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Maupertuis Principle of Least Action: What can future Physics teachers learn by studying a primary source?[+*]

Yaffa Bruxel Rabeno[1]
Graduating in Physics – Federal University of Rio Grande do Sul
Nathan Lima[1]
University of Rio Grande do Sul
Porto Alegre – RS

Abstract

We present a theoretical-methodological reflection for the use of primary historical sources mainly in the context of teacher education, in order to privilege the development of what Freire calls epistemic autonomy. This reflection is presented in dialogue with discussions in the field of Science Studies, especially the so-called Ontological Turn of Anthropology, and the Philosophy of Language of the Bakhtin Circle, presenting three contributions: the emphasis on the epistemic autonomy of students, the search for overcoming the dichotomy between technical and pedagogical knowledge, and the adoption of a non-structuralist epistemological conception, aligned with the methodological discussions arising from Laboratory Anthropology. In order to exemplify the potential and richness of discussions of primary sources from this theoretical-methodological framework, we present the translation from French to Portuguese of a primary source, As Leis do Movimento e do Repouso, written by M. de Maupertuis in the 19th century. XVII, in which its Least Action principle is formalized for the first time. Next, we comment on possible ways of reflection and dialogue in the classroom. Thus, the article brings contributions both in the theoretical sense and in the sense of offering a primary source in Portuguese, with possible reflection paths for the didactic context.

[+] O Princípio de Mínima Ação de Maupertuis: O que futuros professores de Física podem aprender estudando uma fonte primária?

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[1] E-mails: yaffarabeno@hotmail.com; lima.nathan@gmail.com

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I. Introduction

Despite its long existence, the research field of History, Philosophy, Sociology and Science Education (HFSEC) has achieved some of its milestones in the 1980s and 1990s, such as the foundation of *Science & Education* (MATTHEWS, 1992) and the International Group of History, Philosophy, Sociology and Science Education, the IHPST. Over the three following decades, much research has been published advocating a diversity of perspectives, methods and objectives to be achieved. For a review of the major approaches in Physics Teaching, for example, Teixeira, Greca and Freire's work (2012) presents an overview of the international literature.

Moreover, not only but especially in Brazil, some authors have recently been problematizing the importance of articulating teaching activities that make use of History, Philosophy and Sociology of Sciences with explicit didactic and curricular perspectives, making evident the underlying political-pedagogical commitments (MOURA; GUERRA; CAMEL, 2020). That is, the researchers argue that it is not enough to work with HFSEC. It is necessary to analyse whether the theme, approach and discussions align with epistemological and pedagogical perspectives of interest. Just as an example, depending on how it is structured, an activity with the history of science can reinforce mythical conceptions about the history of science (ALLCHIN, 2004) and even the so-called "myths" about science (AULER; DELIZOICOV, 2001); or even that an epistemologically sophisticated approach can reinforce the conception of an apolitical science whenever the social dimension of the scientific enterprise is neglected (MOURA, 2021).

In line with these concerns, this article has two objectives. Firstly, to present a theoretical-methodological reflection for activities with history and philosophy of science to enhance their fostering of what Freire calls *epistemic autonomy* (FREIRE, 2013a, 2013b), with implications for teacher training contexts above all. In particular, starting with discussions of the so-called ontological turn within Anthropology (KOHN, 2015) and Bakhtin's Theory of Concrete Utterance (BAKHTIN, 2016; LIMA *et al.*, 2019; SOUZA, 2002), we emphasise the importance of allowing students (future teachers) to make contact with primary historical sources (original articles, diaries, letters) and, from their study, address the discussion on Physics (the physical concepts present in the text) and about Physics (what Physics or science in general is and how it relates to other fields of culture and human endeavour).

This theoretical-methodological proposal contributes in three ways. First, it reverses what is usually done: instead of presenting a historical episode and immediately tensioning the discussion towards a given reflection, it starts with the students' first reflections to organise possible analyses of the historical episode – which aims to privilege epistemic autonomy. Secondly, it starts with the same episode to discuss Physics (the contents, the concepts) and about Physics (the nature of science), which is in agreement with the idea of breaking the dichotomy between technical and pedagogical contents, as has been recently advocated by teacher training guidelines (MASSONI; BRUCKMANN; ALVES-BRITO, 2020), as opposed to old models such as 2+2 (2 years of pedagogical courses and 2 years of physics courses) (ARAUJO; VIANNA, 2010). This means that, instead of the future teacher learning Physics separately from pedagogical concerns, in this training model, the content itself is integrated with broader discussions that are inherent to the teaching career. Third, it does not commit itself rigidly to a previously specific epistemological conception. By allowing for the encounter of text and reader (and their reflections, experiences and previous readings), it makes room for a new learning about what science is, which aligns with post-structuralist perspectives in history and science teaching[2] (LIMA, 2021).

It should be noted that there are many authors who defend the importance of the use of historical texts and primary sources in science and physics teaching. Oliveira (2011) discusses how reading a historical text mobilises reflections about the Nature of Science. Silva and Guerra's book (2015) brings different primary sources and comments that make it possible to understand the construction process of scientific concepts. Some proposals use primary sources to enable the meaningful learning of concepts (BOSS; SOUZA FILHO; CALUZI, 2009). The literature also emphasises the importance of working with primary sources in an investigative approach, to align with contemporary pedagogical trends (BATISTA; DRUMMOND; FREITAS, 2015). Karam and Lima (2022) defend the importance of using primary historical sources in learning physics, and Karam (2021) presents methodological reflections for choosing primary sources in the classroom. Fonseca et al. (2017), in addition to presenting a broad theoretical discussion on the subject and presenting different works that align with this perspective, also present a didactic proposal in which both conceptual and philosophical aspects can be integrated. Our proposal, therefore, aligns with the discussions present in the literature about the importance of adopting primary sources in teaching. We hope to contribute, in the context of the first objective, with the reflection on this topic from a different theoretical perspective (and possibly complementary to what has been discussed).

The second objective of the article is to present a primary historical source, the original paper *The Laws of Motion and Rest* written by M. de Maupertuis in the seventeenth century and translated by us from French into Portuguese[3]. This text has an important meaning for the history of Mechanics since it is one of the first articles to propose the notion of a Principle of Minimum Action, which would later be present in more recent formulations

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[2] Structuralist perspectives are usually committed to a universal proposition that organizes the reading of history. The notion of a "structure of scientific revolutions," for example, shows how the same structure, or metanarrative, is used to evaluate different episodes. Post-structuralist perspectives, on the other hand, abdicate such a universal proposition.

[3] In this English version, translated from French into English.

of Mechanics (MOREIRA, 1999), such as what we now call Lagrangian Mechanics. In particular, this principle also has a strong role in the development of Quantum Theory, being present in discussions by Louis de Broglie and Erwin Schrödinger (LIMA; KARAM, 2021). From this text, we discuss which possible didactic paths could be opened to address Physics and about Physics from the presented text and the mobilised theoretical-methodological framework. In doing so, we intend, firstly, to exemplify how, from a primary source, we can adopt an epistemic-methodological posture that allows us to be open to building new conceptions about science, as well as to contribute directly to the didactic context, since the translated and commented primary source can be used in mechanic courses typical of undergraduate physics.

This paper is structured as follows: in section 2, we present our theoretical-methodological reflections (which are associated with objective 1). In section 3, we present the primary source translated into Portuguese. In section 4, we make comments and reflections on the use of such a source in the context of teacher training (which is associated with objective two). In section 5, we present our final considerations.

II. Initial theoretical and methodological reflections

Educating implies training someone to be an ideal subject in a certain model of society and, therefore, curriculum design and execution are practices loaded with political values and commitments (SILVA, 2010). In this sense, it has been discussed that the complexity of the contemporary world has demanded more than ever a new epistemic-political pact, which, indeed, values science, but also centers the different understandings and concerns of communities, making visible knowledge and values that, historically, have been erased. Such conception we call non-narcissistic education (LIMA; GUERRA, 2022), alluding the discussions made by Eduardo Viveiros de Castro on the radically decolonial character of Anthropology (CASTRO, 2018).

Advancing the pedagogical discussion on a non-narcissistic approach, an important point to be highlighted is the rescue of Paulo Freire’s concept of epistemic autonomy (2013b); it implies the maturation of the gnosiological curiosity about reality. This maturation, or overcoming of a naive curiosity, demands methodological rigour, knowledge structuring, and advancement in the domain of the typical semiotic means of scientific representation, which corroborates the importance of the formal context of education, where these practices can be elaborated and organized more frequently.

Moreover, considering that the catastrophes we are experiencing today (climate change, pandemic, social inequality) are the result of a paradigmatic crisis of a consumer society, as many authors have already discussed (KRENAK, 2019; SANTOS, 2008), the pedagogical proposal that encourages epistemic autonomy, in the sense given by Freire, has a deep subversive sense. Since the consumer society objectifies nature and human beings, treating the former as natural resources and the latter as human resources, encouraging epistemic autonomy means breaking with the system's undelying logic.

There is a parallel between the usual school model and the factory structuring. While a *designer* conceives and proposes a shoe, countless workers perform steps that they did not conceive, therefore not seeing themselves as creators of those objects. Similarly, in the traditional model of teaching, someone conceived the curriculum and the knowledge. Teachers and students only perform the indicated tasks and thus do not see themselves as creators of knowledge, but feel alienated from it (AULER, 2018).

