NEWTON DA COSTA ON TRUE CONTRADICTIONS:
FROM APORIAS TO REALITY

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Abstract.   Dialetheism is the thesis that some contradictions are true. In this paper, we explore claims to such a thesis to be found in some early philosophical writings of Newton da Costa, most notably in his book *Ensaio Sobre os Fundamentos da Lógica* (hereafter mentioned as the *Essay*). We present and explore da Costa's claims that, in a certain sense, abstract notions such as 'set' and 'truth' may clearly involve true contradictions. Additionally, we reconstruct da Costa's argument to the effect that some contradictions may also be true of concrete reality. In a nutshell, da Costa suggests that some paradoxes, the aporias, are so deeply problematic that they may be pointing to reality as the source of the contradiction; i.e. aporias may ground a form of metaphysical dialetheism. This goes further than most dialetheistic claims found in the current literature. We compare these claims to a standard approach to dialetheism as Graham Priest's, and conclude that da Costa offers a version of dialetheism too in the Essay. This opens the door for interesting explorations of some valuable yet often overlooked contributions of da Costa's thought to the contemporary debate on dialetheism.

Keywords: dialetheism ● paraconsistency ● Newton da Costa ● aporias ● paradoxes

1. Introduction

According to the typical textbook definition, a system of logic is paraconsistent if it violates some form of the so-called *principle of explosion*:[1]

$$\alpha, \neg\alpha \nvdash \beta.$$

Still following the tradition, that means, informally, that a contradiction does not entail every formula, i.e., we may have inconsistency without triviality; a paraconsistent logic is said to *be able to block the trivializing power that contradictions have in systems*

such as classical or intuitionistic logics. As a result, it is said that theories engendering contradictions may still be actively and profitably pursued whenever we use a paraconsistent logic as their underlying logic. All of that, of course, is well known from the literature on paraconsistent logics (see da Costa et al. (2007), Szmuc et al. (2018) for further discussion).

However, from a philosophical point of view, using the formal tools of a paraconsistent logic to live without triviality in an environment where contradictions appear is only the tip of the iceberg. Formally, blocking the explosion principle is achieved by providing an interpretation for the language of the system in which both α and $\neg\alpha$ receive designated values, but where β is not designated, for some α and some β formulas of the language. So, although it looks *prima facie* like some contradictions are made true in those models, there is still wide disagreement about the precise *meaning* of such contradictions; basically, one may read the truth values in such models in many distinct ways, going from truth and falsity to some alternative accounts in terms of information or evidence. As a result, the reading of contradictions may vary from being literally true to being a result of conflicting information (see Carnielli and Rodrigues (2019), Barrio and da Re (2018), Arenhart (2021; 2022) for more on the idea that contradictions must be interpreted).

In this paper, we shall explore a rather neglected line of approach to the meaning of a contradiction and its relation to paraconsistent logics that was formerly advanced by Newton da Costa in his "Essay on the foundations of logic", first published in 1979 (in Portuguese). In particular, our interest here is confined to the third part, titled "Hegel's thesis", where da Costa offers views on the meaning of a contradiction that may be seen as anticipating some dialetheist claims, — *dialetheism*, recall, is the thesis that some contradictions are true. We hope to bring to light some surprisingly remarkable theses by da Costa as they were presented in the Essay; curiously, da Costa himself did not revisit some of these claims in his later research. We do not intend to make a purely exegetical reading of da Costa's aims and neither of how his thinking may have changed throughout the years; our sole concern is to bring to light da Costa's ideas that we believe could contribute to the current debate on the nature of paraconsistency and dialetheism.

In da Costa's views, as we shall present, evidence for dialetheism, if there is any, should come from science. That is, rather than having an a priori philosophically motivated account of contradictions to approach paraconsistent logic, da Costa suggests that we use science to motivate the embrace of paraconsistency. What is actually surprising in da Costa's approach in the Essay is that he then goes on to advance a claim to the effect that the description provided by some of our best scientific theories may be contradictory, so that use of a paraconsistent logic would be mandatory, and the reading of the contradictions involved is purely alethic. Another way to put what will be discussed here is as follows: da Costa may be seen as discussing in clear terms the

conditions under which a contradiction in science may be taken as true.

In order to better illustrate how da Costa's ideas may match with current dialetheism, we shall also draw a parallel with a recent presentation of dialetheism by Edwin Mares (see Mares 2004). Mares distinguishes *semantic* and *metaphysical* versions of dialetheism. *Semantic dialetheism*, the soft version, holds that contradictions appearing in our theories or pieces of discourse are always eliminable in favor of a consistent description, using a different vocabulary, *without any loss of accuracy in the result*. The *metaphysical dialetheist*, on the other hand, holds that there are cases where contradictory theories cannot be made consistent without sacrificing the accuracy of the description. As we shall see, da Costa draws a parallel distinction to the one proposed by Mares.

