

# REFERRING TO NOTHING

OTÁVIO BUENO

*Department of Philosophy, University of Miami, Coral Gables, FL, USA*

*otaviobueno@mac.com*

<https://orcid.org/0000-0002-9161-4205>

---

**Abstract.** Typical accounts of reference demand that referring terms denote existent objects. This assumption is shared by theories across a variety of areas of philosophy, in particular, direct reference views in philosophy of language; neo-Fregean conceptions in the philosophy of mathematics, and easy-ontology approaches in metaphysics. In this paper, this assumption is resisted and the significance and the possibility of referring to the nonexistent is highlighted. After identifying difficulties in all these three theories and resisting a free-logic approach, ontologically neutral quantifiers, which do not require the existence of what is quantified over, are suggested as providing a better conception. It is concluded that the difficulties raised to the previous theories do not affect the ontologically neutral approach, while the approach, properly conceived, allows for nonexistent objects to have properties.

**Keywords:** reference • quantification • direct reference • neo-Fregeanism • easy ontology • nonexistent objects

---

RECEIVED: 01/04/2024

ACCEPTED: 01/09/2024

## 1. Introduction

It is a typical assumption of accounts of reference to require that referring terms denote what exists: one cannot refer to what does not exist, since, in this case, there is nothing to be referred to. This feature is shared by a variety of theories of reference, in particular direct reference theories (and, to a certain extent, descriptivism). This is also an assumption shared by certain metaphysical theories, such as neo-Fregeanism, *via* the connection they impose on truth and reference: a true statement, in an extensional context, in which a referring singular term is used guarantees the existence of the objects that are referred to. (In fact, this is the central form of argument for the existence of mathematical objects found in the heart of neo-Fregeanism; see Hale & Wright 2001). Similar, albeit importantly different, considerations are also found in easy approaches to ontology (Thomasson 2015).

In this paper, this assumption is resisted, and it is argued for the importance and possibility of referring to the nonexistent. This is done, first, by highlighting difficulties faced by each of these three approaches as well as by proposals articulated in terms of free logic. The latter rightly aim to carve out a space between descriptivism



and direct reference theories (Sainsbury 2005). It is argued that free logic is unable to deliver the technical machinery that is ultimately needed for the task. As a better resource, ontologically neutral quantifiers are recommended (Azzouni 2004 and Bueno 2005). These are quantifiers that do not require the existence of what is quantified over, and it is argued that none of the difficulties raised to the previous approaches are faced by the conception in terms of ontologically neutral quantification. Finally, in contrast to Jody Azzouni's (2010) approach, I argue that it is important to make sense of how nonexistent objects, despite their nonexistence, still have properties, so that in referring to the nonexistent one can still refer to different objects. In the end, it is possible to detach reference from existence so that it is perfectly possible and coherent to refer to something without requiring the existence of what is referred to.

## 2. The Primacy of Reference: Mill

The link between reference and existence is clearly articulated in a Millian, direct reference, view. In his *System of Logic*, John Stuart Mill (1843) notes:

Every proposition consists of three parts: the Subject, the Predicate, and the Copula. The predicate is the name denoting that which is affirmed or denied (Mill 1843, p.19; see Sainsbury 2005, p.4).

The denotation in question is of something. In the case of proper names, reference is made directly to the objects that are talked about, independently of the way in which they may have been characterized, that is, independently of the attributes or properties the objects may have. As Mill points out:

Proper names [...] denote the individuals who are called by them; but they do not indicate or imply any attributes as belonging to those individuals (Mill 1843, p.34; see Sainsbury 2005, p.4).

This is the key trait of the direct reference view: names denote directly. Yet, why does the denotation of a proper name require the existence of what is denoted (or referred to)? Because otherwise there is no way of specifying what it is that one is referring to. Since proper names “denote the individuals who are called by them” but “do not indicate or imply any attributes as belonging to those individuals” (Mill 1843, p.34), there is nothing but the sheer existence of these individuals to secure reference to them.

But is Mill committed to such an existentially loaded view? It might be argued that he is not. After all, as he insists: “All names are names of something, real or imaginary” (Mill 1843, p.27; see Sainsbury 2005, p.6). The emphasis on the imaginary in this context is significant. It could be argued that Mill need not be committed here to the existence of imaginary (nonexistent) objects, but only to the claim that we can

imagine, of a name that does not denote, that it does denote (Sainsbury 2005, p.7, note 4).

