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PRESENTATION OF THE SPECIAL ISSUE OF PRINCIPIA ON THE INTERPRETATION OF PARACONSISTENT LOGICS

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Abstract. We present this special issue of *Principia* on the interpretation of paraconsistent logics. After a brief discussion on the very idea of interpreting paraconsistent logics according to different perspectives, we introduce the source of the papers in this issue. Finally, we shortly present each paper that appears in this issue.

Keywords: paraconsistent logic • contradiction • interpretation

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Dedicated to the memory of Prof. Newton C. A. da Costa

1. Introduction

A system of logic is paraconsistent, according to usual definitions, if it violates the principle of explosion:

$$\alpha, \neg\alpha \nvdash \beta.$$

In other words, not everything follows from a contradiction. Although some technical difficulties involved in such a definition have been raised recently (see a discussion in Szmuc, Pailos, and Barrio 2018), there is no doubt that, in a paraconsistent logic, contradictions do not lead to triviality.

In this special issue of *Principia, an international journal of epistemology*, we bring together papers that contribute to the philosophical understanding of paraconsistency in general; that is, to make clearer, in philosophical terms, the general phenomenon that is made available by paraconsistent logics that contradictions are no

longer necessarily regarded as problematic, as they lose at least part of their traditionally troublesome aspects. One may spot some contradictions in one's theories or set of beliefs, and still, no particular logical embarrassment needs to arise from that.

To properly present the issue, we start by briefly discussing what kind of philosophical difficulties lie behind the maintenance of contradictions in our theories or systems of beliefs. That is done in section 2 by a quick mapping of some possible ways a paraconsistent logician relates to the contradictions that are tamed in a paraconsistent environment. This mapping will evidence that one may do that in a number of different and, perhaps, conflicting ways. Following that, in section 3, we briefly present the source from which most of the papers collected here came from: the second edition of the *Workshop on the Interpretations of Paraconsistent Logics*. And last, but not least, in section 4 we present the papers composing this special issue.

2. Philosophical challenges of paraconsistency

The philosophical problems concerning paraconsistent logics appear when one notices that we are being asked to assume that contradictions may work as our premises in inferences. By quickly looking at the literature, one easily finds different attitudes towards contradictions in the context of paraconsistent logics, ranging from more epistemically-oriented understanding of contradictions, such as 'inconsistency toleration', or 'accommodation of inconsistencies', to alethic readings such as 'acceptance of contradictions' (as being true) or 'belief' in (the truth of some) contradictions.

All of those types of relations to contradictions are also connected with the very idea that one may use a paraconsistent logic to keep rationality intact in the presence of contradictions, which are so damaging to systems of logic accepting the principle of explosion. Attempts to bring some order to such attitudes have been advanced in the literature of the philosophy of paraconsistent logic.

Three prominent ways of classifying the approach one has to the contradictions tamed by paraconsistency are: geographical (based on the different schools of paraconsistency), the distinction of various levels of commitment to paraconsistency, and the idea that paraconsistent logics require philosophical interpretations, which again may vary in degrees of commitment to contradictions. Let us sketch these options very quickly.

Geographical differences One way to classify the different approaches to paraconsistency may be framed in geographical terms: there are different schools of paraconsistency, the Australian, the Brazilian, the Belgian, the Canadian... This view is quite widespread, and relates distinct approaches to what a contradiction means and how we relate to it in terms of research programs mostly developed in some countries,

under the guidance of some main figures (see Tanaka (2003) for more information and additional discussions).

What distinguishes such schools, according to Tanaka, is how seriously one takes the contradictory claims:

> Arguably the most radical school, the Australian school of paraconsistency, led by Priest and Sylvan (né Routley), claims that there are some true contradictions and that *the* logic is paraconsistent. [...] The Belgian school, led by Batens, and the Brazilian school, led by da Costa, argue against the Australian school. They question the existence of true contradictions. More importantly, they not only reject the idea that *the* logic is paraconsistent but also deny that there is a uniquely correct logic. (Tanaka 2003, p.29)

So, the different approaches are distinguished by two levels: whether the members of a school are entitled to believe in true contradictions, and whether they believe in logical monism (the idea that there is only one true logic, *the* logic). It is said that members of the Australian school not only believe that there are true contradictions, but also, that there is one paraconsistent logic that correctly describes reasoning with such. The members of the Brazilian and Belgian school, on the other hand, are said not to believe that there are true contradictions, so that use of paraconsistent logics is restricted to deal with inconsistencies that should not be read literally as true; they must be understood as something else (perhaps they may be accommodated, or seen as temporary features of a defective theory, but never as definitive members of our ultimate belief set).