Someone with epistemic autonomy, on the other hand, is a *de facto* subject, with subjectivity, with agency over the world, able to understand and modify it. They are no longer a passive agent of the economic or cultural system. Thus, in a world of perpetual catastrophes and crises, non-narcissistic education resists the hegemonic ideology advocating the epistemic autonomy of the subjects involved. The radicalism of the defence of autonomy is highlighted, for example, in bell hooks's works (2013) and by the discussion of post-colonial and post-abyssal education by Boaventura de Sousa Santos (2019).

In this sense, it is important to discuss what is necessary to promote such autonomy. As we mentioned in the introduction, many researchers already discuss the importance of overcoming the propedeutic teaching the history of science, heading towards investigative approaches (BATISTA; DRUMMOND; FREITAS, 2015). Agreeing with the authors, we also emphasize the possibility of working in an investigative perspective taking as a starting point the reflections, conceptions and axiological positions of the students themselves. That is, from a primary source, taking *what do we learn about nature and what do we learn about science from the text studied?* as a guiding question, and letting students trace their initial reflections.

Although it is a simple practical change, it is committed to a deeper epistemological change. One can, in general, adopt a more axiomatic perspective on science, defending *a priori* what science should be, as epistemologists such as Popper and Lakatos do, or one can adopt a more descriptive perspective, finding the characteristics of science through the observation of its practices, as anthropologists of the sciences usually do, especially after the advent of the Athropology of Laboratories (WOOLGAR, 1982)[4].

This descriptive perspective is methodologically committed to not erasing the speeches and discourses that arise from the encounter with the text in favour of previous theoretical conceptions (LATOUR, 2005). This is of course not denying the existence of previous conceptions, but they are not taken as the primary theoretical categories that dictate the path to be followed (STENGERS, 2018; VENTURINI, 2010). Thus, since there is no rigid initial theoretical framework, each text, in each class, enables a new reflection on science.

[4] Levi-Strauss differentiates between what he calls centripetal research (from previous theories and models towards analysis of episodes and practices) and centrifugal research (from the episode towards theoretical constructs).

Each episode and each meeting is a unique learning – which meets the proposal of the Cultural History of Science and of Science Studies (MOURA; GUERRA, 2016)

This practice, in addition to fostering reading and writing culture in students, which is an extremely important practice in scientific culture, aims to encourage students to develop their reflections and descriptions, based on their theoretical framework, readings and experiences. This implies a break from the traditional culture of Physics Teaching, in which reading and writing are neglected in favour of solving exercise lists.

Once past the stage of reading and initial reflection of the students, the teacher participates in the debate bringing, in turn, their reflections, their theoretical constructions and analysis. The teacher tensions the debate and enriches the discussion based on their trajectory within formal education. Thus, we emphasize that the teacher's role is fundamental and cannot be neglected in this proposal, since epistemic autonomy demands conceptual enrichment and increased methodical rigour, which the teacher must provide.

On the other hand, the students' discussions may become known and gain space in the description of the episode studied. In this context, there is not a single reality to be described, but different realities, with different networks of concepts and actors. These different realities co-exist and legitimise themselves in their communities, as discussed in the studies of the ontological turn of Anthropology (KOHN, 2015). The difficulties of the contemporary world, however, demand that we create points of contact between these realities, and establish consensuses that pave the way for the construction of a common world.

Still, our proposition starts with the assumption that the text, although materially finite, enables a virtually infinite number of interpretations and connections with other texts in the great field of culture, as discussed by Bakhtin (VENEU; FERRAZ; REZENDE, 2015). Thus, the starting point should be the different interpretations constructed by students from their "vision surplus" (BAKHTIN, 1990), that is, from their unique place in existence. The teacher's role, in this sense, is to tension the debates from their particular vision surplus, which is buit from their academic and intellectual upbringing, as well as their valued life experiences.

Thus, the didactic process leads to a new conception of the text and enables everyone (students and teachers) to develop a new conception on Physics and about Physics. While referring to the well-established traditional culture of science, which the teacher is responsible for representing and translating the elements of this community – acting as an intercultural translator (SANTOS, 2019), the values, reflections and analysis of the entire class are allowed to tension the debate. The result is a plural, diverse understanding that respects epistemic autonomy while not ignoring the knowledge established in the scientific context. To summarize, we present a table showing the steps of such a didactic process, as conceived in our proposal.

Table 1 – Systematization of the didactic path from the theoretical proposal (each stage can last as long as necessary and possible within the concrete context).

Step	Description
1 – Teacher chooses an episode	The choice of a historical episode is usually associated, first, with the curriculum itself to be implemented; but also with the teacher's interests in exploring that episode. As Karam (2021) points out, not every episode is useful in the classroom. Therefore, the chosen episode indeed reveals a certain intentionality, a desire to explore certain concepts of Physics or about Physics.
2 – Teacher prepares possible paths and reflections	Before presenting the primary source to the students, the teacher deepens their analysis of the work, resorts to secondary sources and, based on the teacher's knowledge and in dialogue with the specialised literature, traces possible reflections on the concepts to be addressed and on the nature of science.
3 - Students read and write their reflections	Students are exposed to the primary source, without elaboration from the teacher. They are allowed to trace their reflections, write and systematize them. Students are allowed to present their elaboration and interpretation of the text.
4- Dialogue	Students and teacher bring their reflections in a first dialogue. The teacher tensions the debate by presenting the reflections from the scientific communities.
5- Study from the collective constructions	From this, one can choose ways for deepening the work, studying commentators or authors that allow reflections on specific aspects of what was worked.
6- Closing: establishing consensus	Finally, the activity closes with a new moment of dialogue, in which the set of raised reflections is systematized, pointing out the consensuses obtained and the remaining divergences.

To exemplify the richness and reflective depth that the primary sources allow, we follow by introducing a primary source, *The Laws of Movement and Rest*, translated by us from French and subsequently commenting on the possible reflections the teacher could engage with the students. We do not aim to discuss a real didactic intervention, but only to exemplify how – from our vision surplus – we would tension the debate into a didactic experience. Our reflections, as we have reinforced, do not and can never exhaust the meaning of the text, and the didactic process is only completed with the concrete debate with the students' reflections. However, we understand that the discussion allows us to exemplify possibilities and paths of debate, also showing how understandings are taken from the concrete case and not from previous theoretical categories (without denying the existence of such categories).

III. The Primary Source

Pierre-Louis Moreau de Maupertuis was a French scientist, who lived in the 18th century, and devoted himself to different topics of mathematics, physics and natural science:

> *Pierre-Louis Moreau de Maupertuis (1698-1759) was elected to the Académie des Sciences de Paris in 1723 and to the Royal Society of London in 1728. An ardent supporter of Newtonianism in the Académie, he considered himself "the first who dared, in France, to propose attraction as a principle to be examined". In 1744 he was invited by Frederick II to reorganise the Berlin Academy of Sciences, of which he was President from 1746 until his death. Two of his well-known scientific achievements are the empirical proof of the flattening of the Earth's poles and the formulation of the physical principle of minimum action, in which he fought intense disputes with the disciples of Descartes and Leibniz. Maupertuis' conceptions of the generation of organisms appear in three of his works: the Vénus physique of 1745, the System of Nature of 1752, and Letter XIV – On the Generation of Animals, component of the Letters of 1752. Important references to the role of chance, providence and natural laws in generation appear in his 1750 Essay on Cosmology, a work that we will use for the analysis of the author's relations between cosmology and natural history* (RAMOS, 2003, p. 43).

Maupertuis introduced the Principle of Minimum Action in 1744 and 1746 for both optics and mechanics. Although other scientists such as Leonhard Euler (1707-1783) and Joseph-Louis de Lagrange (1736-1813) may have made some updates on the principle, it was not until the 19th century that the principle of least action gained its real and most general form by William Rowan Hamilton (1788-1856) – Hamilton's principle.

THE LAWS
OF MOTION AND REST
derived from a Metaphysical Principle.

By M. de MAUPERTUIS

I presented the principle on which the following work is based, on April 15, 1744, in the Public Assembly of the Royal Academy of Sciences of Paris (Académie Royale des Sciences des Paris), as evidenced by the Acts of this Academy.

Later that year, Professor Euler wrote his excellent book: Methodus inveniendi lineas curvas maximi minimive proprietate gaudentes. In its added supplement, this illustrious geometer demonstrates that, in the trajectories described by bodies under the action of central forces, the velocity multiplied by the element of the curve always makes a **minimum**.

This observation has given me even more pleasure, for it is a beautiful application of my principle to the motion of the planets, of which this principle is indeed the rule.

I will try to extract truths of a higher and more important kind from the same source.

I.
EXAMINING THE EVIDENCE OF GOD'S EXISTENCE
From the Wonders of Nature.

Whether we remain closed in on ourselves, or set out to navigate the wonders of the Universe, we find so much evidence of the existence of an all-powerful and all-wise Being, that it is more necessary to reduce the number [of proofs] than to seek to increase it: that we must at least choose between these proofs, examine their strength or their weakness, and only give each the weight it ought to have: for one can do no more harm to the truth than by wishing to base oneself on false reasoning.

I am not examining here the argument found in the idea of an infinite Being: in this idea too great for us to draw from our mind or any other finite mind, and which seems to prove that a perfect infinite being exists.