However, this move poses a difficulty for Mill, since it is unclear how (direct) reference can be secured given that names are taken to refer independently of any particular specification of the properties (attributes) that the objects that are referred to may have. If the object that is referred to fails to exist, it becomes unclear how to secure reference to the intended object. Neither Hamlet nor Anna Karenina exists but there is a significant difference between referring to one or to the other. That difference is easily explained by noting that they have different properties (e.g., one is a man, the other is a woman). But this requires some mediation: one cannot refer directly to one or the other of these objects without prior specification of at least some of the properties that the objects in question are supposed to have. Thus, in the case of nonexistent objects, direct (non-mediated) reference does not seem to be possible, in contrast with the requirements imposed by direct reference theories.

### 3. The Primacy of Reference: Neo-Fregeanism

More recently, neo-Fregean views have been developed as an attempt to provide a particular philosophy of mathematics, and they are also committed to the link between reference and existence (Hale & Wright 2001). The link is implemented via truth: a true statement, in an extensional context in which a referring singular term occurs, requires the existence of the relevant object. Bob Hale and Crispin Wright articulate the claim clearly:

We start from two ideas. First: *objects* [...] just are what (actual or possible) singular terms refer to. Second: no more is to be required, in order for there to be a strong prima-facie case that a class of apparent singular terms have reference, than that they occur in true statements free of all epistemic, modal, quotational, and other forms of vocabulary standardly recognized to compromise straightforward referential function. For if certain expressions function as singular terms in various true extensional contexts, there can be no further question but that those expressions have reference, and, since they are singular terms, refer to objects (Hale & Wright 2001, p.8).

In other words, reference of singular terms in true extensional statements requires existence of the objects that are referred to: truth demands existence.

Hale and Wright continue:

The underlying thought is that—from a semantic point of view—a singular term just *is* an expression whose function is to effect reference to an object, and that an extensional statement containing such terms cannot be true unless those terms successfully discharge their referential function. Provided,

then (as certainly appears to be the case), there are true extensional statements so featuring numerical singular terms, there are objects—numbers—to which they make reference (Hale & Wright 2001, p.8).

In this way, the existence of mathematical objects follows from true extensional statements containing referring numerical singular terms.

This proposal, nonetheless, does not seem right. Consider, in the context of classical mathematics, the Russell set: the set of all sets that are not members of themselves (that is,  $R = \{x : x \notin x\}$ ). The following statement about  $R$  is true: “The Russell set is extremely large”. This is a true extensional statement that refers to a particular set (namely, the Russell set), but no such set exists (if classical set theory is consistent)! Thus, truth does not require existence (even in an extensional context).

In response, neo-Fregeans may insist that since the Russell set does not exist, the singular term “Russell set” does not refer and, hence, the statement “The Russell set is extremely large” is false, adopting Russell’s own theory of descriptions. After all, given this theory, the statement in question is analyzed as:  $\exists x((Rx \wedge Lx) \wedge \forall y(Ry \rightarrow y = x))$ , in which ‘ $R$ ’ is the predicate “non-self-membered set”, ‘ $L$ ’ is the predicate “is large”.

Nevertheless, this is not right either. Arguably, “The Russell set is a set” is true even if the Russell set does not exist. After all, it is unclear what else the Russell set could be but a set. To deny this just flies in the face of what sets are. Furthermore, arguably, “The Russell set is blue” is false, since sets, in their usual understanding, are abstract objects and, as such, they are neither located in spacetime nor are they causally inactive. Hence, sets are not the kind of things that could have colors, even in principle. This is the case even if the Russell set does not exist. In the end, it is the neo-Fregean link between reference and existence that generates these difficulties. By dropping this link, the grounds for the emergence of these problems vanish.

#### 4. The Primacy of Reference: Easy Ontology

More recently yet, Amie Thomasson has defended an easy approach to ontology (Thomasson 2015). On her view, one can easily settle certain ontological disputes by reasoning from undisputed, uncontroversial truths via apparently straightforward steps—easy inferences—to obtain ontological conclusions (Thomasson 2015, p.21). Typically, she argues, the resolution of debates over certain easy existence questions ends up relying either on conceptual truths or on empirical information, depending on the question under consideration. Nothing more than information of this sort is required for the resolution. As she remarks:

[S]ome existence questions may be resolved conceptually (starting from a conceptual truth and engaging in easy inferences), while others also make

use of empirical work to gain knowledge of the uncontroversial truth fed into the easy inference. Treating existence questions as resolvable by way of easy inferences like these [...] requires of us nothing more than conceptual and empirical work to resolve ontological questions (Thomasson 2015, p.21).