The paper is structured as follows. In section 2, we present in details the terminology that shall be employed throughout the paper. In section 3 we discuss how the terminology is exemplified in the work of da Costa in order to generate metaphysical dialetheism concerning abstract entities. In section 4, we advance what is perhaps the most impressive thesis by da Costa, the claim that a paraconsistent logic may be required by science, given the possibility of true contradictions in reality. In section 5, to round off the idea that da Costa may be offering a version of dialetheism, we briefly draw interesting parallels between da Costa and Graham Priest on the treatment of some prominent paradoxes. This comparative analysis aims to situate da Costa's view in relation to the standard understanding of dialetheism, bringing to light several of his ideas that, although largely overlooked until now, are philosophically significant and may offer valuable contributions to the debate on dialetheism. We conclude in section 6.

2. Terminology

Dialetheism is defined as the thesis that there are sentences of the form α and $\neg\alpha$ that are both true, where $\neg$ is a negation. This is typically motivated by the presence of contradictions involving abstract concepts, such as 'truth' and 'set', giving rise to the many versions of the Liar paradox and Russell's paradox (see Priest 2006a, 2006b, Priest et al. 2024 for more motivations for dialetheism). Paradoxes may also arise in cases of empirical science, as we shall discuss.

Concerning the very idea of a paradox, to begin with, the first important characteristic that we need to account for is that it features contradictions as conclusions of a very peculiar kind of reasoning: paradoxical reasoning. In his treatment, da Costa employs the now widely standard notion of a paradox, as in the treatment offered by Mark Sainsbury:

> This is what I understand by a paradox: an apparently unacceptable conclu-

> sion derived by apparently acceptable reasoning from apparently acceptable premises. Appearances have to deceive, since the acceptable cannot lead by acceptable steps to the unacceptable. So, generally, we have a choice: either the conclusion is not really unacceptable, or else the starting point, or the reasoning, has some non-obvious flaw. (Sainsbury 2009, p.1)

Furthermore, da Costa also adopts the typical distinction —proposed initially by Frank Ramsey — between the following kinds of paradoxes:

Logico-mathematical paradoxes: Russell's paradox, Cantor's paradox.

Semantic paradoxes: the Liar paradox, Grelling's paradox.

Two kinds of solutions are available to paradoxes: *positive and negative solutions*. A solution to a paradox is said to be *negative* if the argument leading to the prima facie unacceptable conclusion is taken as unsound. In this case, according to da Costa, a paradox is just a fallacy in which either some of the premises is false (a "material fallacy"), or else the reasoning is invalid (a "formal fallacy"). This accounts for some of the options raised by Sainsbury (i.e., when the starting point or the reasoning itself has some flaw). However, there is still the option that the conclusion is not really unacceptable, and in these cases we have *positive* solutions. As examples of famous paradoxes with positive solutions, da Costa mentions Peano's space-filling curve paradox and Skolem's paradox. In the first case, recall, the paradox arises from the fact that intuitively a curve cannot fill a plane figure, while Peano's example shows that this is precisely what may happen, and Skolem's paradox arises from the fact that a first-order set theory like Zermelo-Fraenkel, if consistent, has a denumerable model, even though one can prove, inside the theory, that there are non-denumerable sets. Although contrary to our intuitive conceptions, these paradoxes are dissolved when an explanation is given as to whether the conclusion is not really unacceptable.

However, some paradoxes do not result in a conclusion that is only apparently unacceptable: they result in explicit contradictions. That is the case with the Liar and the Russell paradoxes, among others. In these cases, it seems that only a negative solution is reasonable, given that contradictions are not acceptable. That was the mainstream view until rather recently, but the rise of paraconsistent logics changed the picture. With paraconsistent logics available to play the role of the background logic, one no longer needs to fear the presence of contradictions, and such paradoxes no longer require necessarily a negative solution. One may wish to embrace such contradictions, in case one believes to have reasons to do so; as da Costa puts it, "taking the rise of paraconsistent logics into account, a reexamination of the paradoxes becomes mandatory" (da Costa 2008, p.228). It is here that the dialetheist believes to have a point for establishing the existence of true contradictions, given the alleged *ad hoc* character of some consistent solutions to some paradoxes:

> Paradoxes of this kind are apparently valid arguments, often very simple arguments, starting from things that seem obviously true, but ending in explicit contradictions. Unless one can fault them, they establish dialetheism. Though many arguments in the family are, historically, quite recent, paradoxes of the family have been known now for close to two and a half thousand years. It is a mark of their resilience that even now there is still no consensus amongst those who think that there is something wrong with them as to what this is. Better, then, to stop trying to find a fault where none exists, and accept the arguments at face value. (Priest 2007, p.171; see also Priest 2006a, p.83)