When one isolates the two problems, the problem of the appropriate reading of contradictions, and the problem of monism versus pluralism, one sees that part of the debate concerns whether the contradictions are to be thought of as true, or as something else. So, being a paraconsistent logician involves deciding whether one will believe that some of the contradictions appearing as premises may be true, or, in negative, case, providing an alternative reading of the use of contradictions as premises.

Levels of paraconsistent involvement A similar account or division is found in the attempt to divide constructors/developers/users of paraconsistent logic in three distinct levels, depending on the kind of commitment they have to the working of contradictions. As Beall puts it:

- Weak Paraconsistentist: a paraconsistentist who rejects that there are 'real possibilities' in which a contradiction is true; paraconsistent models are merely mathematical tools that prove to be useful but, in the end, not representative of real possibility

> - Strong Paraconsistentist: a paraconsistentist who accepts that there are 'real possibilities' in which contradictions are true, and more than one such 'real possibility' (and, so, not only the trivial one); however, no contradiction is in fact true
> - Dialetheic Paraconsistentist: a paraconsistentist who accepts that there are true contradictions — and, so, that there could be (since our world is a 'real possibility' in which there are some) (Beall 2004, p.6)

The classification ranges from a more instrumentalist use of paraconsistent systems, where the contradictions that are allowed to appear as premises do not represent any real possibility, to a more nuanced approach, a possibilist account, where one sees the mathematical models of paraconsistent logic as describing alternative scenarios where contradictions are made true, but no such scenario is the actual one, to a final scenario where one accepts that true contradictions exist in our world.

Clearly, this is a bit more nuanced than the geographical classification, introducing a distinction not only among those not believing in true contradictions, but also among those believing in true contradictions. That is, the classification allows for two broader groupings of the three options, depending on whether one privileges *true contradictions* or *contradictions in actuality* as the major feature of distinction. Weak and strong paraconsistentists differ from dialetheic paraconsistentists because the latter, but not the former, believe in actually true contradictions, contradictions true about the actual world.[1] On the other hand, strong paraconsistentists side with dialetheic paraconsistentist against the weak paraconsistentist, by accepting that some contradictions are true.

Philosophical interpretations of paraconsistency The idea that we should separate users of paraconsistency between those believing that some contradictions are true and those not holding such beliefs is also present in a more recent debate, holding between proponents of the idea that paraconsistent logics, as formal systems, need to be supplemented with a *philosophical interpretation* (see Carnielli and Rodrigues 2019). According to Carnielli and Rodrigues (2019, p.3790), there are three options when it comes to understand contradictions in paraconsistent logics: one may be

a) a pragmatist about paraconsistency, not discussing the meaning of contradictions,

b) a dialetheist, accepting that some contradictions concerning concrete reality[2] are true, and, finally,

c) an epistemic reading of contradictions, according to which contradictions are not true, they are a result of conflicting evidences found in theories and belief systems.

Again, the classification is different from the previous one, given that it introduces a more specific view on the table when it comes to deny the truth of a contradiction. As Carnielli and Rodrigues put it:

> A third position in paraconsistency, antagonistic to dialetheism, claims that no contradiction is ontological but, rather, all contradictions that occur in scientific theories, belief systems, a number of situations in informal reasoning, and even in semantic and set theoretical paradoxes — that are, strictly speaking, results about languages with certain characteristics — have epistemic character in the sense that they are related to thought and language. This is the position endorsed by us. (Carnielli and Rodrigues 2019, p.3790)

This idea has raised a debate on whether paraconsistent logics are more correctly understood according to one of the possible interpretations, and only one of the interpretations (as Carnielli and Rodrigues proposed in (2019)), or whether different interpretations may be used for different purposes (a view advanced by Barrio and da Re (2018)), resulting in a tolerant approach to interpretations. As we mentioned, this adds a different dimension to the distinction advanced by Beall, which does not countenance contradictions taken on an epistemic dimension, while seemingly recovering and adding some substance to the geographic classification too.