In this way, one can demystify ontological debates, resisting the impression that they are fundamentally intractable and unresolvable.

As an illustration of the sort of considerations that Thomasson has in mind, consider, for instance, the following cluster of arguments (see Thomasson 2015, p.21; I indicate in boldface the premise and conclusions that Thomasson explicitly states; the added ones are presumably implicitly assumed):

**(P1) The cups and saucers are equinumerous.**

(P2) Two items are equinumerous as long as they have the same number.

**(C1) Hence, the number of cups equals the number of saucers.**

(P3) The number of cups equals the number of saucers.

(P4) If the number of cups equals the number of saucers, then there is a number of cups and saucers.

(P5) If there is a number of cups and saucers, then there are numbers.

**(C2) Therefore, there are numbers.**

In this way, according to the easy approach to ontology, just by providing conceptual considerations, the existence of numbers can be established *via* easy inferences.

It is important to highlight that if they work as intended, easy inferences from (C1) to (C2) do require that reference entails existence. After all, by referring to the equality of the number of cups and saucers, the argument above is supposed to extract the commitment to the existence of numbers.

It turns out, however, that on their own easy inferences need not commit one to the existence of objects, whether they are numbers or not. For the number of cups can equal the number of saucers *even if there are no numbers*. Consider the revised (but still easy) argument:

**(P1) The cups and saucers are equinumerous.**

(P2)' Two items are equinumerous as long as there is a one-to-one correspondence between them.

(C1)' Hence, there is a one-to-one correspondence between the cups and saucers.

(P3)' If there is a one-to-one correspondence between the cups and saucers, then there are as many cups as there are saucers.

(C2)' Therefore, there are as many cups as there are saucers.

As the revised argument makes clear, from the equinumerosity of cups and saucers, it follows that there are as many of the former as there are of the latter. It need not follow that numbers exist, though. Nor are functions required to exist as one need not interpret a one-to-one correspondence set-theoretically; such correspondence can be formulated in terms of second-order logic alone. In fact, to express in a second-order language that there is a one-to-one correspondence between  $F$  and  $G$ , all we need to state is that (Boolos 1998):

$$\begin{aligned} \exists R[ \forall x \forall y \forall z (Rxy \ \& \ Rxz \rightarrow y = z) \ \& \ \forall x \forall y \forall z (Rxz \ \& \ Ryz \rightarrow x = y) \\ \& \ \forall x (Fx \rightarrow \exists y (Gy \ \& \ Rxy)) \ \& \ \forall y (Gy \rightarrow \exists x (Fx \ \& \ Rxy)) ]. \end{aligned}$$

As a result, it is possible to resist the ontological implications of arguments based on easy ontology.

So far, proposals as different as Millian, direct reference views in philosophy of language, neo-Fregeanism in philosophy of mathematics, and easy ontology in metaphysics all share the primacy of reference, the commitment to the existence of what is referred to. As indicated, concerns arise in each case regarding how to accommodate reference to the nonexistent:

- (a) It is unclear how direct reference, Millian theorists can distinguish between nonexistent entities.
- (b) It is unclear how neo-Fregeans can accommodate the fact that true extensional statements in which referring singular terms occur do not entail the existence of the relevant objects. After all, the terms may refer to objects, such as the Russell set, that, according to the neo-Fregeans own ontological stance, do not exist.
- (c) It is unclear how easy ontologists can accommodate the fact that easy inferences can be recast in a way that they need not entail the existence of numbers—the recast inferences go through even if numbers do not exist.

These remarks all involve, in their respective ways, reference to nonexistent objects. They suggest that, going forward, what is needed is a way of making sense of reference to the nonexistent.

## 5. Free Logic

One could think that free logic provides the proper setting to resolve this issue, since it has been designed to deal with lack of ontological commitment, including to what fails to exist (for discussion, see Sainsbury 2005 and Lambert 2004). This is a logic free of the existential import found in classical logic. Due to this import, classical logic validates sentences such as the following:

- (i)  $\exists x x = a$
- (ii)  $\exists x (Fx \vee \neg Fx)$

After all, it is assumed that: (a) in every interpretation of the language, each individual constant is assigned to an object in the domain of interpretation, and (b) no interpretations with empty domains are allowed (Sainsbury 2005, p.64). This secures the existence of an object that is  $a$ , for any individual constant  $a$  (ensuring the validity of (i)) and the existence of an object that is either  $F$  or not- $F$  (ensuring the validity of (ii)).

