinction between the usual statement of this principle and the revised statement presented by Silveira and Medeiros (2009). Additionally, the questionnaire responses revealed that some students failed to describe the Hydrostatic Paradox correctly. These observations indicate the need for more extended discussion periods on situations that explore the limitations of Archimedes' Principle. One alternative considered for future applications of the didactic sequence is not to present the revised statement, focusing instead on demonstrating, through the seemingly paradoxical result of the experiment, that the traditional statement does not apply to situations analogous to that one. In this way, the usual statement of the principle would be presented alongside a clear indication of its limits of validity.

Concerning the nature of scientific knowledge, although we did not explicitly emphasize considerations about NoS, important results were noted. On the one hand, participants at times demonstrated a certain tendency toward decontextualized (and anachronistic) comparisons of knowledge from different eras; on the other hand, they appropriately indicated that science is mutable, and scientific knowledge can undergo revisions. Such perceptions are sufficient to assert that the students manifested appropriate conceptions regarding the provisional nature of the scientific enterprise. However, it became clear that it would be desirable to advance in questioning some simplistic views about the NoS that emerged during the intervention, possibly resorting to more explicit approaches, as suggested in the literature.

In summary, considering the results gathered through the research instrument and the teacher's perception of the implementation, the outcome was positive. The discussion of the two versions of the episode allowed the students to exercise their critical thinking by analyzing the physical and historical aspects related to these versions, thus selecting one as the most coherent. Finally, they reached important conclusions regarding the nature of knowledge, as they pointed out the existence of a limit of validity for the studied principle and recognized that it had undergone modifications over the centuries. Through the historically contextualized approach, students were able to understand Archimedes' Principle and apply it, solving situations and evaluating historical narratives intrinsically related to the construction of this knowledge. These results highlight the significant role that the History of Science can assume when used beyond mere illustration, constituting a possible approach to teaching science and about science.

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Appendix I – Excerpts from *De Architectura*

Though Archimedes discovered many curious matters which evince great intelligence, that which I am about to mention is the most extraordinary. Hiero, when he obtained the regal power in Syracuse, having, on the fortunate turn of his affairs, decreed a votive crown of gold to be placed in a certain temple to the immortal gods, commanded it to be made of great value, and assigned an appropriate weight of gold to the manufacturer. He, in due time, presented the work to the king, beautifully wrought, and the weight appeared to correspond with that of the gold which had been assigned for it. But a report having been circulated, that some of the gold had been abstracted, and that the deficiency thus caused had been supplied with silver, Hiero was indignant at the fraud, and, unacquainted with the method by which the theft might be detected, requested Archimedes would undertake to give it his attention. Charged with this commission, he by chance went to a bath, and being in the vessel, perceived that, as his body became immersed, the water ran out of the vessel. Whence, catching at the method to be adopted for the solution of the proposition, he immediately followed it up, leapt out of the vessel in joy, and, returning home naked,[o] cried out with a loud voice that he had found that of which he was in search, for he continued exclaiming, in Greek, Eureka! (I have found it out) (Vitruvius, 1958, p. 238).

[...] he is said to have taken two masses, each of a weight equal to that of the crown, one of them of gold and the other of silver. Having prepared them, he filled a large vase with water up to the brim, wherein he placed the mass of silver, which caused as much water to run out as was equal to the bulk thereof. [...]. He then placed the mass of gold in the vessel, and, on taking it out, found that the water which ran over was lessened, because, as the magnitude of the gold mass was smaller than that containing the same weight of silver. After again filling the vase by measure, he put the crown itself in and discovered that more water ran over then than with the mass of gold that was equal to it in weight [...] (Vitruvius, 1958, p. 239).

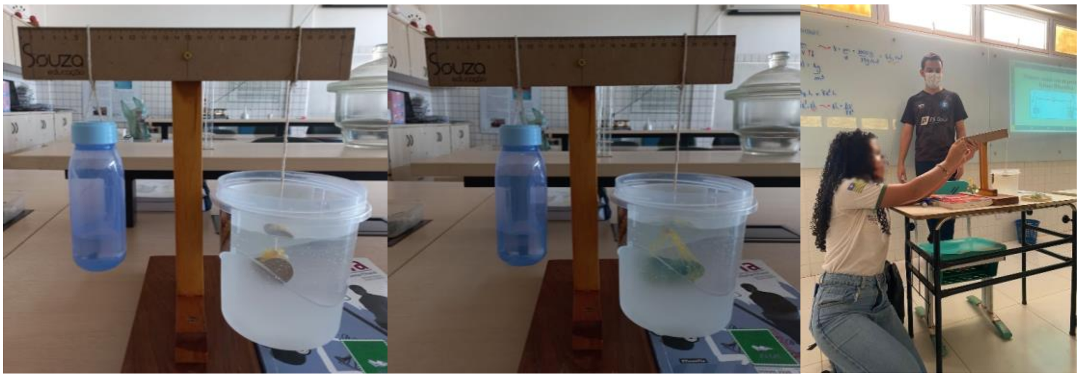
Appendix II – Excerpts from *La Bilancetta*

Excerpt A - Those who read the ancient authors carefully are familiar with the fact that Archimedes discovered the theft by the goldsmith in Hieron's golden crown. However, I believe that, until now, it is not known how that illustrious man proceeded to make this discovery. [...] I would believe, though, that after the news spread that Archimedes had uncovered the theft through water, some contemporary author must have left an account of the event; and that the same, while adding something to the little he had understood from the widespread rumors, said that Archimedes had used water, which became universally accepted. However, the fact that I knew that this method was entirely flawed, lacking the precision required in mathematical matters, led me many times to contemplate how one could rigorously discover, through water, the composition of an alloy of two metals (Galilei, 1986, p. 105).

Excerpt B - [...] after carefully reviewing what Archimedes demonstrates in his treatises [...] a method came to my mind that resolves the problem perfectly. I would believe that this is indeed the method used by Archimedes, as it is not only extremely precise but also relies on demonstrations discovered by Archimedes himself (Galilei, 1986, p. 105).

Excerpt C - Let us then assume that the weight (b) is gold, and that when weighing it in water, the counterweight must be brought to (e); and then, proceeding in the same manner with pure silver, the counterweight is brought to (f), a point that is closer to point (c), as the experiment shows, since silver is less dense than gold. [...] If we now have an alloy of gold and silver, it is clear that, containing silver, it will weigh less [will have a lower density] than pure gold, and, containing gold, it will weigh more [will have a higher density] than pure silver. Therefore, when weighed in air and wanting the same counterweight to balance it when submerged in water, it will be necessary to bring the counterweight to a position closer to the fulcrum (c) than point (e), which corresponds to gold, and farther away than (f), which corresponds to pure silver. Thus, [the counterweight] is located between the markers (e) and (f), and the proportion of the two metals in the alloy will be perfectly determined by the ratio in which [the position of the counterweight] divides the distance "ef" (Galilei, 1986, p. 106).

Appendix III – Hydrostatic Balance developed by the authors and used in the didactic sequence



Source: own authorship.