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Potentially meaningful teaching units on electromagnetic induction: a study on higher education students' conceptualization[+*]

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Abstract

We discuss the results of the implementation of two Potentially Meaningful Teaching Units for approaching the concept of electromagnetic induction in Higher Education basic cycle electromagnetism subjects in two contexts. These unities had nine classes of 100 minutes each, which were split in expositive-dialogic lecture sessions of 40 minutes, when the teacher addressed the content synthetically, and problem-solving sessions of 60 minutes, when students solved problems in groups. Conceptual field theory was used as theoretical framework and the methodology of content analysis was carried out to investigate conceptualization modes developed by students in problem-solving. We restricted data analysis to answers for problems derived from two classes of situations, namely, description of electromagnetic interactions and symbolic representation of the electromagnetic field. In diagnostic evaluation, we identified initial frustrated students' attempts of adapting prior knowledge obtained from electrostatics and magnetostatic contexts for describing electrodynamic fields. Along the UEPS, in both contexts, we highlighted the existence of adaptation patterns of conceptualization modes due to most part of the students to the ones deemed scientifically adequate. Learners seem to start a process of partial detachment of the instrumentalist notion of the

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electromagnetic field and begin to adopt more realist models of this field, besides that, they apparently become more familiar with relational and abstract symbolic representations of the electromagnetic field.

Keywords: *PMTU; Electromagnetic Induction; Conceptual Fields.*

I. Introduction

The descriptive background of Physics Teaching research points for few studies on learning processes related to the concept of electromagnetic induction. Even though works developed on conceptions associated to this concept are relatively sparse, they provide enough data to conclude it is difficult to be learned. Zuza, García and Guisasola (2012) point, in a review of literature about this topic, diverse misconceptions employed by students to solve problems involving the concept of electromagnetic induction. Some of these conceptions are also highlighted by more recent research (GUISASOLA; ZUZA; ALMUDÍ, 2013; ZUZA *et al*., 2014; KUO; WIEMAN, 2016; JELICIC; PLANINIC; PLASINIC, 2017). They are listed in sequence:

- confusing the area of the circuit and the area spanned by the movement of the circuit; attributing to the magnetic field the role of generator of induced electric current; confusing electrostatic fields and non-electrostatic fields; identifying the electromotive force (emf) with potential differences; describing the relations between the concepts of electromagnetic field and electromagnetic force incorrectly; confusing magnetic flux with magnetic field; supposing the flux a concept without physical meaning; associating magnetic flux with magnetic field “fluidity”; confusing the time variation of the magnetic flux with the flux itself; not recognizing the time variation of magnetic flux as a source of emf; confusing the magnetic and electric forces; determinizing the direction of magnetic force incorrectly; not differentiating epistemologically and ontologically between the coulombian and Maxwellian research programs; not relating central concepts in electromagnetic theory; not recognizing electromagnetic induction phenomena; not reaching to relate induced emf, induced current and orientation of the current; not recognizing electromagnetic induction phenomena when there is not induced electric current; supposing the electromagnetic field as a support for force transference; attributing emf to the magnetic flux; establishing relation of direct proportion between the electric current producing the emf and the induced electric current;

A relevant number of students’ difficulties are tied to non-problematizing, linear, excessively narrative, and transmissive teaching processes. In this context, the Potentially Meaningful Teaching Units (PMTU) are presented as alternatives to the traditional teaching model (MOREIRA, 2011), because they are structured on theories whose foundations circle the concept of meaningful learning. A considerable quantity of research has been showing that approaching teaching processes by means of PMTU provides conditions for occurrence of meaningful learning in the predicative form of knowledge related to electromagnetism (PANTOJA; MOREIRA, 2019; PANTOJA; MOREIRA, 2020). Notwithstanding, there are few studies sustaining this methodology facilitates learning processes in the operational form of knowledge.

The operational and predicative forms of knowledge are simultaneously studied by means of the conceptualization, in the theory of conceptual fields (VERGNAUD, 2009). This construct can be understood as how individuals refer to objects and situations of the world through using of concepts. Therefore, we propose the following question to guide this work: "how higher education students develop conceptualization processes in a Potentially Meaningful Teaching Unit to address the concept of electromagnetic induction?" To give an answer this question, we present the results of implementation of two PMTU of the concept of electromagnetic induction in two different contexts. The data analysis was developed taking as theoretical framework the theory of conceptual fields (VERGNAUD, 2009) and as methodological framework, the content analysis (BARDIN, 2008).

II. Theoretical framework

The theory of conceptual fields is a developmental theory which embraces cognitive processes occurring inside and outside school. Its main goal is finding fruitful connections between action and language. This framework takes three fundamental premises: a) cognitive processes are adaptative; b) cognitive development is function both of action and representation; c) activity must be studied in situation (VERGNAUD, 2009).

Gèrard Vergnaud, the author of this theory, indicates the existence of dialectical relation between two ways of knowing, namely, the operational one, underlying individuals' actions, and the predicative one, which is explicit and expressed in symbolic and linguistic forms (VERGNAUD, 2013). In his perspective, both modes of knowledge are relevant for which he calls conceptualization, the act of establishing reference to reality using concepts (VERGNAUD, 2009). Rarely one founds Works considering the operational form of cognition, once most of the research solely emphasizes the study of linguistic processes e representations turned explicit by the students.

Vergnaud approaches cognitive development and learning in a complex manner, by using conceptualization as a central element in this process. Then, by regarding both action and representation, we are driven to consider these two elements simultaneously, which implies action is intertwined with language. Dispraising that means building a behaviorist avatar of this process, from the point of view of Vergnaud (2013).

Another consequence of considering conceptualization as a cornerstone of learning and development is the reorganization of the concept of concept. For Vergnaud (2009), action is seldom addressed to objects, because it is solely oriented towards situations, and these involve sets of interrelated objects. Thus, conceptual learning must take account of situations making concepts useful and meaningful. Classically, concepts are understood as possessing meanings and representations, but this approach is appropriated for investigating the predicative form of knowledge, once language establishes direct reference between signified and signifiers independently of situations (VERGNAUD, 2013). Concepts, under the light of conceptual fields, are composed by three sets: the one of situations making it useful and meaning (referent); the one of operational invariants giving its meaning (signified); and the one of representations symbolizing it (signifiers).

In this context, situations are understood as tasks (well-structured and ill-structured) that can be divided in subtasks. Operational invariants are classified as *theorems-in-action* and *concepts-in-action*. The former are *propositions* considered true about reality, while the latter are categories regarded as pertinent about reality. Inasmuch as propositions are constituted by categories related and these need relations to acquire sense, then theorems-in-action and concepts-in-action cannot be reduced one to the other. By the other hand, linguistic representations, symbolic or gestural, represent situations and operational invariants used by subjects to analyze and master situations.

To appraise both action and symbolization, Vergnaud (2013) brings the concept of *scheme* for analyzing activity, for this one carries with it both mentioned elements. Respecting the thesis pointing that knowledge is adaptation, the author indicates schemes adapt to situations. While the predicative form of cognition is strongly related to enunciating objects by means of language, actions are far more attached to tasks, and once these ones can be expressed linguistically, it is possible to approach both simultaneously through the study of the interaction between schemes and situations (VERGNAUD, 2009).

It is possible to define scheme in four different, but complementary, ways, namely: a) A dynamical and functional totality, that is, a not rigid entity, flexible, serving to the possibility of adaptation (VERGNAUD, 2013); b) an invariant organization of action for a given class of situations, videlicet, a form of organizing action (and symbolization, consequently); c) an entity composed of goals and anticipations permitting the prevision of possibilities for activity, of rules of the kind IF… THEN that compose the generative part of the scheme, of operational composing the conceptual part of the scheme, and of possibilities of inference turning the continuity of the activity possible (VERGNAUD, 2009); d) a function which takes its input values in a temporalized space to *n* dimensions and produces output values in a temporalized space to *n'* dimensions (both *n* and *n'* are too big). The latter definition is important, because it is the one approaching schemes to algorithms (VERGNAUD, 2009; 2013); However, schemes are more general than algorithms, once they do not need to take activity to a conclusion in a finite number of steps, as the latter have.

Vergnaud conceives knowledge as organized in conceptual fields, which are sets of situations and problems that can be addressed using concepts, procedures, and representations of different, but interrelated, types (VERGNAUD, 2009). These conceptual fields are gradually mastered by subjects and can be regarded, literally, as a field of concepts. By this reason, a concept does not acquire its sense solely from one situation and neither a situation is mastered with just one concept, which implies thinking about relations. The study of the evolution of a given individual in a conceptual field requires, then, analyzing how his or her schemes adapt to situations. Therefore, not always the individual possesses schemes developed for this goal. How to proceed in this case?