Nonetheless, if ' $a$ ' is Vulcan—the (nonexistent) planet that was posited to explain the anomaly in the perihelion of Mercury—it is false to state that there exists an object identical to Vulcan ( $\exists x x = a$ ), given that Vulcan does not exist. And if there are no objects in the domain of interpretation, it is not the case that there exists an object that is  $R$  or not- $F$  (and ' $\exists x (Fx \vee \neg Fx)$ ' fails). Moreover, if ' $F$ ' is a vague predicate (such as 'is bald'), it is similarly false to state that there exists an object that is either bald or non-bald. After all, suppose that the object in question is a balding person who is neither definitely bald nor definitely non-bald. The person is, thus, in the penumbra between bald and non-bald, and, thus, ends up being neither bald nor non-bald.

A free logic avoids the existential import imposed by (the semantics of) classical logic by changing the classical quantifier rules as follows (Sainsbury 2005, p.65):

*Universal Instantiation:* From  $\forall x Fx$ , infer  $Fa$ —provided that  $\exists x x = a$  (that is, if there exists an object that is  $a$ ).

*Existential Generalization:* From  $Fa$ , infer  $\exists x Fx$ —provided that  $\exists x x = a$  (i.e., as long as there exists an object that is  $a$ ).

In this way, no existential import results from logic alone. (i) The sentence ' $\exists x x = a$ ' is not logically valid since the object denoted by ' $a$ ' need not exist—the domain of interpretation may be empty. (ii) For the same reason, ' $\exists x (Fx \vee \neg Fx)$ ' is not logically valid either as there may not be any objects in the domain of interpretation and, thus, no objects that are  $F$  or not- $F$ . If the denotation of an individual constant ' $a$ ' can be empty, free logic allows one to refer to an object without assuming its existence. This is a significant outcome considering the tight connection, noted above, between reference and existence that is found in Millian (direct reference) views, neo-Fregeanism, and easy ontology approaches.

There are at least two problems with approaches based on free logic. (a) The overall conception is ultimately incoherent, since in order to be formulated, it presupposes the interpretation of the quantifiers that it is meant to avoid. In fact, to generate classical logic's rules of inference, an ontologically committing reading of the formalism is required. Recall free logic's recapture of two rules from classical logic:

*Universal Instantiation:* from  $\forall xFx$ , infer  $Fa$ , **provided that  $\exists x x = a$ .**

*Existential Generalization:* from  $Fa$ , infer  $\exists xFx$ , **provided that  $\exists x x = a$ .**

Consider the clauses in boldface above and note that the sentence whose validity free logic is supposed to violate— $\exists x x = a$ —needs to be interpreted in a way that undermines free logic's central motivation. That is, the sentence must be read in an existentially loaded way, stating that the object  $a$  exists. After all, if  $a$  does not exist, Universal Instantiation and Existential Generalization are violated. Thus, the same sentence— $\exists x x = a$ —is interpreted as being ontologically loaded (so that classical logic's rules of inference are recaptured) and as *not* being ontologically loaded (so that an ontologically free interpretation can be offered). It is unclear how the semantics for free logic can be coherently articulated given the need for interpreting the same sentence in such conflicting ways: the existentially committing reading undermines the ontologically free one, and vice versa.

It may be argued that in the metalanguage the quantifier is interpreted in an ontologically loaded way, whereas this is not the case of the object-language quantifier. As a result, the two readings of the quantifier can be kept apart. This move, nonetheless, does not work. Both the ontologically free interpretation of the quantifier and the ontologically loaded one are formulated at the metalanguage. The ontologically free reading is implemented by allowing for interpretations of the object language with empty domains. Such domains are those that have no members, that is,  $\neg\exists x(x \in D)$ , where ' $D$ ' is the domain of interpretation. Clearly, this is a metalinguistic device—carried out at the level of the semantics—not a feature of the object language. And an ontologically loaded reading of the quantifier is required in order to secure an ontologically free reading of the object language. Similarly, it is also at the metalanguage that ontologically loaded reading is implemented. And this reading is required to obtain classical logic's quantificational inference rules. Hence, this forces a radical ambiguity in the interpretation of the quantifiers: both ontologically loaded and ontologically free readings are assigned to the same existential quantifier.