With the availability of positive and negative solutions to the paradoxes ending in contradictions (because these are the most relevant ones when it comes to discussing paraconsistency), we are led to a problem: is there any kind of possibility of determining which solution — positive or negative — to adopt? Just as we demand for the negative solutions that they should be non-ad hoc, the same requirement holds for the positive ones (da Costa 2008, p.228 makes these demands clear). This kind of dilemma leads da Costa to introduce new terminology that will be useful for our discussion of how he may be seen as advancing dialetheistic claims. He points to the fact that the typical strategy to face paradoxes ending in contradictions consists in providing for revisions of the conceptual structure of theories deriving such paradoxes; that leads to a substantial conceptual change. However, this is not a definitive argument against directly accommodating the contradictions, which, as the previous quote by Priest indicates, seems to be understood in less revisionary terms (although a revision in logic is mandatory anyway, of course). That is, although we can revise our theory to avoid the contradiction, should we? The tension is captured by the fact that these paradoxes instantiate what da Costa calls 'aporias':

> In the case of some paradoxes it is disputable whether there were good solutions. That happens, in particular, in those cases in which such solutions entail modifications of the basic principles of science; sometimes these modifications are implausible, sometimes their only justification is the elimination of paradoxes. (da Costa 2008, p.224)

That is, whenever substantial conceptual changes are introduced only for the sake of eliminating the paradoxes, one is facing an artificial modification of a theory, and that is a clear sign that we are facing an aporia.

> By definition, the solution of an aporia requires the deep revision of the system of knowledge, due to the unavoidable restriction of accepted principles, or else due to changes in the accepted logic. (da Costa 2008, p.226)

In a very clear sense, this distinction between paradoxes and aporias anticipates the current distinction between semantic dialetheism and metaphysical dialetheism, advanced more recently by Edwin Mares:

> Both semantical and metaphysical dialetheism hold that there are true con-
> tradictions, or at least that it is possible for there to be true contradictions.
> That is what 'dialetheism' means. The difference between the two views con-
> cerns the status of these contradictions (Mares 2004, p.269)

As Mares goes on to make clear, the distinction between semantic and metaphysi-
cal dialetheism involves precisely the possibility of making revisions in inconsistent
descriptions without sacrificing accuracy, as introduced by da Costa. In Mares' own
words:

> Thus we come to the crux of the difference between metaphysical and se-
> mantic dialetheism. The metaphysical dialetheist holds that there are aspects
> of the world (or of some possible world) for which any accurate description
> will contain a true contradiction. Semantic dialetheism, on the other hand,
> maintains that it is always possible to redescribe this aspect of the world,
> using a different vocabulary (or perhaps vocabularies), consistently without
> sacrificing accuracy. (Mares 2004, p.270)

So, bringing it all together: if there are aporias, metaphysical dialetheism is vindi-
cated, semantic dialetheism is not enough. But can we grant that? That is precisely
where da Costa advances the claim that the search for aporias may guide us in the
identification of situations where we face true contradictions.

In fact, da Costa offers a sort of "recipe" of how one should go from an inconsistent
description to decide whether the source of such inconsistency is to be found in reality.
This "recipe" is related to the understanding of the source of aporias. As da Costa
argues, it is up to science to indicate whether there are true contradictions; logic
cannot decide this eminently pragmatical issue. How should we know whether that is
actually the case? We may find a path from aporias to reality as follows: as we pointed
out in the characterization of the nature of aporias, da Costa noticed that the major
distinction between aporias and fallacies consists in the fact that a fallacy involves a
simple formal or material solution: one often finds a mistake in a derivation or detects
false premises, and that is all; aporias, on the other hand, are highly revisionary of
the general system of knowledge. It results that true contradictions should be sought
in the differences of solutions to the different kinds of paradoxes. Aporias may be
indicating a source of a real contradiction.

> ... when a paradox is reduced to a fallacy one should not expect that to over-
> come it, *substantial changes* in the structure of science are required, given
> that in this case it would be an aporia. So, if we are able to detect some fea-
> tures of aporias properly speaking, which distinguish them from fallacious
> paradoxes, we could argue that they possibly reflect objective and real con-
> tradictions; its overcoming will no doubt lead to *radical transformations* in
> science. (da Costa 2008 pp.234–235, our emphases)

So, cases of radical transformations in science to overcome contradictions are, according to da Costa, and also according to Mares' terminology, a good sign of the truth of the underlying contradiction. In da Costa's terms we have some reasonably good evidence that the source of the contradiction is located in reality; in Mares' terminology, we have a vindication of metaphysical dialetheism. As we shall see in the next sections, da Costa makes a case for the occurrence of actual aporias in science, which would amount to a metaphysical version of dialetheism.

3. The abstract cases

We shall proceed, as da Costa does, distinguishing between contradictions that arise in theories dealing with abstract entities, on the one hand, and theories dealing with concrete entities on the other. We begin with the case of theories dealing with abstracta. It is interesting to notice that contradictions relating to the abstract realm are prominent in the current literature on dialetheism.