Fanaro, Otero and Moreira (2009) indicate that action has a double character: contingent and systematic. Action is systematic when individuals already dispose of schemes to adapted to situations, that is, there are clear regularities in conceptualization, which does not mean to say actions is always the same, but it organizes itself invariantly. By the other hand, action is contingent when subjects that do not possess schemes and they must develop conceptualization improvising during they mobilize prior knowledge to reach a solution. When the situation was never solved before, conceptualization becomes an opportunist process, because the subject must use all his or her possible cognitive resources to drive it. When contingency of conceptualization reaches its minimum degree of structuration, namely, the opportunist stage, the subject can try both combine and recombine schemes, mix operational invariants from prior schemes, build new operational invariants, or even create new schemes, et cetera, for coming to conceptualization, that is, the individual makes whatever it takes for identifying objects and properties of reality by means of concepts use.

Based on what was exposed by these authors, we propose the notion of mode of conceptuatization to refer to how individuals articulate and mobilize operational invariants in situation, which allows us to describe and characterize the opportunism and contingency of conceptualization. In this perspective, a scheme can be understood as a systematic mode of conceptualization, organized in invariant way for a class of situations. We aim to identify, in this article, possible modes of conceptualization developed by students when they mastered situations involving the concept of electromagnetic induction. Works on modes of conceptualization in electrostatics (PANTOJA; MOREIRA, 2020) and magnetostatics (PANTOJA; MOREIRA, 2019) were already developed before. The result of these research works point that PMTU in electrostatic and magnetostatic seem to provide conditions for the students to build new modes of conceptualization, that progressively become compatible to scientific representations, in a considerable degree.

III. Methodology

Methodology is split in two parts for discussing separately the construction of the PMTU (didactical methodology) and how data resulting from application of the PMTU were collected and analyzed (investigative methodology).

III.1 Didactic Methodology

PMTU are structured around a set of teaching strategies aiming to provide conditions for occurrence of meaningful learning. Eight programmatic steps orient progressive teaching processes of the specific content to be learned, besides, ten principles related to the concept of meaningful learning underlie such strategies (MOREIRA, 2011). Thus, we intend to increase complexity and relatability among concepts during the teaching process.

The sequential features of a PMTU are described as follows[2]: 1) definition of the specific topic to be addressed (selection of specific content); 2) creation and proposition of problem-situations to conduct students to make prior knowledge explicit (emergence of prior knowledge); 3) proposition of problem-situations in an introductory level to make easier to introduce knowledge to be taught, regarding students' prior knowledge (problematization of prior knowledge); 4) presentation of the content to be taught accordingly to the principle of progressive differentiation (progressive differentiation); 5) resumption of general aspects of the content to be taught/learned in a greater level of complexness, give new examples and apply the principle of integrative reconciliation (integrative reconciliation); 6) conclusion of the unit implementing another process of progressive differentiation that resumes more relevant characteristics of the content, aiming at the same time integrative reconciliation (integrative differentiation); 8) development of summative evaluation process – in this evaluation the teacher must pose problem-situations whose solutions imply comprehension and grasp of meanings (summative evaluation); 9) analysis of the success of the PMTU (MOREIRA, 2011).

Progressive differentiation is the programmatic principle stating that more general concepts must be taught right in the beginning of the instruction and must be progressively differentiated along the whole process in terms of specificity. Integrative reconciliation is the programmatic principle pointing the teacher must explore relations between concepts by highlighting similarities and differences or even by means of reconciliation of real or apparent discrepancies (MOREIRA, 2011).

The PMTU for teaching the concepts of electromagnetic field and electromagnetic induction were developed in nine classes (of two intervals of 50 minutes each). Before these PMTU, another three ones about scalar and vector fields (PANTOJA; MOREIRA, 2017), electric fields (PANTOJA; MOREIRA, 2020) and magnetic fields (PANTOJA; MOREIRA, 2019) were applied. The nine classes were interrelated and aimed at relating the concept of electromagnetic induction to the ones of electric field, magnetic field, electric charge, electric current, electric flux, electric circulation, magnetic flux, magnetic circulation, electromagnetic force, and time variation rate. Table 1 is a timeline synthetizing this process.

Table 1 – PMTU timeline in terms of steps (P) and classes (A).

Step	Description	Class

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[2] The labels for each step, inside parenthesis, were created by us for better referencing.

1	Selection of specific content	0
2	Emergency of prior knowledge	1
3	Problematization of prior knowledge	1
4	Progressive differentiation	2, 3, 4, 5
5	Integrative reconciliation	6,7
6	Integrative differentiation	8
7	Summative evaluation	9
8	PMTU evaluation	X

Each class had a duration of one hour and 40 minutes and was split in two moments, the first one with 40 minutes, in which the teacher presented the specific content to be taught in its final form in an expositive-dialogued manner. The second one, which took one hour, the teacher posed problems to be solved in group by the students (three or four students in each group). The former moment was labeled as expositive-dialogued class (AE) and the second one is called problem solving (RP). There was an exception in the first class, because RP was developed before AE.

The objective of the AE was to guide meaningful construction of new knowledge in predicative form and make easier conceptual organization of cognitive structures. In these ones, programmatic principles of progressive differentiation, integrative reconciliation and sequential organization were applied by means of verbal instruction and conceptual mapping. By the other hand, the objectives of the RP were the ones of stimulating recursive and operational reasoning, facilitate knowledge manipulation in situation, and providing conditions for the students to become aware of their operational invariants. In these ones, problem-situation posing was developed by means of problem-solving of problems proposed by the teacher.

A didactical material for the classes was built for students to use when they deemed necessary[3]. It was a primary reference text for the student to review the content addressed in classroom, but that did not exhaust all possibilities of studying. Besides this production, the proposed studying material included traditional Physics textbooks[4] and articles published in specialized journals framed in Physics Education Research. This was done to reach the transversal feature of diversity of didactic materials (MOREIRA, 2011).

Epistemological and ontological differences between the concepts of force and field were already discussed throughout the course and were being materialized (MARTIN; SOLBES, 2001; FURIÓ; GUISASOLA, ZUBIMENDI, 1998). It is important to stress that the emphasis given to the concept of electromagnetic field is the one of symmetry between electric and magnetic field, because it is common to conceive electromagnetic induction

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[3] The author will be satisfied in sending it by email (glauco.pantoja@ufopa.edu.br). It is written in Brazilian Portuguese.

[4] An example of traditional Physics textbook is the one due to Nussenzveig (2006).

solely as induction of curly electric fields from dynamic magnetic fields (Faraday's law). Otherwise, here we also call attention to the reverse process, the production of rotational magnetic fields from dynamic electric fields (Ampère-Maxwell's law).

The PMTU started with the emergency of prior knowledge (step 2), which searched for diagnostic evaluation, respecting the principle stating the most important isolated variable influencing meaningful learning is prior knowledge. Thus, students had the possibility of turning explicit their prior knowledge or use them in action, in diagnostic evaluation. In the end of the unit was also applied a summative evaluation to investigate possible meaningful learning products built individually during the PMTU (step 7). Formative evaluation was developed along the whole didactic unit. From the answers to diagnostic, formative, and summative evaluations, from the modes of conceptualization evidenced in priorly developed PMTU in magnetostatics (PANTOJA; MOREIRA, 2019) and electrostatics (PANTOJA; MOREIRA, 2020), and from conceptions already registered in literature, it was possible to make certain inferences about modes of conceptualization initially used by students to address the concept of electromagnetic induction in this first evaluation.

In **class 1**, **RP** included an activity with four questions, being one of multiple choice (Ampère-Maxwell's law), whose answer had to be justified, and the other three, written questions (Faraday law's). In the **AE**, it was developed a historical-philosophical approach to the content, highlighting a general view of the construction of the concept of electromagnetic induction. To motivate progressive differentiation in classes 2, 3, 4 and 5, questions like "how do electric motors work?" and "how does occur the production and propagation of radio waves?" were posed to them.

In **class 2**, **AE** included the presentation of Faraday's law in differential (curl) and integral (circulation), while **RP** consisted in three problem-situations, with one involving description of electromagnetic interactions, one on symbolic representation of electromagnetic fields and the other one requiring calculation of electromagnetic fields. In **class 3**, **AE** aimed at progressively differentiating between movement emf and emf induced by electric fields in integral Faraday's law, besides the mechanisms of emf production. At the same time, in **RP** there was proposition of two situations about symbolic representation of electric fields and one about description of electromagnetic interaction.