(b) The strategy to avoid ontological commitment in free logic relies on a shift in the *semantics* for the logic rather than in the object language itself. As we saw, the domain of interpretation for a free logic allows for empty sets: so, there is *nothing* in such sets. But to express such lack of commitment, an ontologically committing reading of the quantifier in the metalanguage is required: no object *exists* in the domain of interpretation. Thus, to secure the existence of an empty set in the domain of interpretation of a free logic, an ontologically committing interpretation of the metalanguage for that logic is needed. There is a tension between what free logic's object language is meant to accomplish (namely, to avoid an ontologically loaded language) and the resources required from the metalanguage (that is, an ontologically loaded language). The metalanguage takes back what the object language aims to achieve. Something needs to go.



## 6. Ontologically Neutral Quantifiers

There is a better alternative to implement the kind of neutrality that free logic searches for: one should use ontologically neutral quantifiers (Azzouni 2004 and Bueno 2005). These quantifiers do not require the existence of what is quantified over. Hence, quantification and existence are effectively distinguished.

Quantifying over what does not exist is usual, ubiquitous, unproblematic. No one supposes the existence of Sherlock Holmes when it is said that he is a detective not a superhero. Nor is a sociologist committed to the existence of moms with 2.4 children when it is asserted that average moms, in a certain population, have that many children. Physicists are likely to demur if one insists that their use of, and quantification over, Hilbert spaces in the formulation of quantum mechanics commits them to the existence of vectors in a multi-dimensional space. They will note that this is just part of the mathematics, not the physics.

If quantification and existence are conflated, one is also led to troublesome conclusions. Speaking about the set of all sets that are not members of themselves (the Russell set), classical set theorists may claim: “There are sets that are too big to exist”, or “Some sets (such as the Russell set) do not exist”. If the quantifiers in these sentences are interpreted as being ontologically committing, the sentences become contradictory. Set theorists would, in fact, be saying: “There exist sets that do not exist”. Clearly, this is not intended. Classical set theorists are not contradicting themselves when they deny the existence of the Russell set. Rather, they are simply identifying, among the sets, one that does not exist.

The problem vanishes if quantifiers are ontologically neutral. In this case, quantifiers only specify the range of objects that are quantified over—all objects in the domain or some of them—not whether these objects exist or not. In this way, one can avoid the conflation of two very different roles in quantification, namely, the determination of the range of quantification (all or some objects) and the specification of what exists. To avoid the conflation, conditions for quantification are introduced that do not require the existence of what is quantified over, and existence is expressed by a suitable predicate in the language—an existence predicate (what else could it be?). The quantificational conditions are:

*Existential quantifier:* some objects in the domain of interpretation are quantified over.

*Universal quantifier:* all objects in the domain of interpretation are quantified over.

Instead of being expressed by quantification, existence is characterized by a predicate, *E*, of existence for which sufficient (but not necessary) conditions are offered.

Otherwise, one ends up begging questions against metaphysical views. Typically, *being causally active* or *being located in spacetime* are sufficient conditions for existence. Objects that clearly satisfy them arguably exist. However, if these conditions are also taken to be *necessary*, platonists will understandably complain since, on their view, abstract objects exist despite the fact that they lack these properties.

As a result, the use of ontologically neutral quantifiers supports a form of agnosticism about such controversial entities. Perhaps these entities exist, perhaps they do not. The issue need not be settled as a requirement to provide an account of mathematics (or other fields in which abstract objects are involved). With the distinction between quantification and existence in place, it is also unproblematic to accommodate claims such as “Some sets do not exist”:  $\exists x(Sx \wedge \neg Ex)$ , in which ‘S’ is the predicate *is a set*, ‘E’ is the existence predicate, and the quantifier is ontologically neutral.

With the use of ontologically neutral quantifiers, the problems faced by the views that have been previously discussed do not arise. (A) The Millian, direct reference theorist, we noted, has problems distinguishing reference to distinct nonexistent objects, such as Hamlet and Anna Karenina, given the commitment to direct reference. When Hamlet and Anna Karenina are referred to, they are not confused, since one relies on the characterizing properties of these objects, namely, the properties that specify what these objects are, and which are provided in the corresponding fiction (see Bueno & Zalta 2005). Since the characterizing properties of Hamlet and Anna Karenina are different, one does not confuse reference to either of them. Reference is mediated by the characterizing properties; it is unclear that there is any other way to refer to such nonexistent objects. And with the use of ontologically neutral quantifiers, despite the quantification over these objects, no assumption about their existence is made.