I will not quote this consent of all men to the existence of a God, which seemed so strong a proof to the philosopher of ancient Rome [1]. I do not dispute whether it is true that some persons deviate from others on this, nor whether a handful of men who thought differently from all the other inhabitants of the Earth could make an exception, nor whether the diversity which can be found in the ideas which those who admit Their existence have of God prevent great profit from being obtained from this consent.

Finally, I will not insist on what we can conclude from the intelligence we find in ourselves, from those sparks of wisdom and a power we see scattered in finite Beings, and which supposes an immense and eternal source from which they derive their origin.

All these arguments seem very strong to me, but they are not the kind I am examining.

From all times, those who have been engaged in the contemplation of the Universe have found there signs of the wisdom and strength of the One who governs it. The more progress that has been made in the study of physics, the more these proofs have multiplied. Some are confused by the characteristics of the Deity which are found at all times in Nature; others, through bad religious zeal, have given some proofs more strength than they ought to have, and have sometimes taken them for proofs which they were not.

Perhaps we would be allowed to relax in the rigour of the arguments if we had no reason to establish a useful principle, but here the arguments are strong enough and the number is large enough so that we can make the stricter examination and the more scrupulous choice.

I will not dwell on the proofs of the existence of the Supreme Being, which the Ancients drew from the beauty, order and arrangement of the Universe. We can see those related by Cicerus [2] and those that he quotes from Aristotle [3]. They knew too little about Nature to have the right to admire it. I am connected with a philosopher who, by his great discoveries, was far more within reach of judging these wonders, and whose reasonings are far more accurate than all theirs.

Newton seems to have been more moved by the evidence he finds in the contemplation of the universe than by all the other evidence he could extract from the depths of his mind.

This great man believed [4] that the motions of the heavenly bodies sufficiently demonstrate the existence of Him who governs them. Six planets, Mercury, Venus, Earth, Mars, Jupiter and Saturn, revolve around the Sun. All move in the same direction and describe roughly concentric orbs: while other star species, comets, describing very different orbs, move in all sorts of directions and wander around all regions of the sky. Newton believed that such uniformity could only be the effect of the will of a Supreme Being.

Less elevated objects did not seem to provide less strong arguments. The uniformity observed in the construction of animals, their wonderful and useful organisation, was to him convincing proof of the existence of an all-powerful and wise Creature [5].

Several physicists, after Newton, found God in stars, insects, plants, and water [6].

Let us not conceal the weakness of some of his reasonings, and to make better known the abuse that has been made of the proofs of the existence of God, let us examine those that seemed so strong to Newton.

The uniformity, he says, of the motion of the planets necessarily proves a choice. A blind fate could not make everyone move in the same direction and almost concentric orbs.

Newton could add to this uniformity of the motion of the planets that all of them move almost in the same plane. The zone in which all of their orbs are enclosed is only about the 17th part of the sphere's surface. Therefore, if we take the Earth's orb as the plane to which we relate others, and regard its position as the effect of chance, the probability that the 5 other spheres should not be included in this area is 17^5 - 1 to 1; that is, 1419856 to 1.

If one conceives, like Newton, that all celestial bodies attracted to the Sun move in the void, indeed, it would hardly be probable that chance had caused them to move as they moved. There remained, however, some probability, and therefore we cannot say that this uniformity is the necessary effect of a choice.

There is more, however: the alternative between a choice or an extreme probability is based only on the impotence, embedded in Newton's system, to commit to a physical cause for this uniformity. To other philosophers who admit a fluid that carries the planets, or that merely moderates their motion, the uniformity of their body does not seem inexplicable: it no longer supposes this singular stroke of chance, or this choice, and does not prove the existence of God any more than would any other motion communicated to Matter [7].

I do not know if the argument Newton draws from animal construction is much stronger. If the uniformity we observe in several were proof, would not this proof be contradicted by the infinite variety we observe in several others? Without departing from the same elements, compare an eagle to a fly, a deer to a snail, and a whale to an oyster, and judge this uniformity. Other philosophers want to find proof of God's existence in a variety of forms, and I do not know which ones are the most well-founded.

The argument based on the suitability of the different parts of the animals to their needs seems more solid. Are not their feet made to walk, their wings to fly, their eyes to see, their mouths to eat, and other parts to reproduce their fellow ones? Does not all this mark intelligence and a design that preceded their construction? This argument struck the ancients as it struck Newton: and it is in vain that the greatest enemy of Providence answers to it, that use was not the objective, that it was the result of the construction of the parts of animals: chance formed the eyes, the ears, the tongue, used for seeing, hearing, speaking [8].

However, could we not say that in the fortuitous combination of the productions of nature, as there were only those in which were found certain relations of expediency, which might subsist, it is no wonder that this expediency is to be found in all the species that now exist? Chance, it would seem, produced an innumerable multitude of individuals. A small number were found constructed so that the parts of the animal could satisfy its needs; in another infinitely greater, there was neither expediency nor order: all the latter perished. Mouthless animals could not live, others who lacked organs for generations could not perpetuate themselves, the only ones who remained are those in which order and convenience were found, and these species we see today are only the smallest part of what a blind fate produced.

Almost all modern authors who have dealt with Physics or Natural History have merely extended the evidence we have drawn from the organization of animals and plants and propelled them into the smallest details of nature. Not to mention here too indecent examples, which would be very common, I will speak only of him [9] who finds God in the folds of a rhino's skin: because this animal being covered by a very hard skin, could not move without these folds. Is not the greatest of truths being attacked in wanting to prove it by such arguments? What would anyone say who denies providence because the turtle's shell has neither folds nor joints? The reasoning of those who prove it by the skin of the rhinoceros is of the same force: let us leave these trifles to those who do not feel their frivolity.

Other species of philosophers fall on the opposite end. Very little touched by the marks of intelligence and purpose we find in Nature, they would like to banish all final causes. They believe that with matter and motion, the world could have been formed as it is. Some see the Supreme Intelligence in everything, while others see it nowhere, they believe that a blind mechanic could have formed the most organized bodies of plants and animals, and work all the wonders we see in the Universe [10].

We see, from all that we have just said, that neither Descartes's great argument, taken from our idea of a perfect Being, nor any of the other metaphysical arguments of which we have spoken made a great impression on Newton. On the other hand, not all the evidence Newton draws from the uniformity and convenience of the different parts of the universe would have seemed proof to Descartes.

We must admit that these proofs are abused: some giving them more strength than they have, others multiplying them greatly. The bodies of animals and plants are very complicated machines, the latter parts of which escape much from our senses, and upon which we are very ignorant as to use and end, that we may judge the wisdom and power necessary to build them. If some of these machines seem to be brought to a high degree of perfection, others seem only sketchy. Several may seem useless or harmful, if we judge by our only knowledge and if we no longer assume that it was an omniscient and omnipotent Being who placed them in the universe.

What use is it, in the construction of some animal, to find appearances of order and expediency, when after that we suddenly stop for some unfortunate conclusion? The serpent, which neither walks nor flies, could not have prevented the pursuit of other animals if a prodigious number of vertebrae did not give his body so much flexibility that it crawls faster than several animals walk. It would have died of cold during the winter if its long, pointed shape had not made it suitable for strengthening itself in the earth. It would have been hurt by crawling continuously, or torn as it passed through the holes where it hides if its body had not been covered with lustful, scaly skin. Is all this not admirable? Nevertheless, what is all this for? To the conservation of an animal, whose tooth kills a man? Oh! We reply, you don't know the usefulness of snakes. They were apparently necessary in the universe; they will contain excellent remedies that you do not know. Let us be silent then, or at least not admire a device so large in an animal that we only know as harmful.

Everything is replete with such reasoning in the writings of naturalists. Follow the production of a fly or an ant: they make you admire the care of providence for the insect's eggs, for the feeding of the little ones, for the animal wrapped in the cloths of the chrysalis and for the development of its parts in its metamorphosis. All this results in the production of an insect, inconvenient to man, which the first bird devours or which falls into the nets of a spider.

While one finds here evidence of the Creator's wisdom and power, is it not to be feared that the other would find there something to strengthen in his unbelief?

Very large minds, as respectable for their piety as for their illumination [11], could not fail to admit that convenience and order do not seem to be observed so exactly in the Universe, no one would be ashamed to understand how this could be the work of an all-wise and all-powerful Being. Evil of all kinds, disorder, vice, and pain, seemed to them difficult to reconcile with the empire of such a Master.

See, they said, on this Earth, the seas cover half of it, on the rest, you will see steep rocks, icy regions, scorching sands. Examine the customs of those who inhabit it: you will find lies, robberies, murders, and, for all vices, more common than virtue. Among these unhappy beings, you will find many despairing in the torments of hunger and misery; many languish in other infirmities, which their duration renders unbearable: almost all oppressed by worries and sorrows.

Some philosophers seem to have been so impressed with this view that, forgetting all the beauties of the universe, they seek only to justify God for having created such imperfect things. Some, to maintain [God's] wisdom, seem to have diminished [God's] power, saying that [God] did all that [God] could do best [12]: Of all possible worlds, this, despite its defects, was still the best. The others, to retain power, seem to be deceiving wisdom. God, according to them, might well make a world more perfect than the one in which we dwell, but would have to use very complicated methods, and had more given how he operated, than the perfection of the work [13]. These use the example of the painter, who believed that a circle drawn without a compass would better prove his skill, which would not have made the figures more composed and regular, described with instruments.