In **class 4**, one more progressive differentiation was carried in **AN**, by means of using integral Faraday's law to approach motors and generators, while in **RP** were posed three conceptual questions to describe electromagnetic interactions using the concept of **induced** electric current. In sequence, in **class 5**, **AE** searched for description of the historical construction of the concept of displacement current, in which it was possible to approach the phenomena of magnetic field induction from time variable electric fields (Ampère-Maxwell's law), and **RP** focused three questions about displacement current, with two of them addressed to symbolic representation of the electromagnetic field and the other one to calculate it.

In **class 6**, **AE** presented the Ampère-Maxwell's law compared to the Faraday's law, which indicates integrative reconciliation when we showed differences and similarities between the two laws, and **RP** brought two questions involving calculation of electromagnetic fields. In **class 7**, **AE** discussed electromagnetic interaction and the sources of electromagnetic field in a deeper way then sixth class, by means of the contrast between static and dynamic fields, and there was also an analysis of the energy balance equation for the electromagnetic field in the two posed situations, which made possible to discuss explicitly the structuring role of Ampère-Maxwell's and Faraday's laws in electromagnetic interaction. There was no **RP** in this class.

In **class 8**, **AE** aimed at deriving the electromagnetic wave equation and the speed of light from the Maxwell's equations, which turned possible to implement a progressive differentiation (introduction of the concept of electromagnetic wave) in a perspective of integrative reconciliation (association with the rest of the content and with structuring equations of electromagnetism), however, there was not **RP**. **Class 9** did not have **AE** and in **RP** there was an individual summative evaluation with five questions, with two of them addressed to Ampère-Maxwell's law and three about Faraday-Lenz's law, and a production of one conceptual map.

We, then, discuss the aspects related to the methods of data analysis (step 8), which search for evaluate the PMTU. Moreover, we describe the context in which research was applied, the instruments for data collection, and the methodological processes of interpretation of these date during the conduction of this investigation.

III.2 Research methodology

This study was applied in two different contexts, both in Brazilian universities, one of them located at the South region of Brazil (**study I**) and the other in the North region (**study II**). The group of the study I was constituted by 17 students enrolled in Physics courses (bachelor and teacher formation), aging between 19 and 29 years old, studying for the first time a discipline of general physics III. The group of the study II was composed of 12 students with same average age of the first group and had been studied similar curricular components to the former group but were enrolled in a course of Physics Engineering. Nine tenths of the last class failed the general physics I course and stated that felt frustrated with the difficulties the institution was facing to carry on the course, what could count as a unmotivating factor for these students since the beginning of the course.

When data were collected, the University in which study I was conducted did not require the submission of research programs (doctorate, in this case) to an ethics commission in humans, because this kind of judgement was made in the context of specific post-graduation commissions related to the course. In study II, the University in which we conducted the study did not have an ethics commission for research with humans, then, for guaranteeing the application of ethical procedures associated to the research, we invited the participants to integrate the investigation as subjects, we explained the research would not guarantee them benefits or damage, we pointed that the participation in the study was voluntary and that they could abandon the investigation whenever they wanted, without suffering any kind of retaliation, and moreover we indicated that there was not any kind of financial compromise among any parts. Finally, we gave them a free and informed consent term, containing this description, for them to assigning in case of concordance. No student declined participation in the investigation throughout the whole process.

Students of both groups had already study contents from disciplines of general Physics I (mechanics) and general Physics II (waves, fluids, oscillations, thermodynamics). Then, we judged possible from the premise that students had relatively consolidated ideas about force and about energy, in addition of relevant knowledge on the concept of gravitational field. Regarding this, a PMTU was built to introduce the concepts of vector field and scalar field, as a preparation for the PMTU that would follow it, namely, the ones of electric field (PANTOJA; MOREIRA, 2020) and magnetic field (PANTOJA; MOREIRA, 2019). We aimed at adjusting the PMTU for each step whenever possible, because the context had clear differences. The PMTU described here are framed at the end of general Physics III course.

The main tools for data collection involved students' conceptual maps, answers to pencil and paper problems produced in the RP and field notes written by the researcher who acted as professor. From this, we compared empirical evidence (textual registers analysis) with possible responses produced by a mode of conceptualization. To carry out the textual register analysis, we conducted the principles of content analysis (BARDIN, 2008). In this work, we focus our analysis to students' answers to pencil and paper problems. The conceptual maps and field notes produced were used as triangulation tools for solving doubts or falsifying hypothesis that could appear during the process of analysis.

Content analysis proposes facilitation in interpretation and diminished uncertainties in inference making in the act of reading textual registers. It had heuristic function in the process of systematization of information collection of messages and it is configurated as an alternative to "intuitive" reading. It can be comprehended as a set of analytical techniques aiming at producing inferences about conditions of production or reception of messages and takes as base indicators obtained by means of objective and systematic procedures of description of the content of the messages (BARDIN, 2008).

A cycle of content analysis has three stages: **pre-analysis**, **exploration of material** and **production of inferences**. Pre-analysis includes *floating* reading of the material; *corpus* selection, respecting the rules of homogeneity (materials are similar), exhaustivity (all tasks were analyzed), pertinence (all tasks are equally relevant) and representativity (the material is representative of the whole situation); construction of hypotheses; production of indexes and indicators; preparation of material for its future exploration. Exploration of material includes application of decisions made in the prior stage, besides implementation of codification operations, decomposition, and enumeration. Inference taking includes interpretation and treatment of results, which implies a more elaborated and synthetic reading than its initial counterpart. In sequence, we describe how each of these steps was implemented.

In **pre-analysis**, for satisfying homogeneity rule, we adopted as *corpus* of the investigation five out of the seven tasks containing pencil and paper problems (there was not RP in classes 7 and 8), once two of them were oriented to situations of calculation of electromagnetic field (PANTOJA; MOREIRA, 2021), something out of the scope of this work. Rules of pertinence, exhaustivity and representativity were satisfied, because all the activities related to the object of study were considered, namely, conceptualization in situations of symbolic representation of the electric field and description of electromagnetic interactions. In floating reading, we searched for indexes and indicators related to explicit (in writing) and implicit (those filling gaps in writing) operational invariants recurrent in students' tasks. We conducted this stage in a way that could relate what students wrote or left implicit with data collected from literature of research in Physics teaching on conceptions of the concept of electromagnetic field. From this, it was created a set of possible theorems-in-action, representing the answers of the students, for conducing exploration of material.

The **exploration of material** intended to organize theorems-in-action evidenced by students in possible modes of conceptualization. In sequence, we searched for corroboration giving support to the use of these modes. Initially, we identified the relation between proposed questions and answers presented by the students, which permitted to attach the goals of situations to objectives elaborated by them. From this, we marked the operational invariants associated to these objectives in students' answers. Then, we organized the excerpts turned explicit by the students to reconstitute possible inferences developed by them. Thus, the relations between explicit propositions were analyzed to infer implicit theorems-in-action and concepts-in-action that could give sense to the relations, but that for some reason are not mentioned. Finally, we built hypothetical reasoning diagrams containing rules of action of the type IF... THEN organizing implicit and explicit operational invariants compatible with the concatenation of ideas presented in presented answers. In the whole process of analysis phrases were used as unit of analysis and the enunciates were taken as unit of context (proposed situation and students answers).

The **production of inferences** is characterized by interpretation of evolution of application, due to students, of conceptualization modes throughout the PMTU to situations, which generated a synthetic meaning presented in the section of results. This was useful for understanding which changes or stabilizations occurred in students' conceptualizations.

Our approach to methodology followed identical formats to the ones employed by Pantoja and Moreira (2020, 2019) for analyzing higher education students' conceptualizations on the concepts of electric field and magnetic field. In sequence, we present the results accordingly to the steps of content analysis.

IV. Results

We present the results of pre-analysis, exploration of material and production of inferences developed in content analysis. In the first one, we bring a list of theorems-in-action (in form of propositions) and concepts-in-action (in form of categories) possibly used by students; in the second one, the organization of the operational invariants in modes of conceptualization, and in the third one, we discuss how the conceptualization modes were applied in situations proposed during the PMTU to analyze how conceptualization was developed in the unit. Between the second and the third stages, we introduced an example of how analysis was carried out. To make notation uniform, we use uppercase letters for situations and lowercase letters for modes of conceptualization and operational invariants (for both cases we use Greek and Latin alphabets).