Although da Costa indicates that changes in the accepted logic are also features of the solutions to the aporias, that by itself does not seem to include *change of the logical theory* itself. Rather, aporias manifest themselves whenever one provides for deep changes in a conceptual structure of a theory so that the changes are implemented even *while keeping the accepted logic* (i.e. classical logic) for the sake of advancing a negative solution. As an instance, the Liar paradox requires deep changes in the way we naively theorize about truth. Tarski's hierarchy of languages, which is the currently most accepted solution to the Liar in the confines of classical logic, is a clear sign that the Liar is an instance of an aporia; "one cannot eliminate it [the paradox] without major changes in logical categories" (da Costa 2008 p.227). This is important: da Costa notices that the artificiality introduced by the Tarskian hierarchy of languages is a clear sign that our intuitive canons have been substantially revised. Russell's paradox is an aporia for the same reason: it requires deep reformulation of the principle of naive abstraction, either through the introduction of the theory of types, or through the use of a cumulative hierarchy in set theory. Nothing of the kind seemed natural for the participants in the debates on paradoxes when they were proposed (in the early 1900s). Only slowly did the axiomatization by Zermelo gain wide acceptance by the mathematicians. Both solutions, of course, introduce radical changes in the very idea of truth and set, respectively.

But then, should we avoid such revisions, retain the naive inconsistent concepts, and go paraconsistent? Clearly, the fact that such aporias exist, and that they seem to require rather artificial changes in our conceptual system seem to speak in favor of a paraconsistent approach. That would change the logic, for sure, but would preserve the intuitive structure of the underlying paradoxical notions. No substantial change

on the general underlying conceptual apparatuses of the theories is required.

So, is this a case where there are true contradictions? Yes, but there is something else to it. In this case, where one deals with abstract objects, da Costa does not believe that there is one correct way to go; he is a pluralist about mathematics. Rather, positive solutions to such paradoxes are, in a certain sense, obvious: they are made possible by the use of some paraconsistent set theory. These set theories make possible the positive solutions to the paradoxes, allowing for the existence of Russell's set, but at the same time, they also have to live side by side with consistent set theories, where these paradoxical sets do not exist. In mathematics, given that we may explore theories using distinct logics, we clearly have distinct objects that are engendered by the respective systems. It is not a matter of correct or incorrect, but one is left with a pragmatic choice of which logic to apply in order to investigate distinct domains. Certainly, one can only investigate Russell's set, for instance, inside a paraconsistent setting (see, for instance, da Costa et al. 2007).

The latter claims may sound odd, but they are more clearly understood in the context of da Costa's version of *plenitudinous Platonism*. This is a generalization of Hilbert's criteria of existence in mathematics. While Hilbert claimed that entities described by consistent entities exist, da Costa claims, more generally, that every nontrivial mathematical system describes an objectively existing abstract structure (see a discussion in Arenhart 2019). In this case, it seems, distinct systems, one with Russell's set and another one without Russell's set, are not actually rivals; they do not compete as descriptions of a unique independently existing realm of sets, but rather, each kind of theory gives rise to its own theoretically dependent (albeit objective) realm. In this sense, dealing with such inconsistent entities is almost an obvious feature of paraconsistent set theories, and this is not the most remarkable reason we would have to adopt such logics (where adoption is understood in pragmatic terms, guided by the fruitfulness of a mathematical system).

In other words, paraconsistent set theories are interesting, they give rise to contradictory sets, which may be studied on their own. However, in the face of da Costa's version of mathematical pluralism, there is a sense in which such true contradictions are not the most impressive ones. The real test for paraconsistent logics comes not from the abstract reign, where entities are created by the use of distinct logics, but from the concrete realm, where we do not have so much of a choice, given the objective features of the entities being dealt with. In this case, if contradictions are present in the concrete world, we have no choice but to adopt a paraconsistent logic. Let us check whether there are any chances that we could find such contradictions.

4. The concrete cases

As we have mentioned, the real challenge for paraconsistency is to find applications in the concrete world, to deal with concrete problems. Logic, by itself, according to da Costa, does not generate aporias. Rather, paraconsistent logics merely evidence that positive solutions to such paradoxes are logically possible. But there remains the problem of choosing between positive and negative solutions, that is, choosing between paraconsistent and non-paraconsistent logics. In the case of systems about abstract objects, positive and negative solutions may live side by side. When it comes to concrete entities, the choice requires a different set of considerations, involving the whole system of knowledge.

According to da Costa, there are some considerations that could change the game in favor of paraconsistency: what if we discover that the concrete world requires paraconsistency, due to its inconsistent character? That could make a quite convincing case for paraconsistent logics (see a discussion in Arenhart 2018). The next question, then, is: is the real world contradictory? Surprisingly, da Costa advances an answer that seems to lead us to believe that the answer is positive. It comes through the grounding of the following claim:

Hegel's thesis: There are true sentences of the form $\alpha \wedge \neg\alpha$.