I do not know if any of the above answers is satisfactory, but I do not believe that the objection is invincible. The true philosopher must not allow himself to be dazzled by the parts of the universe where order and convenience shine, nor let himself be shaken by those where he does not discover them. Despite all the disorders he notices in nature, he will find sufficient characters of the wisdom and power of his Author so that he cannot ignore them.

I am not talking about another kind of Philosopher who asserts that there is no evil in Nature: *That everything that exists is good* [14].

If we examine this proposition, without first assuming the existence of an all-powerful and wise Being, it is not sustainable. If we take it away from the assumption of an omniscient and omnipotent Being, it will be nothing but an act of faith. In the first place, it seems to pay homage to the Supreme Intelligence, but deep down, it tends only to subjugate everything to necessity. It is more a consolation in our miseries than a compliment to our happiness.

I return to the proofs we have drawn from the contemplation of Nature.

Those who gathered most of this evidence did not sufficiently examine its strength or extent. Thousands of things in the universe announce that it is not ruled by a blind power. On all sides, we can see a series of effects contributing to some goal; this only proves intelligence and intent. Moreover, it is for the intent that we should seek wisdom. Skill in execution is not enough, the reason must be reasonable. One would not be surprised, one would blame the worker, and he would be even guiltier if he employed more skill in the construction of a machine which would be useless or whose effects would be dangerous.

What is the use of admiring this regularity of the planets, of moving all in the same direction, almost in the same plane and in roughly similar orbits, if we did not see that it was better to make them also move. Are so many poisonous plants and noxious animals, produced and carefully preserved in nature, adequate to make us know the wisdom and goodness of the one who created them? If these things were discovered in the universe, they could only be the work of demons.

Our view is indeed as limited as it is, we cannot require it to pursue the order and sequence of things far enough. If I could, I would no doubt be as impressed with the wisdom of motives as with intelligence in execution. Nevertheless, in this helplessness we are in, let us not confuse these different attributes. Although infinite intelligence necessarily supposes wisdom, limited intelligence could lose it. In addition, it would be worth as much that the universe owed its origin to a blind destiny as if it were the work of such intelligence.

II. THAT WE SHOULD LOOK FOR EVIDENCE OF THE EXISTENCE

Of God, in the General Laws of Nature. That the Laws according to which Motion is preserved, distributed, and destroyed are founded on the attributes of a supreme Intelligence.

It is not, therefore, in the small details, in those parts of the Universe whose relations we know very little, that we should seek the Supreme Being: it is in the phenomena whose universality suffers no exception, and whose simplicity exposes itself entirely to our vision. This research will indeed be more difficult than that which consists merely in examining an insect, a flower, or anything else of this kind, which nature offers at all times to our eyes. But we can ask for the help of a safe guide in this walk, although it has not yet taken its steps to where we want to go.

So far, mathematics has aimed only at the gross needs of the body or useless speculations of the mind. We never think of making use of it to demonstrate or discover truths other than those concerning extension and numbers. For we must not be mistaken with some Works, which have of mathematics only air and form, and which are only the most uncertain and darkest metaphysics. The example of some philosophers must have either taught that the words Lemma, Theorem and Corollary do not carry mathematical certainties completely, that this certainty does not depend on these great words, or even on the method followed by the geometers, but on the simplicity of the objects they consider.

Let us see if we can make better use of this science. The proofs of the existence of God, which it will provide, will have, above all others, the advantage of the evidence that characterizes mathematical truths. Those who do not have sufficient confidence in metaphysical reasoning will find more certainty in this kind of evidence, and those who do not pay sufficient attention to popular evidence will find in it more accuracy and elevation.

So let us not stop at the simple speculation of the most wonderful objects. The organization of the animals, the multitude and smallness of the parts of the insects, the immensity of the heavenly bodies, their distances and their revolutions are more apt to astonish our mind than to enlighten it. The Supreme Being is everywhere, but not everything is equally visible. We shall see [God] best in the simplest objects: let us look for [God] in the first laws, which [God] has imposed on nature; in those universal rules, according to which motion is conserved, distributed or destroyed, and not in phenomena which are very complicated consequences of those laws.

I could have started with these laws, how mathematicians give them and how experience confirms them. Then seek there the characteristics of the wisdom and power of the Supreme Being. However, like those who gave it to us were based on hypotheses that were not purely geometric, so its certainty does not seem to be based on rigorous demonstrations. I found it safer and more useful to deduce these laws from the attributes of an all-powerful and wise Being. If those I thus meet are the same as those observed in the universe, is this not the strongest proof that this Being exists and is the author of these laws?

However, it may be said, though the rules of motion and rest have hitherto been demonstrated only by hypotheses and experiments, they are perhaps the necessary consequences of the nature of Bodies, and there is nothing arbitrary in their establishment, do you ascribe them to a providence what is the effect of necessity alone?

If it is true that the laws of motion and rest are indispensable consequences of the nature of bodies, this further proves the perfection of the Supreme Being. It is that all things are ordered in such a way that blind and necessary Mathematics perform what the most enlightened and free Intelligence would prescribe.

Some ancient philosophers held that there was no movement. The very subtle use of your mind denies what your senses perceive. The difficulties they encountered in conceiving how bodies move made them deny that they are moving and that they can move. We will not relate the arguments on which they tried to base their opinion, but we will observe that we cannot deny motion except by reasonings that destroy the existence of all objects outside us, which would reduce the universe to our being and all its phenomena to our perceptions.

It is true that we only know movement through the senses: but has there been much that we know differently? The driving force, the power that a moving body has to move others, are words invented to complement our knowledge and which mean only the results of phenomena. Only habit prevents us from feeling all that is wonderful in the communication of the movement. Since we opened our eyes, nothing has struck them as often as this phenomenon. He who has not reflected on it finds nothing obscure in it, and he who thinks much about it despairs to understand anything.

If someone who had never touched a body, and who had never seen it shocked, but who had the experience of what happens, when different colours mix, saw a blue body appear moving towards a yellow body and he is asked about what will happen when the two bodies meet? Perhaps the most likely thing he could say would be that the blue body will turn green as soon as it hits the yellow body. But let him guess either that the two bodies come together to move with a common speed, or that one would communicate to the other part of his speed to move in the same direction with a different speed, or that it would reflect in the opposite direction; I do not believe this is possible.

However, as soon as we reach bodies, as soon as we know that they are impenetrable, as soon as we realize that it takes a certain force to change the state of rest or movement in which they find themselves, we see that when one body moves towards another, if it reaches it, it must reflect, or stop, or slow down, it must move the one who finds it if it is at rest, or who changes its movement, if it moves. However, how are these changes made? What is this power that bodies seem to have to act on each other?

We see parts of matter in motion, we see others at rest. Motion is therefore not an essential property of matter, but a state in which it can be found or not, and which we do not see that it can control itself alone.

The parts of matter that move in nature have therefore received their movement from some strange cause, which hitherto is unknown to me. Moreover, since they are indifferent to motion or rest, those who are at rest remain there, and those who move once continue to move until some cause changes their condition.

When one part of Matter in motion finds another at rest, it communicates to it a part of its motion or all of its motion. In addition, the meeting of two parts of matter, one of which is at rest and the other in motion, or both in motion, is always followed by some change in the state of both. This shock seems to be the cause of this change, though it would be absurd to say that one part of matter, which cannot move by itself, could move another.

To find the first cause of the movement, the greatest philosopher of antiquity resorted to a motionless and indivisible first motor [15]. A modern philosopher not only recognized God as the author of the first motion printed in the matter, but he believed in the Action of God continually necessary for all distributions and modifications of the motion. Not being able to understand how the force to move belonged to the body, he judged it justified to deny that it belonged to [God] and concluded that when a body clashes or presses another body, only God moves it: the impulse is only the occasion that determines that God moves it [16].

These philosophers put the cause of motion in God only because they did not know where to put it: they could not conceive that matter had any efficacy to produce, distribute and destroy motion, they resorted to an immaterial Being. It was necessary to know that all the laws of motion and rest were founded on the most proper principle, to see that they owed their establishment to an all-powerful and wise Being, or that this Being acts immediately, or that [God] gave bodies the power to act upon each other, or that [God] employed some other mine which is still less known to us.

The simplest of the laws of nature, that of rest or equilibrium, has been known for many centuries, but until now it did not seem to have any connection with the laws of motion, which were much more difficult to discover.

These researches were so little to the taste, or so little within the reach of the ancients, that we can say that they still make a very new science today. How, in fact, would the ancients have discovered the laws of motion, while some reduced all their speculations about motion to sophisticated disputes, whereas others knew there was no motion?

The most laborious or most sensible philosophers did not judge that the difficulties connected to the first principles of things were cause for despair about knowing anything about them, nor excuses for dispensing them of any research.

As soon as the true way of philosophising was introduced, we were no longer content with these vain disputes about the nature of motion: we wanted to know from what laws it is distributed, preserved, and destroyed. We felt that these laws were the foundation of all Natural Philosophy.