IV.1 Pre-analysis

To systematize investigation, it was necessary to describe how students mobilize their operational invariants in situation. Once there are different types of situations in electromagnetic theory, it would be necessary to classify them. Based on work aiming to build a conceptual field relative to the concept of electromagnetic field (PANTOJA, 2021), we bring two classes of situations making sense of the concept of electromagnetic field, these are the one of description of electromagnetic interactions (Σ) and symbolic representation of electromagnetic field (Θ). Each of these is structured in different forms and call for mobilization of operational invariants of distinct dimensions.

The class of *description of electromagnetic interactions* (Σ) requires establishment of reference to situations in which there are interactions between electrically charged objects in arbitrary motion (systems of charges and currents) and electric or magnetic fields (or both). It is always necessary to describe how these objects interact mediated by electric of magnetic fields, even if the source is omitted. We present an example of a situation of this kind:

Suppose you have an uniform magnetic field (with respect to position), $\vec{B}$, *limited to a cylindrical volume of radius R. This magnetic field decreases in intensity at a constant rate along time. What is the instantaneous acceleration of a negative charge of value* q *in regions* $r < R$ *and* $r > R$*?*

To solve this problem and others like this one, it is necessary to identify interacting objects (1). One must understand the interaction between objects as electromagnetic in its nature, that is, it is necessary to consider both electromagnetic fields and forces (2). It is necessary to apply laws to describe momentum and/or energy changes carried out in interaction (3). Last, it is indispensable to describe the dynamics of the electric charge carrier (4).

In the presented example, the students need to: identify the source of magnetic field and the negative charge as interacting objects (1); make considerations about the magnetic field and determine the existence of an induced electric field (2); apply Lorentz's force to determine the net force over the negative charge carrier (3); describe the dynamics of the test charge by means of operational invariants due to classical mechanics' conceptual field (4). We divided the four thought operations in dimensions to better analyze the use of operational invariants by the students. This division is presented in table 2.

Obviously, there are other possibilities. It is possible, for example, to "withdraw" the magnetic field and introduce a time-variable electric field in its place, moreover it is possible to take out a rest charge from the situation and to introduce a point electric charge with constant velocity (or even variable velocity), to present open problems or tasks without exact solution, et cetera. What is necessary to be in the situation is a source of electric field or variable magnetic field and an object possessing electric charge.

For the class of situations Σ, we have the following operational invariants: a_1) the *sources* of *electromagnetic field* are *moving electric charge carriers*[5] and *time varying electric* or *magnetic fields*; b_1) *electromagnetic fields* are *fluids;* b_2) *electromagnetic fields* are *real* and *material physical quantities*; b_3) *electromagnetic fields* are *mathematical tools;* b_4) *electromagnetic fields* are *real* and *immaterial*; c_1) *electromagnetic force* is exerted *instantaneously at distance;* c_2) *electromagnetic force* is *transmitted contiguously* along *space*; c_3) *electromagnetic fields* are *support* for *energy transfer;* c_4) *electromagnetic fields* represent *force* by unit *electric charge* on an *electric charge carrier*; c_5) *electromagnetic fields* transfer *momentum and energy* by exerting force and doing work; d_1) *electromagnetic force* is equal to the *electromagnetic field*; d_2) *electromagnetic force is different* from *electromagnetic field*; e_1) *electromagnetic energy* is located in *moving electric charge carriers*; e_2) *electromagnetic energy* is in *moving electric charge carriers* is exchanged by *electromagnetic fields*; e_3) *kinetic energy* due to *moving electric charge carriers* is transformed in *interaction energy* of the *electromagnetic field*; f_1) *induced electromotive force* is produced by *magnetic force* when there is a *moving conductor* in a region of *stationary magnetic field*; f_2) induced electromotive induced is established by an *electric force* exerted by an *induced electric field* in a region where there is a *time variable magnetic field*; f_3) *induced electromotive force* is produced by a *time variation of magnetic flux* when there is a *moving conductor* in a region where there is a *stationary magnetic field*; g_1) *electric fields* do *work* on *moving electric charge carriers*; g_2) *magnetic fields* do not do *work* on *moving electric charge carriers*; h_1) *energy exchanges* in *electromagnetic interaction* are *instantaneous*; h_2) *energy exchanges* in *electromagnetic interaction* are *non-instantaneous*; i_1) *electromagnetic forces* satisfy the *superposition principle*; i_2) *electromagnetic fields* collide with one another; i_3) *electromagnetic fields* satisfy the *superposition principle*.

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[5] We mean is ordered movement, that is, electric currents.

The class of *symbolic representation of the electromagnetic field* (Θ) requires the subjects to establish abstract relations among the field and other concepts relevant to electromagnetic induction phenomena. Symbolic representations do not hold structural relation with the object they aim to represent (MARKMAN, 1999).

Table 2 – Description of dimensions for analysis of operational invariants addressed to the class of situations Σ.

D	Description	Op. Invariants
1	A – Identification of electromagnetic field sources	a_1
2	B – Description of the ontological nature of the electromagnetic field	b_1, b_2, b_3, b_4
	C – Identification of the role of electromagnetic field in electromagnetic interaction	c_1, c_2, c_3, c_4, c_5
	D – Relation between electromagnetic field and electromagnetic force	d_1, d_2
3	E – Localization of electromagnetic energy	e_1, e_2, e_3
	F – Identification of the physical nature of the electromagnetic field	f_1, f_2, f_3
	G – Description of energy exchanges	g_1, g_2
	H – Consideration of interaction velocity infiniteness	h_1, h_2
	I – Application of the superposition principle	i_1, i_2, i_3
4	Elements coming from the classic mechanics conceptual field	

For the *symbolic representation of the electromagnetic* field, students must refer to the (electric or magnetic) field sources, in first place. Then, they must establish reference to points in space. Thus, it is necessary to reference electric and magnetic field vectors (intensity and direction). We present a situation below:

Is it possible to present an example of situation in what it is possible to state, safely, that there is an induced electromagnetic field, due to a time variation of magnetic field?

Solving problems of this kind requires identification of sources of electromagnetic field (1). Solution requires enumeration and characterization of points in space using natural language and theorems-in-action grounded in algebra or vector analysis (2), and mapping meaning of Maxwell’s equation into the problem (3). Electromagnetic fields are described by vector fields, then it is also important to associate vectors to points in space (4) it is fundamental. Last, the establishment of symbolic representation (5) must be employed.

The proposed example involves a non-structured problem, and, because of this, it does not lead to a unique solution. It is necessary to identify a magnetic field source, which can be a source of slowly varying electric current, like a solenoid, for example (1). We must think in points in space to build an Amperian curve (2) and we need to map the meaning of Faraday’s law in this situation (3). This meaning is the one stating the time varying magnetic field is source of induced electric field, but not just it, it is necessary to reach Ampère-

Maxwell's law meaning indicating that electric current produces magnetic field. One needs to think about induced electric field vectors to relate them to the circulation of electric field and to the time variation of magnetic flux in integral Faraday's law or associate them to the partial derivate (with respect to time) to the curl of the electric field in the differential version of this law (4). Last, it is essential to conclude that this arrangement is sufficient to produce the phenomena and describe it (5). Like we did to the class of situations Σ, we divided the though operations in dimensions to make analysis easier. Results are presented in table 3.

For the class of situations Θ, we have the following operational invariants: k_1) *electric flux* is equal to the *electric field*; k_2) *magnetic flux* is irrelevant, because it is *null;* k_3) *electric fields* must have an *irrotational component;* k_4) *magnetic fields* are *solenoidal;* k_5) *electric fields* are *monopolar* when created by *electric charge;* k_6) *magnetic field*s are never *monopolar;* l_1) *magnetic circulation* is equal to the *magnetic field;* l_2) *electric circulation* is equal to the *electric field;* l_3) *electric fields* must be *solenoidal;* l_4) *magnetic fields* are *solenoidal;* l_5) *electric field*s are created by *time varying magnetic fields;* l_6) *magnetic fields* are created by *electric currents;* l_7) *magnetic fields* are created by *time varying electric fields;* m_1) *flux* is equal to the *normal component* of the *field;* m_2) *electric flux* indicates the *electric field* has *radial component* when there are *electric charge carriers;* m_3) *magnetic flux* indicates that the *magnetic field* never has *radial components*; m_4) *electric flux* indicates the presence or absence of *electric charges* producing an *electric field* in a *region of space;* m_5) *magnetic flux* indicates the inexistence of *magnetic charges* producing *magnetic fields* in *space;* n_1) *circulation* is equal to the *tangential component* of the *field;* n_2*) electric circulation* indicates the *induced electric fields* have *circular field lines*; n_3) *magnetic circulation* indicates *induced magnetic fields* have circular *field lines*; n_4) *electric circulation* indicates *time varying magnetic fields* produce *induced electric field;* n_5) *magnetic circulation* indicates that *electric current* and *time varying electric fields* produce *induced magnetic field*.