As we have seen, for abstract objects, that is quite simply true, such sentences are provided inside some set theories and some paraconsistent treatments of the Liar. But is it the case at the concrete level?

In order to present how da Costa develops his answer to that question, we need some further background distinctions. We are no longer in the realm of paradoxes about abstracta, but rather in the realm of contradictions in empirical science. We are looking for some sentences of the form $\alpha \wedge \neg\alpha$ that can be made true by some of our best scientific theories about the empirical world. There are, according to da Costa, two kinds of contradictions in science:

Semiotic contradiction: contradictions that arise due to semiotic factors (syntax, semantics, pragmatics).

Real contradictions: they are "true contradictions in a strict sense, reflecting aspects of reality; a contradiction $A \wedge \neg A$ is real if A and $\neg A$ are true propositions, satisfying Tarski's (T) schema and referring to *real* things" (da Costa 2008, p.233)

Basically, the distinction refers to contradictions that arise as artifacts of the way we manipulate language, and contradictions that result inevitably from our description of the concrete world. What da Costa really wants is: is there evidence for real

contradictions? Recall that according to da Costa, an aporia is a good sign that we are facing a true contradiction. As we have seen, there are aporias that point to true contradictions in the abstract realm (Russell's paradox, the Liar). Now the challenge is to investigate to what extent aporias can point to contradictions in the concrete realm.

The idea that overcoming an aporia requires a radical transformation of science is tantamount to Mares' indication that the attempt to provide for a consistent description, in the case of a semantical dialetheist, will lead to loss of accuracy. So, the recipe provided by da Costa suggests that finding aporias will lead us to find possible sources of true contradictions or, of contradictions having their source in reality. Candidates for aporias are available in the sciences, with the following list being provided by da Costa:

- Wave-particle duality in quantum mechanics: quantum entities exhibit, on different occasions, wave behavior and particle behavior, with a single description being unable to account for all the kinds of phenomena produced by the theory.

- The Copenhagen interpretation: understood as based on Bohr's complementarity interpretation, it restricts what one can say about the phenomena (basically, denying that one may describe the quantum world when no measurement is being performed), renouncing to a complete description of reality.

- Zeno's paradoxes: as an outcome of Zeno's paradox, discrete space and time cannot be finite. The usual solution, which is the mathematical continuum, is far from the naive ideas of space and time, being a "rather artificial theoretical construction".

Another kind of evidence for an inconsistent world appeals to the current state of theoretical underdetermination in QM: there are plenty of incompatible interpretations of QM (Bohm, Copenhagen, GRW, Many worlds...). The fact that we seem to need diverse incompatible descriptions indicate that the source of the difficulty may be found in reality itself. We may be attempting to advance a consistent description of an inconsistent world:

> The micro-physical aporias lead alternative, mutually exclusive, attempts of solutions. So, would it not be sensible to suppose that the interpretative difficulties arise from the fact that we are looking for consistent descriptions of an inconsistent reality? (da Costa 2008, p.236)

But what are we to make of all these claims? Contradictions in these cases are real or not real? Well, da Costa emphasizes that, by definition of aporia, overcoming an aporia requires substantial change in the so-far established system of knowledge and da Costa wishes to use this as one indication that the aporia has its source in

reality. Still, this is not foolproof: science typically eliminates contradictions through scientific revolutions. The argument shows, again, that contradictions need not be eliminated as a matter of logic. The issue is to be solved by the scientific community, related to pragmatic factors in logical and scientific theory choice. However, the very possibility of true contradictions (opened by paraconsistent logic) now shows that the elimination of contradictions is not mandatory, and also, that it is impossible to refute the thesis on a priori grounds. More than that: it is easier to prove Hegel's thesis than to refute it (one single true contradiction vindicates Hegel's thesis, while no amount of negative solutions will ever prove that there are no true contradictions).

That is how one would motivate paraconsistent logics: by identifying situations where elimination of a contradiction would promote deep conceptual revision. What we may be doing, in these cases, says da Costa, is to try to account for an inconsistent reality with consistent concepts, and we may be losing something about reality itself in this process.

Given that we shall rely on a motivation originating in da Costa's writings, we start by presenting his views on how to connect paraconsistency and empirical science. The point was raised by da Costa mostly in his (2008), but it is also found in what looks like a minor topic in a discussion section closing his paper on paraconsistent set theory (see da Costa 1986). This is in fact a very neglected passage of this otherwise known paper, but it connects perfectly well with da Costa's more openly philosophical excursions on the philosophy of logic in general (as developed in da Costa 2008 and available for a long time only in Portuguese),[2] and paraconsistent logics in particular.