The great Descartes, the boldest of the philosophers, sought these laws and deceived himself. However, as if the times had finally brought this matter to a kind of maturity, they suddenly found themselves springing up on all sides, these laws unknown for so many centuries. Huygens, Wallis and Wren found them at the same time. Several mathematicians after them, who looked for them in different ways, confirmed them.

However, not all mathematicians who agree today in the most complicated case agree in the simplest case. All agree on the same distributions of Motion in the Shock of *Elastic Bodies*, but they designate different [laws] for *hard bodies* and some claim that one cannot determine the distributions of motion in the shock of these bodies. The constraints they encountered there made them decide to deny the existence and even the possibility of hard bodies. They affirm that the bodies we take for such are only elastic bodies, whose rigidity makes imperceptible the bending of their parts and their straightening.

The experiments would have been carried out on bodies commonly called hard, which proves that these bodies are only elastic. When two globes of ivory, steel or glass collide, although after the shock we find them as their first figure, perhaps they have not always preserved it. We can be sure of this by the eyes if we paint one of the globes with some colour that can be erased and that stains the other: it is seen by the size of the spot that the globes flattened during the shock, although afterwards, no noticeable change remained on its surface.

Metaphysical reasonings are added to these experiments: it is stated that hardness, taken in the strict sense, would require in-nature effects incompatible with a certain *Law of Continuity*. It is said that when a hard body encounters an unshakable obstacle, it suddenly loses its speed, without going through any other degree of decrease or without converting it into a contrary speed, and that a positive speed becomes negative, without having gone through rest.

Nevertheless, I admit I do not feel the force of that reasoning. I do not know if we know enough about how the Movement is produced or extinguished to be able to say that here the law of continuity has been violated: I do not even know what that law is. When we suppose that speed increases or decreases in degrees, would there not always be faults from one degree to another? In addition, would imperceptible flaws not violate continuity as much as the sudden destruction of the universe would?

As for the experiences reported; they show that one can confuse *hardness* with *elasticity*; but they do not prove that one is only the other. On the contrary, as soon as one has reflected on the impenetrability of bodies, it seems that it is no different from their hardness; or that hardness is a necessary consequence. If in most bodies the parts of which they are composed separate or fold, this is only because these bodies are clusters of other bodies: the simple bodies, the primitive bodies, which are the elements of all others, must be hard, inflexible, and unalterable.

The more we examine elasticity, the more it seems that this property depends only on a particular structure of bodies, which leaves between their parts intervals in which they can curve.

It would seem, therefore, that we would be better grounded in asserting that all primitive bodies are hard than in asserting that there are no hard bodies in Nature. However, I do not know if the way we know the Bodies does not allow us any affirmation. If we want to admit it, we will agree that the strongest reason we had for admitting only elastic bodies was the inability to find the laws of communication of the Movement of hard bodies.

Descartes admitted this Body and believed he had found the laws of its Movement. He started with a very similar principle: *That the amount of movement is always the same in nature.* He deduced false laws from this because the principle is not true.

The Philosophers who came after him were struck by another conservation: it is what they call the *Living Force*, which is the product of each mass by the square of its velocity. These did not base their laws of motion on this conservation; they deduced this conservation from the laws of motion they saw as a result. However, as the conservation of the Living Force occurs only in the collision of elastic bodies, it has been confirmed in the opinion that there are no bodies other than these in nature.

The conservation of movement is only true in some cases. The conservation of the Living Force occurs only for certain bodies. Nor can it pass for a universal principle, nor as a general result of the laws of Motion.

If we examine the principles upon which some Authors who gave us these laws were founded, and the paths they followed, we shall be surprised to see that they succeeded so happily. And one cannot help believing that they relied less on these principles than on experience. Those who reasoned more justly recognized that the principle they used to explain the communication of the movement of the elastic bodies could not be applied to the communication of the movement of the hard bodies. Finally, none of the principles, which have been used hitherto, either for the laws of motion of the rigid body or for the laws of motion of the elastic body, extends to the laws of repose.

After so many great men who have worked on this subject, I hardly dare to say that I have discovered the universal principle, on which all these laws are founded; which also extends to rigid bodies and elastic bodies; on which depends the motion and repose of all corporeal substances.

It is the principle of the *least amount of action*: a principle so wise, so worthy of the Supreme Being, and to which Nature seems so constantly attached; that [Nature] not only observes it in all its changes but that in its permanence, [Nature] still tends to observe it. *In the shock of bodies, movement is distributed in such a way that the amount of action that the change assumes is the least possible. At rest, bodies that remain in equilibrium must be situated so that if there were any small movement for them, the amount of action would be minimal.*

The laws of Motion and Rest deduced from this principle are precisely the same as those observed in Nature: we may admire their application in all phenomena. The movement of animals, the vegetation of plants, and the revolution of the stars are only the consequences: the spectacle of the universe becomes much greater, much more beautiful, and much more worthy of its Author when we know that a small number of laws, the most wisely established, suffice for all these movements. It is then that one can have a correct idea of the power and wisdom of the Supreme Being; not when we judge by some small part, of which we know neither the construction, nor the use, nor the connection it has with others. What satisfaction for the human spirit, as it beholds these laws, which are the principle of motion and the rest of all the bodies of the universe, to find there the proof of the existence of the One who governs it!

III.
SEARCH
Laws of Movement and Rest.

Bodies, at rest or in motion, have a certain force to persist in the state in which they are: this force belonging to all parts of matter is always proportional to the quantity of matter, which these bodies contain, and is called their *inertia*.

The impenetrability of bodies and their inertia made it necessary to establish some laws, to bring together these two properties, always opposed in nature. When two bodies meet, unable to penetrate, the rest of the one and the motion of the other, or the motion of both must be altered: but this alteration depends on the force with which the two bodies collide, let us examine what *the shock* is. Let us see what it depends on and if we cannot get a clear enough idea of its strength, let us at least see the circumstances that make it the same.

It is assumed here, as assumed by all those who have sought the laws of motion. Let the bodies meet directly, that is, that their centres of gravity move in a straight line that is the direction of their movement and that in the shock this line passes through the place of their touch and is perpendicular to it. This last condition always occurs if bodies are globes of homogeneous matter, as we consider them here.

If a body moves with a certain speed, it finds another body at rest; the shock is the same as if the last body, moving with the speed of the first, finds it at rest.

If two bodies move toward each other, they meet; the shock is the same as if one of the two were at rest, another met with a speed equal to the sum of the speeds of the one and the other.

If two bodies moving to the same side meet; the shock is the same as if one of the two were at rest, and the other met with a velocity equal to the difference between the velocities of the one and the other.

In general, then, if two bodies meet, or one of the two is at rest, or both move towards each other, or both move towards the same side: whatever their velocities, if the sum or difference of these velocities (what is called the *respective velocity*[5]) is the same, the shock is the same. *The magnitude of the shock of two given bodies depends only on their respective velocities.*

The truth of this proposition is easy to see, in conceiving the two bodies carried in a moving plane, whose speed destroying the speed of one of the two would give the other the sum or difference of the speeds they had. The collision of the two bodies in the plane would be the same as in a stationary plane.

Let us now look at the difference that hardness, or elasticity of bodies, makes in the effects of shock.

Perfectly hard bodies are those whose parts are inseparable and inflexible, of which, consequently, the form is unalterable.

Perfectly elastic bodies are those whose parts, once bent, straighten, return to their first position and return the body to its first shape. As for the nature of this elasticity, we do not undertake to explain it. It is enough here to know the effect.

I am not talking about soft bodies or fluid bodies, they are just clusters of rigid or elastic bodies.

When two hard bodies meet, their parts are inseparable and inflexible, the collision can only change their velocity. The two bodies press and push each other until the speed of one is equal to the speed of the other. *Aftershock, hard bodies walk together at a common speed.*

Nevertheless, when two elastic bodies meet, while they are pressing and pushing each other, shock is also used to bend their parts. And the two bodies remain applied against each other until their elastic [material], stretched by the shock as far as it can be, separates them as they fall apart; and cause them to move apart as fast as they approached. For the respective velocity of the two bodies to correspond to the only cause that stretched its elastic material, decompression must reproduce an effect equal to that, which produced compression

5 [T.N.] Today, we would call it relative velocity.

as a cause: that is, a respective velocity, in the opposite direction, equal to the first. *The respective velocity of the elastic bodies is therefore, aftershock, the same as before.*

Let us now look for the Laws, according to which movement is distributed between two bodies that collide, whether these bodies are hard or elastic.

We shall deduce these Laws from a single Principle, and from this same Principle, we shall deduce the laws from their rest.

GENERAL PRINCIPLE

Whenever there is any change in Nature, the Amount of Action required for that change is as small as possible.

The Quantity *of Action* is the product of the mass of the bodies, their velocity and by the space they travel. When a body is transported from one place to another, the action is greater the greater the mass, the faster the velocity, and the longer the space through which it is transported.

PROBLEM I.

Finding the Laws of Hard Body Movement

Considering two hard bodies, whose masses are A & B, which move to the same side, with the speeds a & b: A faster than B, so that it reaches and clashes with it. Be the common velocity of these two bodies after the shock $= x < a \;\&> b$. The change in the Universe is that the body A that moves with speed a and that at a certain time crosses a space $= a$, starts to move with speed x and just travels through space $= x$: the body B that would move with speed b, and cover only a space $= b$, moves with speed x, travels in a space $= x$.