(B) As we saw, Neo-Fregeans have problems with true extensional statements containing singular terms that refer to objects that they take not to exist, such as “The Russell set is extremely large”. With ontologically neutral quantifiers, statements of this kind pose no problem since one can quantify over objects independently of any commitment to their existence. And since the Russell set is determined by its characterizing property, namely,  $R = \{x : x \notin x\}$ , whether it exists or not, it is indeed extremely large given that so many sets are not members of themselves.

(C) As we also saw, it is not clear how easy ontologists can make room for the fact that easy inferences can be recast without entailing the existence of numbers. Recall the easy inferences (Thomasson 2015, p. 21):

(P1) The cups and saucers are equinumerous.

(C1) Hence, the number of cups equals the number of saucers.

(C2) Therefore, there are numbers.

Even granting the soundness of the arguments, with ontologically neutral quantifiers, it does not follow that numbers exist, as one can quantify over numbers independently of their existence. The satisfaction of the existence predicate is still needed before any commitment to the existence of numbers is in place.

(D) In contrast with what happens with free logic, with ontologically neutral quantifiers, the reading of ' $\exists x x = a$ ' is not ambiguous—being ontologically committing in some contexts and not in others. The sentence simply states that some object is  $a$ . This is not a logical truth as  $a$  need not be in the domain of interpretation. For instance,  $a$  may be something that cannot be quantified over—it is not an object—such as an ontological category in Jonathan Lowe's view (see Lowe 2006). Nor is a logical truth the related sentence stating that  $a$  exists—namely, ' $Ea$ ', in which ' $E$ ' is the existence predicate. After all,  $a$  need not exist. Moreover, differently from what goes on with free logic, there is no tension between metalanguage and object language (one expressing ontological commitment, the other not): quantifiers in neither of these languages are ontologically committing. Ontological commitment is expressed via the existence predicate instead.

Finally, Azzouni insists that only existing objects can have properties (Azzouni 2004; 2010). In order to avoid the difficulty of being forced to conflate different nonexistent objects, such as Hamlet and Anna Karenina, Azzouni argues that sentences are used to implement the intended distinction. On his view, it is the “aboutness illusion” (the illusion that there are objects our claims are about) that forces us to think that the differences in question emerge from the relevant objects.

Nonetheless, this move does not work. Sentences can be used to describe the difference of the relevant objects, but what distinguishes the objects is the fact that they have different properties. The sentences are only an expression of the properties under consideration: the objects themselves are different—even if they fail to exist. There is no illusion in thinking that objects guide our discourse about them. They need not exist to accomplish that: their properties do the work.

## 7. Conclusion

Referring to nothing becomes problematic in several influential views—direct reference theories, neo-Fregeanism, easy ontology proposals—due to the link they make between reference and existence. With ontologically neutral quantifiers, as opposed to the use of free logic, however, the resulting problems can be avoided: there is no difficulty in referring to and distinguishing objects that do not exist. The result is a promising and agnostic view about language and ontology. Even though far more needs to be said, I hope enough was said to suggest that this is a view worth taking seriously.

## References

- Azzouni, J. 2004. *Deflating Ontological Consequence: A Case for Nominalism*. New York: Oxford University Press.
- Azzouni, J. 2010. *Talking about Nothing: Numbers, Hallucinations, and Fictions*. New York: Oxford University Press.
- Bueno, O. 2005. Dirac and the Dispensability of Mathematics. *Studies in History and Philosophy of Modern Physics* 36: 465–490.
- Bueno, O. & Zalta, E. 2005. A Nominalist's Dilemma and its Solution. *Philosophia Mathematica* 13: 294–307.
- Hale, B. & Wright, C. 2001. *The Reason's Proper Study: Essays towards a Neo-Fregean Philosophy of Mathematics*. Oxford: Clarendon Press.
- Lambert, K. 2004. *Free Logic: Selected Essays*. Cambridge: Cambridge University Press.
- Lowe, J. 2006. *The Four-Category Ontology: A Metaphysical Foundation for Natural Science*. Oxford: Clarendon Press.
- Mill, J. S. 1843. *System of Logic*. London: Parker.
- Sainsbury, R. M. 2005. *Reference without Referents*. Oxford: Clarendon Press.
- Thomasson, A. 2015. *Ontology Made Easy*. New York: Oxford University Press.

## Acknowledgments

For helpful discussions on the issues examined in this work, my thanks go to Jonas Arenhart, Jerzy Brzozowski, Ivan Ferreira da Cunha, Cezar Mortari, Marco Ruffino, Amie Thomasson, and the audience at the 13<sup>th</sup> Principia International Symposium in which an earlier version of this article was presented.