The passages concerning a programme for implementing paraconsistency —in the sense of having to choose a paraconsistent logic as the underlying logic of our best theories— appear in the final section of his paper on paraconsistent set theory (see da Costa 1986). There, da Costa presents in a very summarized fashion his views on the aims and prospects for a widespread adoption of paraconsistent logics as the working logic in science and philosophy. The major problem a paraconsistent logician should be concerned with, in da Costa's views, becomes: how to properly motivate the need for the official adoption of paraconsistent logics? This should be understood as a requirement of more than mere exploration of a possibility, so that paraconsistency can effectively offer solutions to some genuine problems that other kinds of system cannot. So, the problem is more basic than offering an interpretation to paraconsistent logics once those are adopted. It is rather related to the question why we should adopt those logics to begin with. The answer, as we shall see, is directly related to some form of dialetheism. But let's not get ahead of ourselves.

The programme is advanced in the following terms:

> The main concern to paraconsistent set theory is not to make possible the existence, and thereby the investigation, of some sets which cause trouble

> in naive set theory, such as Russell's set ... On the contrary, the most important characteristic of paraconsistent set theories is that they allow us to handle the extensions of 'inconsistent' predicates which may exist in the real world or are inherent in some universes of discourse in the fields of science and philosophy. According to several dialecticians, for example, there exist real contradictions in the world, and we need paraconsistent logic to handle them ... Analogously, contradictions must be taken into account in some psychoanalytic theories ... (da Costa 1986, pp.369–370)

He elaborates the same point in other words:

> What I am trying to say is that the paraconsistent programme should not be judged solely by the mathematico-formal features of the paraconsistent set theories (for example, if they allow one to demonstrate the existence of infinitely many 'pathological' sets, if Russell's set does exist and, supposed its existence, if it is identical or not to the universal set), but above all by their aptness to cope with *concrete* problems. That is, problems originated from the vicissitudes of inquiry, in the domains of science and of philosophy... (da Costa 1986, p.370)

What does it amount to? It indicates in general lines the requirements that should be met in order for the widespread adoption of paraconsistent logics in science and philosophy. As it becomes clear from the quoted passages, da Costa is rather quick to recognize that paraconsistent systems — in particular, systems of paraconsistent set theory — offer positive solutions to problems involving contradictions related to abstract entities, such as semantic and set theoretic paradoxes (e.g. the Liar paradox and Russell's paradox). But this is not the most important part of the motivation. Inconsistent entities, at least in the abstract realm, seem to come along with the paraconsistent theories they are part of. Russell's set or the Liar sentence, entities giving rise to the need of paraconsistent apparatuses, are part of theoretical constructions adopting paraconsistent systems, which can handle such inconsistencies and allow us to investigate them. But still, they do not seem to create the most compelling case for paraconsistency, or, at least, not as compelling as da Costa would like to have, and as he envisages in another domain: as a mathematical theory, paraconsistent set theory can deal with concrete problems, problems concerning real entities in the concrete world. As the quotes above evidence, the paraconsistent programme will prove its worth when contradictions about the concrete enter our best science and demand a paraconsistent approach. So, more than looking for contradictions in the abstract realm, we need to envisage applications of paraconsistent logics to concrete realms.

That leads us directly to the next obvious problem for a fully-fledged philosophy of paraconsistency: are there contradictions in our best theories about concrete reality? Aporias seem to indicate that there are. The major difficulty is that these contradictions are typically eliminated, even if there is loss of accuracy. What da Costa

suggests is that there may be a point where such eliminations will no longer be the best option, due to their failure to account for the underlying inconsistencies of reality. When that point is recognized, there will be an explicit need for paraconsistency offering solutions that are not open to the classical logician.

5. da Costa and Priest on dialetheism

Claiming that da Costa advances a form of dialetheism can be quite contentious for both dialetheists and non-dialetheists alike. Priest (2007, p.167) asserts that da Costa does not come close to endorsing dialetheism, while prominent members of the Brazilian School of Logic emphasize that da Costa never defended dialetheistic theses. Nevertheless, it seems that no one would doubt that Priest is a dialetheist. So, while arguing that Priest advances dialetheism might be somewhat uncontroversial, it is by no means trivial to claim that da Costa considered endorsing a form of dialetheism in his *Essay*.[3] Taking this into account, in this section we will briefly discuss the positions of Priest and da Costa in relation to the common understanding of dialetheism. Under the prevailing interpretation of dialetheism, we shall claim, if Priest is considered a dialetheist, then the ideas put forward by da Costa in the chapter "Hegel's thesis" should also be classified as a form of dialetheism.

This debate clearly depends on how dialetheism is understood. Here, we are merely adhering to the definition of dialetheism as it appears in the relevant literature, in line with Priest's own conception of the term. As defined above, dialetheism is the view that there are sentences (propositions or any other truth-bearer) of the form α and $\neg\alpha$ that are both true — that is, the view that there are true contradictions (the dialetheias). (Mares 2004, Priest 2006a, 2006b, 2007, Priest et al. 2024). Dialetheism is the view that some sentences (or language-like entities) are dialetheias. (see Priest 2019). As Priest (2006a, p.299) urges, dialetheism is a thesis about language or language-like entities.