This change is therefore the same as would have happened, if while the body moves with speed a, and traverses a space $= a$, it would have been carried backwards in an immaterial plane, which would have moved $a - x$, by a space $= a - x$: and that the body moving with speed b, traverses the space $= b$, would have been transported in an immaterial plane, which would have moved with $x - b$, by a space $= x - b$.

Now that the bodies A e B move with adequate velocities in the moving planes, or are they at rest, the motion of these planes carrying the bodies being the same: the Quantities of Action, produced in Nature, will be $A(a - x)^2, \& \; B(x - b)^2$; the sum of which must be as small as possible. So we have

$$Aaa - 2Aax + Axx + Bxx - 2Bbx + Bbb = Mínimo$$

or

$$-Aadx + 2Axdx + 2Bxdx - 2Bbdx = 0$$

From which we get as a common speed

$$x = \frac{Aa + Bb}{A + B}$$

In this case, when the two bodies move on the same side, the amount of movement destroyed and the amount produced is equal: the total amount of movement remains, after the shock, the same as before.

It is easy to apply the same reasoning to the case where bodies move toward each other: or just consider it b as negative compared to a, and the common velocity will be

$$x = \frac{Aa - Bb}{A + B}$$

If it was at rest before the shock, $b = 0$; and the common velocity is

$$x = \frac{Aa}{A + B}$$

If a body encounters an unshakable obstacle, we can consider this obstacle as a body of an infinite mass at rest: If, therefore, B is infinite, speed $x = 0$.

Now let us see what should happen when bodies are elastic. The bodies I am going to talk about are the ones that have perfect elasticity.

PROBLEM II.

Finding the Laws of Elastic Body Movement?

Considering two elastic bodies, whose masses are A and B, which move to the same side, with the velocities a and b, A faster that B, so that it reaches and collides with it: and be α and β the velocities of the two bodies after the shock: the sum or difference of these velocities after the shock is the same as before.

The change in the Universe is that the body A, which moves with speed a, and which at a certain time crosses a space $= a$, starts to move with speed α, and only travels through a space $= \alpha$: the body B, which only moves with speed b, and only travels through a space $= b$, starts to move with speed β, and travels through a space $= \beta$.

This change is, therefore, the same as would have happened, if while the body A moves with speed a, and traverses space $= a$, it would have been carried backwards in an immaterial plane, which would have moved with $a - \alpha$, by a space $= a - \alpha$, and which the body B moves with speed b, and wandered through space $= b$, it would have been carried in an immaterial plane, which would have moved with $\beta - b$, by a space $= \beta - b$.

Now let the bodies A and B move with suitable velocities in the moving planes, or be them at rest, the motion of these planes carrying the bodies, being the same: the Quantities of Action, produced in Nature, shall be, $A(a-\alpha)^2$ & $B(\beta-b)^2$; the sum of which shall be the least possible. So we have

$$Aaa - 2Aa\alpha + A\alpha\alpha + B\beta\beta - 2Bb\beta + Bbb = Mínimo.$$

or

$$-2Aad\alpha + 2A\alpha d\alpha + 2B\beta d\beta - 2Bbd\beta = 0$$

Now, for the elastic bodies, being the respective velocity after the shock the same as before; we have $\beta - \alpha = a - b$, where $\beta = \alpha + a - b$ and $d\beta = d\alpha$: that being replaced in the previous equation, gives the velocities

$$\alpha = \frac{Aa - Ba + 2Bb}{A+B} \text{ and } \beta = \frac{2Aa - Ab + Bb}{A+B}$$

If the bodies move toward each other, it is easy to apply the same reasoning: either just consider b negative compared to a, and the velocities will be

$$\alpha = \frac{Aa - Ba - 2Bb}{A+B} \; E \; \beta = \frac{2Aa + Ab - Bb}{A+B}$$

If one of the bodies was at rest before the shock, $b = 0$ and the velocities are

$$\alpha = \frac{Aa - Ba}{A+B} \text{ and } \beta = \frac{2Aa}{A+B}$$

If one of the bodies is an unwavering obstacle, considering this obstacle as a body B of an infinite mass at rest; we will have the speed that is $\alpha = -a$, which is to say that the body A will bounce back with the same speed as it had when hitting the obstacle.

If we take the sum of the Living Forces, we will see that after the shock it is the same as it was before, that is:

$$A\alpha\alpha + B\beta\beta = Aaa + Bbb$$

Here, the sum of the Living Forces is preserved after the shock, but this conservation occurs only for elastic bodies, and not for rigid bodies. The general principle, which extends to both, is that *the amount of action required to cause any change in nature is the least possible.*

This Principle is so universal and fruitful that we derive from it the Law of Rest or Balance. There is no longer any difference here between hard and elastic bodies.

PROBLEM III.

Finding the Law of Body Rest?

I consider here the bodies attached to a lever and to find the point around which they remain in equilibrium. I am looking for the point around which, if the lever receives any small movement, the Quantity of Action will be as small as possible.

Let it be c the length of the lever, which I suppose immaterial, at the ends of which are placed two Bodies, whose masses are A and B. Be z the distance of the body A to the point sought and $c - z$ the distance of the body B: it is obvious that if the lever has any small movement, the bodies A and B will describe small arcs similar to each other and proportional to the distances of these bodies at the point we are looking for. These arches will therefore be the spaces traversed by the bodies and, at the same time, will represent their velocities. The Quantity of Action will therefore be proportional to the product [of the mass] of each body by the square of its arc or (since the arcs are similar) to the product [of the mass] of each body by the square of its distance from the point around which the lever revolves: that is, Azz and $B(c - z)^2$, the sum of which must be as small as possible. So we have

$$Azz + Bcc - 2Bcz + Bzz = Minimo.$$

Where we got

$$z = \frac{Bc}{A + B}$$

Which is the fundamental proposition of statics.

References

1. *Cicer. Tuscul. I.13*
2. *Tuscul. (I-28) & 29.*
3. *De Nat. of gold. II. 37. 38.*
4. *Newt. Opticks III. Book. Consultation 31.*
5. *Theol. Astron. of Derham. , Theol. Phys. of the same. , Lesser's Theol. of insects.*
6. *Fabricius' Theol. of Water.*
7. *See Mr. Dan. Bernoulli on the inclination. Planes of the orbits of the planets.*
8. *Lucret. lib. 4*
9. *Philos. Transaction No. 470*
10. *Descartes. Princip. Descartes' Man.*

11. Medit. Chret. & Metaph. by Fr. Malebranche Medit. VII.

12. Leibnitz. Theod. II. go. N. 224. 225.

13. Malebranche Medit. Chret. & Metaph. VII.

14. Pope. Ensaio sobre o homem.

15. Aristot. Physic Lib. VIII

16. Malebranche. Interviews about the metaphor. Maintenance

IV. Discussion about the primary source: what does it allow us to learn about Physics and about the Physics?[6]

The purpose of this section is to present our reflections on the primary source presented (which corresponds to step 2 of what was proposed in Table 1). These would be the reflections the teacher would take to the dialogue to tension the debate from the primary source. With this discussion, we hope to exemplify the wealth of reflections that arise from reading the primary source as well as contribute to teachers who intend to use this source in a Mechanics course for teacher training.

The primary source reached the research group when we learned that contemporary versions of Maupertuis' Principle of Minimum Action had historical importance in the development of Quantum Mechanics, whether in its relativistic formulation for Louis de Broglie's theory or non-relativistic for Schrödinger (LIMA; KARAM, 2021). Therefore, we decided to make a rescue of the original proposition, arriving at Maupertuis' own text. From the reading of the primary source, and in dialogue with the specialized literature, we systematized four possible axes of reflection.

a) Physics: what can we learn about Minimal Action from the text?

Nowadays, the principle of minimum action is usually presented in the context of Lagrangian Mechanics, through Hamilton's Principle (MARION, 2004):

$$\delta \int L dt = \delta \int (K - V) dt = 0 \qquad (1)$$

In which L is the lagrangian, given by the difference between the kinetic energy (K) and the potential energy (V). The integral is determined in relation to time t. Hamilton's principle, however, is later than Maupertuis's. Few books present this principle: an example is

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[6] The present work is configured as a theoretical proposition, without the objective of bringing an experience report. However, an example of discussion based on Maupertuis' text was held at the State Meeting of Physics Teaching, held in 2023 at the Federal University of Rio Grande do Sul. The seminar recording can be found at https://www.youtube.com/watch?v=cSPPbtKh7Qw&t=16s.

the book by Landau and Lifchitz (1969). In its contemporary form, the principle is expressed as follows:

$$\delta \int p.ds = 0 \tag{2}$$

That is, the integral of the moment p along the trajectory must be a minimum. This principle is more restrictive than Hamilton's principle and can be obtained as a particular case. One of the main difficulties when working with this principle is precisely the domain of calculus of variations, which is not simple and is usually only approached in the later Physics courses. Thus, the traditional approach of Mechanics course textbooks can be challenging. As has been known for decades, understanding the concept or description of the phenomenon before the mastery of mathematical formalism can be fundamental to understanding the subject (PIAGET, 1976). It is noteworthy that Maupertuis was able to approach the subject without using calculus of variations. If we use the principle as we know it today in cases such as collisions (elastic and inelastic) and static equilibrium, it is possible to describe the minimum action of systems without having to use the calculus of variations. That is, the three examples given by Maupertuis can be very important to introduce the subject, to give the notion of what is the minimization of action and to pave the way for formal discussion.