Table 3 – Description of dimensions for analysis of operational invariants addressed to class of situations Θ.

D	Description	Op. invariants
1	A – Identification of sources of electromagnetic field	a_1
2	Elements from conceptual field of analysis (mathematics)	-
3	K – Physical Interpretation of the electric flux and magnetic flux	k_1, k_2, k_3, k_4, k_5
	L – Physical Interpretation of electric circulation and magnetic circulation	$l_1, l_2, l_3, l_4, l_5, l_6, l_7$
4	M – Relation between flux and electromagnetic field	m_1, m_2, m_3, m_4, m_5
	N – Relation between circulation and electromagnetic field	n_1, n_2, n_3, n_4, n_5
5	Elements from conceptual field of analysis (mathematics)	-

We follow to the second stage of analysis, in which we describe the modes of conceptualization possibly used by the students in situations. In sequence, we present an example of how operational invariants and modes of conceptualization where schematized. It is important to highlight that the two lists of operational invariants contain both meanings coherent with scientific conceptual reference structure and incoherent with it (alternative conceptions). Throughout description of results, we discuss which ones are scientific and the ones that are alternative.

IV.2 Exploration of material

We aimed to organize operational invariants in modes of conceptualization to better understand learning processes carried out by students. As highlighted in the section of theoretical, these modes are descriptive structures for opportunism and contingency of conceptualization and are useful for describing this process in novel situations. They are forms of organization of action and involve reference to these ones by means of concepts. We produced them from students' answers and from findings on students' representations obtained from literature.

We formulated six modes of conceptualization addressed to situations of description of electromagnetic interactions (Σ): instant action-at-distance (σ_1); fluid electromagnetic field (σ_2); electromagnetic field as support (σ_3); instrumentalist electromagnetic field (σ_4); microscopic scientific electromagnetic field (σ_5); e macroscopic scientific electromagnetic field (σ_6). The operational invariants associated to these modes are expressed in table 4.

Table 4 – Modes of conceptualization associated to description of electromagnetic interactions.

	A	B	C	D	E	F	G	H	I
σ_1	a_1		c_1	d_1	e_1			h_1	i_1
σ_2	a_1	b_1	c_2	d_2	e_2			h_2	i_1, i_2
σ_3	a_1	b_2	c_3	d_2	e_2		g_1, g_2	h_2	i_1, i_3
σ_4	a_1	b_3	c_4	d_2	e_2	f_1, f_2	g_1, g_2	h_2	i_1, i_3
σ_5	a_1	b_4	c_5	d_2	e_3	f_1, f_2	g_1, g_2	h_2	i_1, i_3
σ_6	a_1	b_4	c_5	d_2	e_3	f_3		h_2	i_1, i_3

It was supposed the level of conceptualization of structures σ progressively approaches scientific knowledge accepted for the theory of electromagnetic field. Mode σ_1 admits in electromagnetic interaction occurring instantly at distance, that is, an electric charge carrier or electric current conductor exerts force directly on the other, without influence of an electromagnetic field. Mode σ_2 introduces the concept of field, however in a substantialist ontological and epistemological perspective (BACHELARD, 1996), in other words, the electromagnetic field is essentially a fluid. Mode σ_3 differs from σ_2 in the conception of materiality of the field, because the first one approaches of the cartesian ether (WHITAKKER, 1910), a material medium which supports propagation of force. Mode σ_4 adopts an instrumentalist perspective (BUNGE, 2010) for the electromagnetic field once the former considers the latter just a mathematical tool for calculation of electromagnetic forces. Modes σ_5 and σ_6 are closer to scientific thought because they consider the electromagnetic field a real and immaterial physical dynamic variable (which transports energy and *momentum*). Both are different in approach: while σ_5 includes microscopic explanations (using fields and forces), σ_6 entails macroscopic explanations (using flux and emf)

There are, also, five classes related to situations of *symbolic representation of the electromagnetic field* (Θ), which are: tautologic electromagnetic field (θ_1); electromagnetic field equal to field equation (θ_2); pictorial electromagnetic field (θ_3); relational electromagnetic field (θ_4); integrated pictorial-relational electromagnetic field (θ_5). The operational invariants associated to these modes are expressed in table 5.

Table 5 – Modes of conceptualization oriented to symbolic representation of the electromagnetic field.

	A	K	L	M	N
θ_1	a_1	-	-	-	-
θ_2	a_1	k_1, k_2	l_1, l_2	m_1	n_1
θ_3	a_1	k_3, k_4	l_3, l_4	m_2, m_3	n_2, n_3
θ_4	a_1	k_5, k_6	l_5, l_6, l_7	m_4, m_5	n_4, n_5
θ_5	a_1	k_3, k_4, k_5, k_6	l_3, l_4, l_5, l_6, l_7	m_2, m_3, m_4, m_5	n_2, n_3, n_4, n_5

It is supposed a growing level of conceptualization in the passage from θ_1 to θ_5. Students who master θ_5 know, basically, how to work with integral formulation, from both symbolic (algebraic and analytical) and pictorial (geometrical) point of view. Mode θ_1 represents the concept of electromagnetic field solely related to electric charges, without reference to field equations. Mode θ_2 establishes reference to field equations by confusing them with the electromagnetic field itself (ARAUJO; VEIT; MOREIRA, 2007; GUISASOLA et al., 2008). Mode θ_3 symbolizes electromagnetic fields in a pictorial perspective, that is, reference emphasizes orientation of electric and magnetic fields. Compared to θ_3, mode θ_4 indicates the relations between the electromagnetic field and its sources (NOUSIANEN; KOPONEN, 2017), what implies relational characteristic. Mode θ_5 includes elements from θ_3 and θ_4, being the most complete mode of conceptualization. We understand that both θ_3 and θ_4 are good didactic objectives for a course on general Physics III, then, we consider both equivalent. However, the excellent point would be θ_5.

IV.3 An example

We show one example of how the analysis was developed for each answer of each task. Situation 1 of task 4, which refers to energy transformation, was solved by student F of study I. The statement of the task requires the *description of a way of transforming chemical energy of fat in electric energy*[6]. This situation can involve a description of electromagnetic interaction because it requires at least two elements interacting electromagnetically: a source of electromagnetic field and a conductor object. Then, the situation is from class Σ.

There is no closed solution for this problem because it is ill-structured. There are two forms of involving the concept of electromagnetic induction for approaching the situation and both involve the concept of generator. The first one considers generator's rotor as a source of magnetic field and the stator as an electric current conductor coil, while the other one involves the reverse process, that is, a mobile conductor and a fixed source of magnetic field. The resolution will be different for both cases if approach is microscopic, but it is basically the same in the macroscopic case.

When we choose microscopic approach, we must be conscient that in the first case the magnetic field varies in time and induces an electric field and in the second there are not induced electric fields, because there are not variable magnetic fields in time. With these two parameters, the movement of the coil is solidary to the one of the electric charge carriers and these have their direction modified by the magnetic force, which implies in a Hall emf and, consequently, there is an equivalent electric field which helps to solve the problem (NUSSENZVEIG, 2006). In macroscopic approach, both situations can be summarized to the analysis time variation of magnetic flux. Student F chooses the last option, as can be seen in his answer:

> *Using a **coil**, a bicycle, and a **magnetic field**. To make the **wheel** of the bicycle spin, we apply a **torque**. By transmitting this **torque** to the **coil**, we make it **spin**. Then, it will occur a **variation of the magnetic flux** and, consequently, an **induced emf** (and an **induced electric current**). That is, besides burning **calories**, we would be generating **energy**.*

The following explicit concepts-in-action are identified in boldface in his answer: coil; magnetic field; wheel; torque; spin; variation of magnetic flux; induced emf; induced electric current; calories; energy. The explicit theorem-in-action *induced electromotive force* is produced by a *time variation of magnetic flux* when there is a *moving conductor* in a region where there is a *stationary magnetic field* (f_3) is clear in his answer. The concept of conductor is an implicit concept-in-action. Analysis of predicative knowledge is restricted to study only explicit features posed by students in textual forms, but it is possible to go further and consider some conscient or inconscient elements implicit in students' actions when they

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[6] One must understand the idea was used in a wide sense, once it is not necessarily fat the only source of chemical energy during cycling in strict sense, muscles can, and generally use, energy from carbohydrate due to food or glycogen.

solved this problem. This information can be inferred from gaps left in their explicit knowledge which can be understood as implicit operational invariants.