As we have seen, according to da Costa, paraconsistent logic allows for the existence of true contradictions at an abstract and formal level, particularly in certain paraconsistent mathematical theories. In this field, da Costa is explicit about the existence of true contradictions.

> Regarding the abstract-formal domain, we have already seen that an object can possess contradictory properties. This happens with certain abstract objects referred to by paraconsistent theories. Thus, there are true contradictions of abstract and formal nature. (da Costa 2008, p.232)

Therefore, at an abstract level, we can already identify a clear dialetheist position in da Costa's essay, since dialetheism requires nothing more than endorsing the view that there are true contradictions.

An additional point for those who accept the distinction between semantic and metaphysical dialetheism would be to investigate which kind of dialetheism is at play in the abstract formal domain. According to Mares' conception of dialetheism, what determines whether dialetheism is semantic or metaphysical is whether it is possible to provide a description without sacrificing accuracy. Da Costa's understanding of aporia, as we have seen, may point to this distinction. As we have seen, according to da Costa, aporias — as Russell's and Liar paradoxes — compel us to a deep revision, resulting in a loss of accuracy (i.e. loss of the previously prevalent naive approach to such concepts) through a consistent redescription. So, if we take into account the division between metaphysical dialetheism and semantic dialetheism, we find a form of metaphysical dialetheism in da Costa's work, even within the abstract domain.

Although da Costa advances a clear defense of dialetheism in the abstract and formal domain, as we have seen, the real challenge, according to him, lies in dealing with true contradictions in concrete reality. In da Costa's approach, paradoxes are taken as a means to investigate true contradictions in reality. The first step in this investigation is to distinguish between fallacies and aporias. To illustrate this distinction in a concrete context, da Costa brings the catalog paradox.[4] This paradox is overcome, based on classical logic, through a common strategy using the proof by *reductio*. In this way, it is easily demonstrated that no such catalog exists. Unlike the catalog, we cannot conclude that there is no sentence of the Liar paradox. As we have seen, the Liar paradox is considered an aporia, as overcoming it requires profound transformations in both logical and linguistic categories.

In the concrete domain, on the other hand, an aporia may point to a contradiction in concrete reality. As we have seen in the previous section, da Costa brings some candidate aporias from the empirical science, noting that overcoming them requires deep transformations in theories (quantum mechanics becomes compartmentalized on some dualities, like particle/wave, complementarity, and so). Based on this, da Costa argues that such aporias in science may reflect real and objective contradictions in the concrete world. Here again we see the mark of aporias that link them to metaphysical dialetheism: in da Costa's view, we cannot get rid of contradiction through some re-description to ensure the consistency of the theory, without deep transformations or a loss of a previous standard of achieving complete explanations. So, da Costa's approach is characterized by metaphysical dialetheism in the concrete realm as well.

Still, one might disagree that da Costa is advocating for a dialetheistic view, considering that he does not explicitly assert the existence of true contradictions inherent in concrete reality. However, notice that, by parity of reasoning, no such demand is raised for other dialtheists. Given that the acknowledgment of contradictions in the abstract realm is enough to label Priest and Mares, among others, as dialetheists, it seems that the same could be true of da Costa.

Priest (2006a, sec.20.6) addresses the question of whether dialetheism implies that there are contradictions in the world. If we consider the dialetheistic views categorized under semantic dialetheism, as framed by Mares, the answer is clearly negative. However, for the dialetheism supported by Priest, the answer is positive, though in "one quite unproblematic sense": if some contradictions are true, then there must be something in the world that makes it true. Nevertheless, in Priest's view, dialetheism does not require a strong metaphysical commitment to true contradictions in the world.

> It is therefore mistaken to claim that 'the dialetheist claims that some contradictions are ontological in the sense that they are due to some 'inner contradictory essence of reality'. Indeed, [...] it is not even clear that the claim that there are contradictions, in reality, makes sense. For it to do so, one has to endorse some kind of correspondence theory of truth, holding reality to comprise facts or fact-like entities. I have never endorsed such a view. Indeed, the only theory of truth I have ever advocated [...] is anything but such a realist theory. So let me say it one more time: it is not clear that a philosophically substantial claim to the effect that there are contradictions in reality makes sense; and even if it does, I am not committed to it. (Priest 2019, p.589)

According to Priest, the adoption of dialetheism does not imply the existence of true contradictions in reality itself. Priest does not endorse a strong form of metaphysical realism that involves ontological contradictions, in the sense of the "inner contradictory essence of reality". In a nutshell, the "unproblematic sense" of true contradictions is enough to grant a dialetheistic view.