For this to be done, however, the teacher must use the examples of Maupertuis but needs to reconstruct his arguments in a contemporary conception, because – as Silva and Martins (2007) well pointed out – the mathematical argument used by Maupertuis is conceptually wrong. However, if we analyse the examples used by him, with the contemporary conception, we can write the minimization of action falling into a simple differential calculus problem, which is more usual for students of basic education.

Taking into account that we are talking about a mechanics course for undergraduate courses, the teacher is teaching the subject to students who know differential and integral calculus. In Classical Mechanics courses, variational calculus is usually introduced so that Hamilton's principle can be discussed. Using Maupertuis's examples allows one to use only differential calculus (as Maupertuis himself does) so that there is a didactic advantage in this aspect, as students will be able to use mathematical tools that they already know. In this sense, the episode becomes especially interesting to introduce the discussion and build a bridge for the introduction of Lagrangian and Hamiltonian mechanics.

b) Importance of error for Physics

Taking advantage of the discussion by Silva and Martins (2007), the debate on the role of error in the scientific enterprise is extremely conducive. In general, epistemological narratives tend to privilege the importance of experiments and predictions that work, which provide an increase in the explanation of empirical content (POPPER, 2002). When we look at historical sources, we recognize, however, that scientific development is based on errors as well. Although Maupertuis presented a conception that today we understand as wrong, his proposal of the existence of a magnitude minimized along the trajectory, associated to *momentum*, remained present in Mechanics and proved very fruitful, including for the development of Quantum Theory.

The writings of thinkers such as Paul Feyrabend and Thomas Kuhn can help us reflect on how non-rational, or extra-scientific, conceptions can contribute to scientific development. In particular, the discussion on the genetics of concepts by Foucault (2018) emphasises that in the genesis we do not find certainty, solid and correct ground, but doubt, error and deception. How can science promote so many results from mistakes? This is an extremely challenging issue that the historical episode imposes on us and that deserves to be debated in the didactic context.

Examples of how the discussion about the role of error can be introduced in the pedagogical context already exist in the literature. For example, Bagdonas, Zanetic and Gurgel (2018) rescue the problem of the cosmological constant in the classroom and show that many students thought that a person as intelligent as Einstein could not make mistakes. Thus, the proposal explicitly shows the potentiality of bringing this type of discussion to tension the conceptions about the nature of science.

c) Religious and Scientific Thought

Another important and surprising aspect of the text is its proximity to religious thought. The main purpose of the text was not to find a new physical law but to defend the existence of God. For Maupertuis, the existence of a single principle, from which we can derive the expressions of elastic, inelastic and equilibrium collision is proof that there is an intelligence behind the organization of the Universe.

Thus, contrary to the popular discourse that science and religion always hold a position of antagonism, Maupertuis's text reveals the possibility of religious thought motivating, inspiring and directing the scientific enterprise. This and other possibilities of a relationship between science and religion are discussed in the science education literature (BAGDONAS; SILVA, 2015). The scheme presented by Bagdonas and Silva (2015) allows us to look at these different possibilities from a theoretical framework based on the philosophy of science.

In the specific case of Maupertuis, we see that religious thought, or the search for the justification of God's existence, is closely connected to the search for a unifying principle behind the explanation of all physical phenomena. For him, it is not the diversity of characteristics that proves the existence of a creator, but precisely the possibility of unifying the most different movements in a single principle. Arguments like this, about the relationship between the multiplicity of experiences and a deeper underlying reality, permeate the entire history of science (FEYRABEND, 1988).

This discussion can be taken to the classroom to tension the discussions so that students reflect on how to interpret Maupertuis's work. In particular, in the context of

Mechanics, it is valid to bring to the debate the fact that Newton himself had a large number of mystical writings (FORATO, 2008) – which is usually neglected in the didactic context.

d) Role of non-scientific thinking today

Directly connected to the previous issue, the text introduces the reflection on whether the interrelationships between science with other areas of knowledge remain relevant in the current context and, even more, whether non-scientific knowledge itself has an important value for societal development and care. That is, the episode also opens space to put the myth of the superiority of technocratic decisions in suspicion (AULER; DELIZOICOV, 2001).

We can start with the previous discussions to further expand the reflections and, in particular, bring the specific debate generated on the Nature of Science to think about the contemporary context. Although it is not necessary to do this in every study of a historical episode, the present case allows such a connection to be made. Moreover, as we argue, each episode presents a unique set of possibilities for reflection. In this case, the presence of strong religious thinking based on a very important concept for Classical and Quantum Mechanics gives us the possibility to rethink the current political-epistemic pact and reflect on the role of specialists and non-specialists in contemporary society.

The idea is not to present a definitive answer to this question, but precisely to make it possible to reflect on the different epistemological positions and the political commitments they carry. In doing so, we allow the teaching of an extremely technical topic (principle of minimum action) to transit through epistemological and political reflections, giving rise to an analysis of our contemporary society. In this sense, we reinforce the potential of this approach as a promoter of epistemic autonomy and non-alienated subjects. We do not need to avoid discussing Physics to enter the political dimension of science, nor do we need to follow the current hegemonic didactic model in which the social and cultural crossings of science are ignored.

Regarding this particular topic, we see two main positions (which can represent extreme poles in a continuous spectrum of positions). Some authors defend the fundamental importance of science for the development of humanity, for the mitigation of inequities and for overcoming climate mutations. Bruno Latour (2020), for example, explicitly defended the scientific network as opposed to other forms of knowledge circulation in the contemporary world, especially referring to the post-truth context inaugurated with Donald Trump's election.

Other authors, however, reinforce the importance of rescuing worldviews developed in other communities, outside the scientific context, approaching a more integrated conception of nature, distinctive from the typical extractive conception of the modern world. Pignarre and Stengers (2011) discuss, for example, the contribution of contemporary magical thinking to move away from the worldview of the consumer society. That is, again, Maupertuis's text gives rise to a deep reflection with a direct impact on our perception of contemporary problems.

As we intend to highlight in this section, the reflections arising from the reading of Maupertuis's text are the result of the propositions made by the author himself in conjunction with the vision surplus of the interpreters (BAKHTIN, 2016). Other historical episodes may not have such an obvious relationship with the question of religiosity or the importance of error. On the other hand, other interpreters may favour other aspects of the text, ask other questions and have other concerns.

Therefore, we emphasise the importance of letting students have first contact with the primary source and trace their reflections. Eventually, some reflections will fall within the scope presented here (with lesser or greater dialogue with the literature), or perhaps other reflections and questions will be raised. From the encounter of the text with the vision surplus of students and teachers, a unique, singular, and deep knowledge of Physics and about Physics can be built, respecting the epistemic autonomy of all subjects.

V. Final Considerations

In this paper, we present a theoretical-methodological reflection on the use of primary sources in Physics and science teaching, especially in the context of teacher training. Starting with Freire's concept of epistemic autonomy, we defend the importance of starting with the interpretations of primary sources made by the students themselves. In doing so, different views are allowed to be raised and debated. In dialogue with Bakhtinian philosophy, we understand that a text gives rise to virtually infinite interpretations and, therefore, the didactic context can be a space for the encounter and dialogue of these reflections.

The teacher, in turn, from his vision surplus, organises his reflections to tension and enrich the debate, contributing to the analysis of the historical episode, to the learning of Physics and about Physics. In doing so, in addition to valuing epistemic autonomy, we seek to overcome the dichotomy between technical and pedagogical knowledge, and we adhere to a post-structuralist description of the nature of science, favouring the description of episodes, in dialogue with what has been discussed in the context of Social Studies of Sciences, especially in the Ontological turn of Anthropology, and in the Cultural History of Science.

To exemplify the potentiality and richness of the use of primary sources from this conception, we present a primary source translated from French, *The Laws of Movement and Rest,* by Maupertuis. In this text, the author's conception of the principle of minimum action is presented for the first time, which had an important influence not only on Classical Mechanics but also on Quantum Theory.

Next, we present our reflections on the episode, which would correspond to the teacher's analysis, which would be taken to tension discussions with students. We present our analysis in four axes, indicating how – from the text – we can benefit from learning Physics and about Physics. As we reinforce throughout the work, the learnings listed here are specific to the episode adopted and refer to the vision surplus of the authors. Other episodes and other interpreters will privilege other aspects and dimensions of Physics.

We hope, therefore, to contribute primarily to the pedagogical debate on the use of history and philosophy of science from new theoretical frameworks for Science Education. In addition, we understand that the work contributes to the translation and analysis of a primary source of historical interest, with important contributions to fundamental concepts in Physics. Our analysis also raises questions that can be explored in the didactic context and presents possible works that can contribute to the reflection on such questions.

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References

ALLCHIN, D. Pseudohistory and Pseudoscience. **Science & Education**, v. 13, n. 3, p. 179-195, 2004.

ARAUJO, R. S.; VIANNA, D. M. A história da legislação dos cursos de Licenciatura em Física no Brasil: do colonial presencial ao digital a distância. **Revista Brasileira de Ensino de Física**, 2010.

AULER, D. **Cuidado! Um cavalo viciado tende a voltar para o mesmo lugar**. Curitiba: Appris, 2018.