From this, which implicit operational invariants could be inferred in this analysis? Student F states explicitly in question 2 of the same task that "electric current produces magnetic field" and this was used as secondary information to deduce the operational invariant the *sources* of *electromagnetic field* are *moving electric charge carriers*[7] and *time varying electric* or *magnetic fields* (a_1). This operational invariant appears implicit in task 1 because the student relates an emf with an induced electric current when he says that *"...consequently, an induced emf (and an induced electric current)",* which leads us to conclude that insofar the coil is an electric current conductor it produces electromagnetic field.

The operational invariant *electromagnetic fields transfer momentum and energy by exerting force and doing work* (c_5) practically summarizes the conclusion obtained by the students' reasoning. Let us show:

> *[To convert chemical energy of fat in electric energy] ... we apply a torque [in the bycicle's wheel]. When we transmit this torque to the coil, we make it spin. By this [spinning movement] there will be a [magnetic field] flux variation [in time] and, consequently, an induced emf... That is, besides burning calories, we would be generating energy.* (Our notes)

It is implicit in students' conceptualization that chemical energy is first transformed in kinetic energy (implicit concept-in-action) and that this is transmitted to the coil. The movement of the coil in the region of magnetic field provokes a time variation of the magnetic flux, which implies that the magnetic field is mediating the interaction. This produces an in induced emf (and electric current) that is related to electric energy. The student concludes saying that in the middle of the process we lost energy of one type (chemical) and gain another form of energy (electric), that kinetic energy is in the coil (electric charge carrier) and that it is transformed by means of an electromagnetic field. By the other hand, he does not come in detail on how the relation between electromagnetic force and work done in the task is established (what would be done if he used the mode σ_5).

We search in task 2 of the same task which possible operational invariant the student F could use for dimension D. The theorem-in-action *electromagnetic force is different* from *electromagnetic field* (d_2) can be inferred because the student states that "… the electric current will generate a magnetic field which, affecting the magnet, will generate a torque". We therefore understand the student differentiates the two concepts once the magnetic field can apply a torque, a quantity derived from force.

We deduce the operational invariant *kinetic energy* due to *moving electric charge carriers* is transformed in *interaction energy* of the *electromagnetic field* (e_3) from the same

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[7] We mean is ordered movement, that is, electric currents.

reasoning we inferred c_5. Here, the emphasis is put in locating energy and there it is on the role of the magnetic field in interaction. Both dimensions are relevant because not necessarily the student supposes the field mediation processes of energy exchange implies guaranteeing him to admit the energy is in the field. In this case, it becomes clearer, but there are others in which is difficult to deduce.

The last operational invariant about which we discuss is the one of dimension H. This theorem-in-action seems to be implicit in students' answer once he develops a causal line of explication for energy exchange. The spin movement produces a change of flux, this induces an emf, and the latter generates electric current, then, we think it is probable that he conceived the process in a dynamic way, and he tries to turn it explicit. We present, in sequence, the "linearization" of the implicit and explicit thought operations produced by F in our analysis which entail action rules of the typo IF… then:

> ***IF*** *we need to generate electric energy from chemical energy,* ***THEN*** *we need to transform one into the other.* ***IF*** *we will consume chemical energy from fat,* ***THEN*** *we will transform it in rotational kinetic energy by means of a torque.* ***IF*** *a coil has rotational kinetic energy* ***& IF*** *this coil is inside a region of magnetic field,* ***THEN*** *it will occur time variation of magnetic flux through the coil.* ***IF*** *there is time variation of magnetic flux,* ***THEN*** *an emf is produced in the coil.* ***IF*** *emf is produced by energy,* ***THEN*** *it must be like it.* ***IF*** *there is induced emf in the coil* ***& IF*** *the coil is an electric current conductor,* ***THEN*** *there will be production of electric current in the coil.* ***IF*** *there is electric current in the coil,* ***THEN*** *energy was transferred to it.* ***IF*** *this process is mediated by a magnetic field,* ***THEN*** *it must transport energy.*

Then, we concluded the theorems-in-action a_1, c_5, d_2, e_3, h_2 are implicit and the f_3 is explicit in students' F conceptualization. From the table 4, we infer the mode of conceptualization σ_6 was used by the student in this situation. It does not seem to be direct (explicit) or indirect (implicit) reference established by student F with respect to the ontological feature he attributes to the magnetic field, therefore we preferred not to go further in analysis for dimension B so that we did not need to run the risk of forcing an interpretation, once in principle we admitted that there is no empirical basis to interpret the occurrence of any operational invariant, be it implicit or explicit, of this dimension in student's answer. The superposition principle did not apply to this case, then, this was another dimension we did not deduce anything for simply not being necessary any kind of reference to it.

IV.4 Inference making

In this work, we analyze evidence students' use of conceptualization modes and take them as indicators of meaningful learning once they involve non-random and non-literal acquisition of new knowledge. Not always meaningful learning products and processes include scientifically coherent knowledge. Alternative concepts are examples of that because they are derived from meaningful learning, despite being scientifically incorrect. In this section, we discuss the diverse modes presented by students in situation and sustain that during PMTU they progressively approach from scientifically accepted knowledge. We chose to present the results of each class so we can compare the groups of both studies.

Table 6 – Modes of conceptualization due to students of study I.

T	A_D			2			3	4			5			A_S		
S_P	1	2	3	1	2	3	1	1	2	3	1	2	3	1	2	3
A	-	-	-	θ_4	σ_5	-	MAIORIA FALTOU	σ_6	σ_6	σ_6	-	θ_4	θ_4	θ_4	σ_4	-
B	σ_5	σ_5	-	θ_4	σ_5	θ_4			σ_6	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
C	σ_5	σ_5	θ_4	θ_4	σ_5	θ_4		σ_5	σ_6	σ_6	-	-	-	θ_4	σ_6	σ_6
D	σ_6	σ_6	-	θ_4	σ_5	θ_4		σ_6	σ_6	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
E	-	σ_3	-	θ_4	-	θ_4		σ_1	σ_1	-	-	-	-	-	σ_4	σ_5
F	-	-	-	-	-	-		σ_6	σ_5	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
G	-	-	-	θ_4	σ_5	θ_4		σ_5	σ_6	σ_6	θ_1	-	θ_4	θ_4	σ_6	σ_6
H	σ_5	σ_5	-	θ_4	σ_5	θ_4		σ_6	σ_1	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
I	σ_5	σ_5	-	θ_4	σ_5	θ_4		σ_6	σ_1	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
J	-	σ_2	-	θ_4	σ_5	θ_4		σ_6	σ_6	σ_6	-	θ_4	θ_4	θ_4	σ_5	σ_4
K	σ_6	σ_5	-	θ_4	σ_5	θ_4		-	-	-	θ_4	-	-	θ_4	σ_4	-
L	-	σ_3	-	θ_4	σ_5	θ_4		σ_5	σ_6	σ_6	-	-	θ_4	θ_4	σ_4	σ_6
M	-	-	-	θ_4	σ_5	θ_4		σ_6	σ_1	σ_6	-	θ_4	θ_4	θ_4	σ_2	σ_6
N	-	-	θ_2	θ_4	σ_5	θ_4		σ_6	σ_1	σ_6	-	θ_4	θ_4	θ_4	σ_6	σ_6
O	σ_6	σ_4	θ_4	θ_4	σ_5	θ_4		σ_6	σ_6	σ_6	-	θ_4	θ_4	θ_4	σ_5	σ_4
P	σ_6	σ_5	θ_4	θ_4	σ_5	θ_4		σ_5	σ_5	σ_5	-	θ_4	θ_4	θ_4	-	σ_6
Q	σ_3	σ_3	-	θ_4	σ_5	θ_4		σ_6	σ_6	σ_6	-	θ_4	θ_4	θ_4	σ_4	σ_4

The tables 6 and 7 expose a background of the modes of conceptualization evidenced throughout PMTU on electromagnetic induction in studies I and II, respectively. In these tables, we use the letter T to indicate tasks proposed in PMTU and the symbol S_P for denoting the problem situation being analyzed. Each student is codified by a Latin letter in the second row and each cell contains the mode of conceptualization used in each task. We represent the cognitive biases of functional reduction, functional fixation, and conceptual error by the colors green, blue and red respectively. We use blank spaces for identifying a question which was not answered. The two PMTU had the same teacher.

Table 7 – Modes of conceptualization due to students from study II.