However, da Costa goes beyond Priest by investigating true contradictions in reality in the "problematic sense" — that is, through an investigation that entails more robust metaphysical assumptions related to realism. Recall da Costa's notion of "Real Contradiction", which concerns true contradictions that reflect aspects of reality; contradictory propositions, $\alpha \wedge \neg\alpha$, are true as far as they refer to real things (da Costa 2008, p.233).

The investigation of the existence of true contradictions in reality is a posteriori, and aporias are a kind of key to true contradiction in reality. However, efforts to address certain aporias in empirical science suggest that we can neither conclusively affirm the existence of contradictions in reality nor definitively deny the possibility that true contradictions might exist. Even though logic cannot decide such issues, the door is open for future science to stop revising theories and embrace at least one true contradiction in concrete reality.

Although da Costa does not explicitly affirm the existence of true contradictions in reality in a "problematic sense", this does not undermine his dialetheist ideas. As observed, even Graham Priest does not claim that such true contradictions actually

exist in reality. More important than that, however, is the very concept of dialetheism itself. Dialetheism is fundamentally a thesis about language, as Priest urges, and it does not require an ontological commitment to true contradictions in the concrete world. Therefore, based on the common understanding of dialetheism, a clear dialetheist defense can be identified within da Costa's philosophy of paraconsistency as developed in the Essay.

6. Conclusion

Let us now close the paper with some additional comments on how the very possibility of true contradictions in reality is to be accounted for in terms of da Costa's proposal. According to him, logical laws are not only laws employed by Reason to deal with some context, they are also understood as descriptions of general features of reality, as investigated by a given discourse or scientific theory. Contradictions are true or real relative to a systematization of the theory, where one may choose a paraconsistent logic. Logical theory choice depends on science. Choice depends on reality.

> In brief, logical laws are regulative norms of a rational context, but they also produce, in a certain sense, general relations holding about reality. Constitutive reason, let us say, constructs reality based on real features, providing elements for operative reason to work. Rational categories, although they are elaborated by constitutive reason, somehow translate, mediately or immediately, directly or indirectly, features of the real world. (da Costa 2008, p.244)

'Constitutive reason' here is to be understood as a set of basic ontological categories (objects, properties, facts) that a logical language describes, and which obeys general properties as described by a system of logic. It is partly imposed by reason in a scientific context, and also partly abstracted from the context. If the context indicates that inconsistent objects are present (and this is signalled by the presence of an aporia), then, it seems, paraconsistent logics can give a definitive contribution to the objective description of reality, and they are fully justified.

> Summarizing, the paraconsistent programme, at least in its connection with set theory, has two kinds of motivation: one mathematico-formal, related to 'abstract' problems, and another 'concrete', linked to actual scientific and philosophic issues. Perhaps the second kind of motivation is more fruitful than the first, as a source of relevant paraconsistent insights. (da Costa 1986, p.371)

However, so far, those cases indicated as aporias have been treated as anomalies rather than as a manifestation of an inconsistent reality. The precise distinction between such cases remains the biggest challenge for paracosistentists. Anyway, as we

have seen, the issue goes through the notion of dialetheism. Concerning the semiotic contradictions that appear in some paradoxes, like Liar, Russell, Zeno paradoxes, da Costa grants that true contradictions may be obtained for sure. So, as we have seen, taking the common understanding of dialetheism, there is a clear defense of dialetheism in da Costa's philosophy of paraconsistency, by claiming that there are true contradictions in abstract and formal domains.

Nonetheless, from da Costa's perspective, a significant motivation for paraconsistent logic arises from the possibility of true contradictions inherent in concrete reality. Accordingly, da Costa pursues a more radical line of inquiry than even Priest's dialetheism by investigating the presence of such contradictions in the empirical world — an issue related to a "problematic notion" that Priest avoids endorsing. This more radical inquiry aligns with da Costa's paraconsistency program: the core motivation behind paraconsistency becomes truly meaningful when we go a step further and locate the locus of contradictions in concrete reality, guided by our best empirical sciences. While this remains a research program in development, da Costa offers significant philosophical insights that point us toward where the sources of these contradictions might be found.

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Notes

[1]There are differences regarding adjunctive and non-adjunctive approaches to paraconsistency, but we shall not be concerned with those differences here; see da Costa et al. (2007), Szmuc et al. (2018) for further discussion on different formulations of paraconsistent logics.

[2]A French translation was published in 1997, see da Costa 1997.

[3]Note that asserting that da Costa developed dialetheist theses in his book *Essay on the Foundations of Logic* is quite different from claiming that da Costa is a dialetheist. As mentioned before, this paper does not aim to provide an exegesis of da Costa's thought, but rather to highlight some dialetheist ideas — which carry considerable philosophical weight — that he developed in his *Essay*.

[4]The paradox of the catalog arises from the assumption of a library that has a catalog of all, and only those, catalogs that do not list themselves.

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