AULER, D.; DELIZOICOV, D. Alfabetização científico-tecnológica pra quê? **Ensaio: Pesquisa em Educação em Ciências**, v. 3, n. 1, p. 122-134, 2001.

BAGDONAS, A.; ZANETIC, J.; GURGEL, I. O maior erro de Einstein? Debatendo o papel dos erros na ciência através de um jogo didático sobre cosmologia. **Caderno Brasileiro de Ensino de Física,** v. 35, n. 1, p. 97-117, 2018.

BAGDONAS, A.; SILVA, C. C. Enhancing Teachers' Awareness About Relations Between Science and Religion. **Science & Education**, v. 24, n. 9, p. 1173-1199, 2015.

BAKHTIN, M. **A Estética da Criação Verbal**, 1990.

BAKHTIN, M. **Os Gêneros do Discurso**. São Paulo: Editora 34, 2016.

BATISTA, G. L. de F.; DRUMMOND, J. M. H.; FREITAS, D. B. Fontes primárias no ensino de física: considerações e exemplos de propostas. **Caderno Brasileiro de Ensino de Física**, v. 32, n. 3, p. 663-702, 2015.

BOSS, S. L. B.; SOUZA FILHO, M. P.; CALUZI, J. J. Fontes Primárias e aprendizagem signficativa: aquisição de subsunçores para a aprendizagem do conceito de carga elétrica. *In*: ENCONTRO NACIONAL DE PESQUISA EM EDUCAÇÃO EM CIÊNCIAS, VII, 2019. p. 1-12.

CASTRO, E. V. de. **Metafísicas Canibais**: elementos para uma antropologia pós-estrutural. São Paulo: Ubu, 2018.

FEYERABEND, P. **Farewell to Reason**. New York: Verso, 1988.

FONSECA, D. S. *et al.* Pressão atmosférica e natureza da ciência: uma sequência didática englobando fontes primárias. **Caderno Brasileiro de Ensino de Física**, v. 12, n. 1, p. 145, 2017.

FORATO, T. C. de M. A filosofia mística e a doutrina newtoniana: uma discussão historiográfica. **Alexandria: Revista de Educação em Ciência e Tecnologia**, v. 1, n. 3, p. 29-53, 2008.

FOUCAULT, M. **Microfísica do Poder**. Rio de Janeiro: Paz e Terra, 2018.

FREIRE, P. **Pedagogia do Oprimido**. Rio de Janeiro: Terra e Paz, 2013 a.

FREIRE, P. **Pedagogia da Autonomia**: saberes necessários à prática educativa. Rio de Janeiro: Paz e Terra, 2013 b.

HOOKS, B. **Ensinando a transgredir**: a educação para prática da liberdade. São Paulo: Martins Fontes, 2013.

KARAM, R. Considerações metodológicas sobre o uso de fontes primárias no ensino de Física. **Revista Brasileira de Ensino de Ciências e Matemática**, v. 4, n. 3, 2021.

KARAM, R.; LIMA, N. W. Using History of Physics to teach Physics? *In*: GUISASOLA, J.; MCLOUGHLIN, E. (Org.). **Connecting Research in Physics Education with Teacher Education 3**. Dublin: The International Commission on Physics Education, 2022. p. 22-38.

KOHN, E. Anthropology of Ontologies. **Annual Review of Anthropology**, v. 44, n. 1, p. 311-327, 2015.

KRENAK, A. **Ideias para Adiar o Fim do Mundo**. São Paulo: Companhia das Letras, 2019.
LANDAU, L. L.; LIFCHITZ, E. **Mécanique**. Moscou: Éditions Mir, 1969.

LATOUR, B. **Reassembling the Social**: An Introduction to Actor Network Theory. Oxford: Oxford University Press, 2005.

LATOUR, B. **Onde Aterrar? Como se orientar politicamente no Antropoceno?** Rio de Janeiro: Bazar do Tempo, 2020.

LIMA, N.; KARAM, R. Particle velocity = group velocity: A common assumption in the different theories of Louis de Broglie and Erwin Schrödinger. **American Journal of Physics**, v. 89, n. 5, p. 521-528, 2021.

LIMA, N. W. *et al.* A Teoria do Enunciado Concreto e a Interpretação Metalinguística: Bases Filosóficas, Reflexões Metodológicas e Aplicações para os Estudos das Ciências e para a Pesquisa em Educação em Ciências. **Investigações em Ensino de Ciências**, v. 24, n. 3, p. 258-281, 2019.

LIMA, N. W. Histórias Plurais para a Construção de um Mundo Comum: como História, Filosofia e Sociologia das Ciências na Educação em Ciências Podem Contribuir para Construção do Mundo Pós-Pandemia. **Revista Brasileira de Ensino de Ciências e Matemática**, 2021.

LIMA, N. W. ; GUERRA, A. Superando Narciso: histórias das ciências para adiar o fim do mundo. **Revista Brasileira de História da Ciência**, v. 15, n. 2, p. 386-399, 2022.

MARION, J. **Classical Dynamics of Particles and Systems**. New York: Brooks/Cole, 2004.

MASSONI, N. T.; BRUCKMANN, M. E.; ALVES-BRITO, A. A Reestruturação Curricular do curso de Licenciatura em Física da UFRGS: construção de novas identidades na formação docente inicial do século XXI. **Revista Educar Mais**, v. 4, n. 3, p. 512-541, 2020.

MATTHEWS, M. R. History, philosophy, and science teaching: The present rapprochement. **Science & Education**, v. 1, n. 1, p. 11-47, 1992. Disponível em: https://doi.org/10.1007/BF00430208

MOREIRA, I. de C. Maupertuis (1698-1759) e o Princípio de Mínima Ação. **Revista Brasileira de Ensino de Física**, v. 21, n. 1, p. 172-186, 1999.

MOURA, C. Para que história da ciência no ensino? Algumas direções a partir de uma perspectiva sociopolítica. **Revista Brasileira de Ensino de Ciências e Matemática**, v. 4, n. 3, 2021.

MOURA, C. B. de; GUERRA, A. Cultural History of Science: A Possible Path for Discussing Scientific Practices in Science Teaching? **Revista Brasileira de Pesquisa em Educação em Ciências**, v. 16, n. 3, p. 749-771, 2016.

MOURA, C. B. de; GUERRA, A.; CAMEL, T. A natureza da ciência pelas lentes do currículo: normatividade curricular, contextualização e os sentidos de ensinar sobre ciências. **Ensaio: Pesquisa em Educação em Ciências**, v. 22, p. 1-27, 2020.

OLIVEIRA, O. B. de. Em defesa da leitura de textos históricos na formação de professores de ciências. **Pro-Posições**, v. 22, n. 1, p. 71-82, 2011. Disponível em: https://doi.org/10.1590/s0103-73072011000100007

PIAGET, J. **Para Onde Vai a Educação**. Rio de Janeiro: Livraria José Olympio Editora, 1976.

PIGNARRE, P.; STENGERS, I. **Capitalist Sourcery**: Breaking the Spell. London: Macmillan Publishers Limited, 2011.

POPPER, K. **The Logic of Scientific Discovery**. New York: Routledge, 2002.

SANTOS, B. de S. **Um Discurso sobre as Ciências**. 5. ed. São Paulo: Cortez, 2008.

RAMOS, M. C. Origem da vida e origem das espécies no século XVIII: as concepções de Maupertuis. **Scientiae Studia,** v. 1, n. 1, p. 43-62, 2003.

SANTOS, B. de S. **O Fim do Império Cognitivo. A Afirmação das Epistemologias do Sul**. Belo Horizonte: autêntica, 2019.

SILVA, A. P. B.; GUERRA, A. **História da Ciência e Ensino**: Fontes primárias e propostas para sala de aula. São Paulo: Editora Livraria da Física, 2015.

SILVA, A. P. B.; MARTINS, R. de A. Maupertuis e o princípio mecânico de ação mínima:

uma análise crítica. **Revista Brasileira de Ensino de Física**, v. 29, n. 4, p. 625-633, 2007.

SILVA, T. **Documentos de Identidade**: Uma Introdução às teorias do currículo. Belo Horizonte: Autentica, 2010.

SOUZA, G. T. **Introdução à Teoria do Enunciado Concreto do círculo Bakhtin/Volochinov/Medvedev**. 2. ed. São Paulo: Humanitas/FFLCH/USP, 2002.

STENGERS, I. A proposição cosmopolítica. **Revisto do Instituto de Estudos Brasileiros**, n. 69, p. 442-464, 2018.

TEIXEIRA, E. S.; GRECA, I. M.; FREIRE, O. The History and Philosophy of Science in Physics Teaching: A Research Synthesis of Didactic Interventions. **Science & Education**, v. 21, n. 6, p. 771-796, 2012.

VENEU, A.; FERRAZ, G.; REZENDE, F. Análise do Discurso no Ensino de Ciências: Considerações Teóricas. Implicaçoes Epistemológicas e metodológicas. **Ensaio: Pesquisa em Educação em Ciências**, v. 17, n. 1, p. 126-149, 2015.

VENTURINI, T. Diving in magma: how to explore controversies with actor-network theory. **Public Understanding of Science**, v. 19, n. 3, p. 258-273, 2010.

WOOLGAR, S. Laboratory Studies: a comment on the state of the art. **Social Studies of Science**, v. 12, p. 481-498, 1982.