T	A_D			2			3	4			5			A_S		
S_P	1	2	3	1	2	3	1	1	2	3	1	2	3	1	2	3
A	σ_5	σ_5	-	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_5	θ_4	σ_6	σ_6
B	-	-	-	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_5	θ_4	σ_4	σ_6
C	σ_4	σ_4	θ_4	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_5	θ_4	σ_5	σ_5
D	σ_3	σ_3	θ_4	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_5	-	-	-
E	σ_3	σ_4		θ_4	σ_5	θ_4	σ_4	-	-	-	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
F	σ_3	σ_3	θ_4	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_4	θ_1	σ_6	σ_6
G	σ_3	σ_3	-	θ_4	σ_5	θ_4	σ_4	-	-	-	θ_4	θ_4	θ_4	θ_4	-	-
H	σ_5	σ_5	θ_4	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6
I	σ_5	σ_5	-	θ_4	σ_5	θ_4	σ_4	-	-	-	θ_4	θ_4	θ_4	θ_3	σ_6	σ_6
J	σ_5	σ_5	-	θ_4	σ_5	θ_4	σ_6	σ_6	σ_6	σ_6	θ_4	θ_4	θ_4	θ_1	σ_6	σ_6
K	-	-	-	θ_4	σ_5	θ_4	σ_4	-	-	-	θ_4	θ_4	θ_4	θ_4	σ_6	σ_6

In the specific case of cognitive biases, functional reduction can be understood as the act of reducing a multicausal phenomenon to a sole cause (VIENNOT; RAINSON, 1999) and functional fixation as the incapacity of using a real or thought object in a distinct function from that the subject considers to be its' "natural function" (EYSENCK; KEANE, 1994). Conceptual error is defined as the use of certain scientifically accepted mode of conceptualization with some incorrect detail.

In **diagnostic evaluation** on **study I**, respectively the only situation of type Θ, some students seem to not mention the electric field in space. Two students (C and O) seem to use a relational mode of conceptualization (θ_4). One student (N) evidences a cognitive bias of functional fixation which leads him to use a mode of conceptualization that confuses field and field equations (θ_2). The student P determines an electric field using Ampère's law for a context in which it does not apply, that is, the mode θ_4 is inadequate to master a situation involving the concept of static magnetic field, another case identified of functional fixation.

With respect to the situations of type Σ, four students evidenced modes of conceptualization of scientific-microscopic type (σ_5), three seem to present the scientific-macroscopic type (σ_6), with one of them (K) presenting functional reduction in the first task. By the other hand, in the second situation, student K seems to use the substantialist mode of conceptualization (σ_2) and three students (E, L, Q) apparently use the ones of support type (σ_3) because they represent the situation as transportation of electric current by the magnetic field (THONG; GUNSTONE, 2008). Four students (I, P, H e K) possibly use the functional reduction cognitive bias because they reduce the problem to the action of an electric field (I and P) or to the action of a magnetic field (H and K). Besides, it is evidenced juxtaposition of modes of conceptualization (KALKANIS; HADZIDAKI; STAVROU, 2003) based both in external prior knowledge (possibly obtained in High School) and cognitive structures built during prior PMTU on the concepts of electric field and magnetic field. Relatively to the situation of type Θ in **study II**, four students (C, D, F and H) seem to associate the existence of an electric field to a time variable magnetic field in a relational way (θ_4), something different from the first study. In general, these students apparently try to establish relations between what they learned and presented situations. This can be considered an indicative that meaningful learning processes occurred in prior PMTU.

Respectively to the description of electromagnetic interactions, there is some similarity with the first study: four students seem to present the model σ_3, which sees the field as support for energy transfer. Other situation like this is reference to static magnetic field correctly used in inadequate situations. It appears to be a situation of functional fixation for students A, H, I and J, but, at the same time it points to a possible pattern of meaningful learning in operational form of knowledge because they used learned theorems-in-action for solving new problems, even in distinct contexts. By the other hand, the student C shows signs of attributing merely mathematical importance to the concept of electromagnetic field (σ_4).

From analysis of diagnostic evaluation, we concluded the students have some relevant prior knowledge relative to experiences that occurred before the context of the course, but we can identify a considerable quantity of what is used with what they learned in prior PMTU, for solving new situations, without adapting these contents to the context of electrodynamics, which leads to conceptual error, of epistemological and ontological nature, despite it involves active use of previous knowledge. This evidences the necessity of highlighting differences between static and dynamic features of the electric and magnetic fields.

In **task 2** respective to **study I**, students presented relations coherent with the mode of conceptualization θ_4, which is more relational and abstract, in first and third questions (class Θ). The differences among answers were associated to the way it was organized the conduct expressed in ideas and its' explicitness level. In the second situation (class Σ), we found 13 students out of the 17 who showed signs of using the mode of conceptualization σ_5. By the other hand, five of them seem to present distinct procedural errors[8] (B, D, H, I, N), two showed signs of functional reduction of the electromagnetic force to the magnetic force between magnetic field and electric charge, including by inadequately computing it by means of the Ampère's (G and M). Two students do not answer to these questions.

Relatively to **study II**, two students evoke modes of conceptualization which are more abstract-relational in nature to discuss the electromagnetic field (θ_4) in situations 1 and 3. With respect to situation 2, the background is like the one of the first study, namely, seven students evidence the utilization of σ_5 and four use the same mode with a simple causality bias. This bias is expressed when they indicate the nullity of the electric force by the absence

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[8] Erros simples devido à fadiga, distração ou outro aspecto, que simplesmente são "automaticamente" sanados ao longo da produção textual. Por exemplo: esquecimento de um valor que volta na próxima linha da equação; colocação de um dois no lugar de um elevado ao quadrado que, no próximo passo volta a ser o que era antes do "erro". Não são erros conceituais.

of a magnetic field in the point in which is placed the proof electric charge, a case of functional reduction.

These data suggest that the PMTU may have given a greater emphasis to symbolization of electromagnetic field because students tend to establish stronger and meaningful relations with the sources (NOUSIANEN; KOPONEN, 2017) in situations like that. We highlight here the importance of emphasizing relations between electromagnetic field and its orientation in space in the same proportion as stress is put on the sources to provide students more conditions of building integrated modes of conceptualization (θ_5).

In **task 3** of **study I**, more than three fifths of the class was absent in the class. They pointed their absence was due to necessity of studying for a very hard exam on differential equations. We understand that difficulties can occur during the process. Because of this, we understood it was necessary to not consider this activity because answers due to students in the class would not represent the ones of the class.

In **study II**, four students seemed to use the instrumentalist mode of conceptualization (σ_4), which takes the field as mathematical tool. This mode describes well electromagnetic interaction but does not regard the epistemological and ontological details related to the concept of electromagnetic field. By the other hand, seven students evidenced modes using the electromagnetic field which focused on time variation of magnetic flux (σ_6). We faced it as a considerable gain in terms of learning if we consider their evolution throughout the PMTU.

In **task 4** of **study I**, the second problem situation led students to think about the counter-torque due to the magnetic field generated by induced current on a stator on the rotor containing an electromagnet, in the case of the electric generator. The concept of magnetic field is important in this situation because it is responsible for offering this counter-torque which opposes to the time variation of the flux through the rotor, something coherent with the principle of energy conservation. Five students describe the electromagnetic interaction for this system without using the concept of electromagnetic field (mode σ_1), which we understand as point of concern. By the other hand, the seven students that were in this class in **study II** showed signs of using the mode σ_6. In our perspective, this was positive.

Task 5 of **study I** was conducted before the class on displacement current. We decided to do this to analyze how students would transfer knowledge obtained about induction of electric fields from time variable magnetic fields to the reverse case (time variable electric fields generating induced magnetic fields). Most part of the students seemed to evidence transference for situations 2 and 3 (class Θ) once this content was directly related to other one already studied and learned. In the second problem situation, 12 out of 17 students presented answers compatible with the mode of conceptualization θ_4, while in the third one, there were 14 out of 17. By the other hand, in the first question, which asked for the meaning of the concept of displacement current, 10 students did not answer, six showed signs to use the mode θ_4 and one seemed to use θ_1. We attributed the low adhesion to potential confused caused by the term “displacement current” in this context.

The same task was proposed in **study II**. In the first and in the second problem situations the 11 students showed signs of using relational modes of conceptualization (θ_4). In the third question, seven students apparently used θ_4 (relational-abstract) and four, θ_5 (integrated). We evaluate this finding in a positive way because there is considerable difference with respect study I and there was solid stabilization of concepts until that moment. This data makes stronger the hypothesis stating the term “displacement current” can cause confusion in students minds for being unknown.

In summative evaluation on **study I**, 16 out of the 17 students indicate signs of using modes of conceptualization θ_4 for situations of class Θ, which evidences capacity of representing the electromagnetic field by establishing reference to relational and abstract features, for example, relations among field equations, electromagnetic field, and sources (NOUSIANEN; KOPONEN, 2017). For situations of class Σ, four students presented conceptualization varying from instrumentalist (σ_4) and realist (σ_5 e σ_6) modes of conceptualization. Three of them are more inclined to microscopic modes (σ_5), while one is oriented to macroscopic modes (σ_6). Nine students tend to stabilization in modes of conceptualization σ_6 and three in modes of σ_4 type. This points to the possibility of development of progressive mastering of the concept of electromagnetic field, something difficult to achieve (ZUZA; GARCÍA; GUISASOLA, 2012).

By the other hand, there are some signs of cognitive biases. One example is the conceptual confusion of associating an electric field to the time variation of magnetic flux when the magnetic field is stationary. For situations like that, there is an equivalent electric field attributed to the Hall effect occurring (NUSSENZVEIG, 2006). This one is responsible agent for the work done once the magnetic force cannot do it. Another case of cognitive bias is the one of simple causality which expresses itself when a student considers the area of a coil the only relevant fact for calculation of magnetic force. Naturally, these mistakes occur during learning processes and both evaluation of inadequateness and *feedback* are important as forms of learning. In our judgment, these students were apt to discuss these conceptual aspects by achieving establishment of reference to reality by means of concepts (VERGNAUD, 2009).

For situations of class Θ in **study II**, six students apparently used the relational mode (θ_4), two students approached the electromagnetic field through general aspects without understanding reference to field equations (θ_1), while one student presented a conceptual error which related time variation of flux to the production of electric field when what is in context is a magnetic stationary field. Faraday’s law relates time variation of magnetic field to the production of an electric field, what is included in integral formulation; however, when there are stationary magnetic fields, it is possible to occur flux variation by changes in the orientation of the circuit or of the area elapsed by its movement (ZUZA; GARCÍA;

GUISASOLA, 2012). An example is the case of generators with a source of stationary magnetic field, when moving coils are placed in regions of magnetic field produce movement emf. This is a reason why the mode σ_5 ends up being better than the σ_6 in situations like that because with the former there is not any induced electric field, and the explanation becomes more precise.

For situations of class Σ, eight students base themselves on the idea of time variation of flux and on the notion of induced electric current to make statements on the electromagnetic field (mode σ_6), while one uses microscopic modes to talk about emf and induced electric current in the two questions (mode σ_5). It is known that students usually have low familiarity with models of this nature and prefer explanations dealing with field flux variation instead of the ones evoking the concept of electromagnetic force (ZUZA et al., 2014). Based on presented data on students' modes of conceptualization, we consider positive the evolution experienced by students during summative evaluation and throughout PMTU.

V. Conclusions and discussion

In this work we aimed to answer the following research question: "how higher education students develop conceptualization processes in a Potentially Meaningful Teaching Unit to address the concept of electromagnetic induction?". To achieve this goal, we applied two similar PMTU for teaching the concept of electromagnetic induction in two classes of higher education in two different contexts. We used the Theory of Conceptual Fields (VERGNAUD, 2009) and we gave our contribution to the theoretical framework developed by the author by discussing the opportunism and the contingency of conceptualization. We discussed and indicated both advances and throwbacks due to students of these classes in terms of conceptualization of the concept of electromagnetic induction. The classes of situation included in this study were the ones of symbolic representation of the magnetic field (Θ) and description of electromagnetic interactions (Σ).

Results indicate that are greater consolidation of modes of conceptualization of relational-abstracted type (θ_4) respectfully to situations of class Θ. This is evidence that students acquired greater familiarity in relating in an abstract way the concept of electromagnetic induction to the ones of electromagnetic field, electric flux, magnetic flux, electric circulation, magnetic circulation, electric charge, and electric current. We consider this a fruitful result, because students started the course with apparently superficial knowledge on electrodynamics and achieved relatively high levels of conceptualization.

In correlated research in contexts of electrostatics and magnetostatics difficulties in identification of electromagnetic field sources and in its relationship with electric and magnetic field were found (FURIÓ; GUISASOLA; ZUBIMENDI, 1998; GUISASOLA; ALMUDÍ; FURIÓ, 2005; NOUSIANEN; KOPONEN, 2017). Then, we started from the principle that the mastering of symbolic representations is usually harder for students, and we teach accordingly to that; therefore, the PMTU gave greater emphasis on relational aspects of the field because it was supposed more complex to be learned. Results are positive in this context, however, the ability of dealing with analogic representations compared to symbolic ones seems to depend on the subject, instead of being of a general validity. Then, we understand that a better approach must search for integration between symbolic and analogic representations so that reaching the construction of modes of type θ_5 would be easier for students.

Data analysis indicated the students gradually get rid of the cognitive biases' binds when analysis situations of type Σ. We highlight the fact that students' conceptualizations ended nearer the most advanced poles of conceptualization (modes σ_4, σ_5 and σ_6). From this data it is derived another problem of investigation, the one of formulating situations which provide conditions for establishment of explicit relationship between modes σ_5 and σ_6.

In investigations on higher education students' conceptions, it was identified a tendency to a more macroscopic bias on students' conceptualization (ZUZA et al., 2014), which is corroborated in this work. In other research it is pointed out that students have difficulties in understanding the field as a physical quantity, which means more than a mathematical tool FURIÓ; GUISASOLA; ZUBIMENDI, 1998; GUISASOLA; ALMUDÍ; FURIÓ, 2005). We stress that this conception is resistant to change, because we strongly insisted in discussion of ontological and epistemological features of the electromagnetic field as mediator of interaction and, even though, the mode σ_4 appears to be frequent. By the other hand, subjects reach understanding that electromagnetic force and field are distinct physical magnitudes, the first depends on proof charges and sources, the last one is a function of points on space, and that there is not necessary parallelism between the vector describing the force and the one representing the field, as would occur for the electrostatic case. We perceived the advances in mastering a conceptual field are progressive if we put the problem in this perspective.

One case in study 1 which was notable was the one due to a student (M) that in the last class returned to evidence a mode of conceptualization of type σ_2 (substantialist). This is not such a surprise once data of this kind are discussed since the time in which research on conceptual change started. It is possible to say about this that meaning learning involves modification of old meanings and construction of new structures that coexist with prior knowledge, including alternative conceptions, in different conceptual profiles evoked in distinct sociocultural contexts (MORTIMER, 1996). Moreover, conceptualization strongly depends on situations we already master and often unites incoherent elements (VERGNAUD, 2009). This is consonant with student M because he seems to use a mode of type σ_2 in the last class of the semester. This phenomenon shows us clearly the statement due to Vergnaud on the mastering of a conceptual field to demand time whose magnitude can vary from years to decades (VERGNAUD, 2013). Meaningful learning is, then, a non-linear process once it involves constantly advances, throwbacks, ruptures, and continuities.

Another aspect we would like to stress is motivational. Both classes were studying the discipline of Physics II again and, even though the content seemed to be unknown for them and their reasoning on the electromagnetism appeared to be fragmented in initial diagnostic evaluation, they presented considerable evolution in learning (PANTOJA; MOREIRA, 2020). Specially in study II, students showed signs of being widely unmotivated and they alleged a list of reasons why they were like that: orthodox methodologies encouraging rote learning; problems settlement of the course on the University; absence of laboratories and professors; lack of coherence in documents related to the course. At the end of the PMTU, a huge number of students of both classes stated to the professor they were extremely satisfied with learning they could developed in this context and asked form him to teach them Physics IV.

Last, we highlight the contribution Science Teaching Research the use of the concept of mode of conceptualization for describing the opportunism and contingency of conceptualization. Addressing long-term competencies is something aimed by the Theory of Conceptual Fields (VERGNAUD, 2013), however in a considerable number of times we do not have prior schemes and we need to build them, therefore, approaching conceptualization as a process of continuous gradation between opportunism and systematics (FANARO; OTERO; MOREIRA, 2009) allowed us to tackle the question from the notion of mode of conceptualization.

Addressing knowledge in operational form by means of PMTU is another point we stress in this article as implication for research in Physics\Science Teaching because a great number of PMTU aimed at analysis meaningful learning processes in predicative perspective by using the Ausubelian of meaningful learning as theoretical framework for evaluation of learning. Ausubel restricts the analysis to predicative knowledge (VERGNAUD, 2013), while the theory of Conceptual Fields allows analysis of cognition both in predicative and operational forms of knowledge. Then, Vergnaud offers a more complete form of analysis of learning, that yet is not often explored in research in Science Teaching.

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