One difficulty with this way of putting the problem (which does not affects the two previous classifications, or, at least not so obviously) is that it seems to labor under the assumption that paraconsistent logics are first available as formal systems, and as such, stand in need of an additional philosophical layer of interpretation. Independently of whether one is a monist about interpretations (as Carnielli and Rodrigues in (2019)) or a pluralist (as Barrio and da Re (2018)), one faces the challenge as one of attributing some philosophical meaning to different logics available beforehand. Against this way of framing the difficulty, Arenhart (2021, 2022) has argued that it puts the problem in terms that are not fair to the different positions put into conflict. That is, a dialetheist is someone who believes that some contradictions are true, and goes looking for an appropriate logic for that thesis. One does not start with a random paraconsistent logic and attempt to provide for a dialetheist reading of it. In fact, many paraconsistent logics are just inappropriate for dialetheism, making the idea that one should 'interpret' them in dialetheic terms unpalatable from the start (see Arenhart 2021 for details). Dialetheism, as a philosophical claim, and even the idea that one may face conflicting evidence in some situations, are not different ways to read systems of logic, but rather, are different kinds of phenomena requiring a paraconsistent logical modeling.

Where do we go from here? As one can see, one may frame the challenge of understanding contradictions in the context of paraconsistent logics in many different

ways. Not all of them capture the same nuances, but the contrast between dialetheistic views and non-dialetheistic views is a constant between them all. The very idea that paraconsistency needs an interpretation, and how that interpretation would look like, may be seen as a source of philosophical dispute. Given all of these many intricacies, the present special issue offers papers that address some of these issues and contribute to the still ongoing debate.

3. The source of the papers

Most of the papers collected in this special issue were presented at the second edition of the *Workshop on the Interpretations of Paraconsistent Logics* (WILP, on the Brazilian acronym). The workshop was proposed as a platform where those different topics concerning the understanding of paraconsistency and contradictions could be presented and debated in a friendly environment. The first edition of the workshop took place online, during the Covid-19 pandemic, in 2020, while the second edition took place in Florianópolis, in 2022.[3]

The guiding theme behind the II WILP was precisely a celebration of Newton da Costa's developments on paraconsistent logics and of his philosophy of paraconsistency, although the participants were allowed to discuss any other theme related to paraconsistency and its philosophy. It was likely the last workshop to be graced by the distinguished presence of Newton da Costa in person, before his passing in April 2024. Those attending the workshop had the privilege of seeing da Costa explain with his usual enthusiasm recent features of his research on the intersection of paraconsistency and the foundations of physics. It is to him that we dedicate this special issue.

4. The papers in this issue

The papers present in this special issue engage with many of the topics we have mapped before concerning the relation of paraconsistency with the nature of contradictions and the latter's proper understanding.

Abílio Rodrigues, in his paper *'What is this thing called dialetheism?'* discusses precisely the major topic present in all of the different approaches to understanding paraconsistency we have presented here: *dialetheism*. As we have seen, Rodrigues, along with Carnielli in Carnielli and Rodrigues (2019) characterized dialetheism as a thesis involving true contradictions in reality. Now, Rodrigues comes back to such a claim in order to make the meaning of dialetheism more precise. As he argues, there are in fact no textual evidence that dialetheism, at least in Graham Priest's version, requires contradictions to be true of concrete reality, and, as a second point,

he also discusses what would it take for one to be one such dialetheist. As a result of such exploration, Rodrigues claims that dialetheism either reduces to an implausibly strong thesis or else to a thesis that clearly resembles the claims advanced by the epistemic reading of paraconsistency, where contradictions are not a result of reality, but of our theories, language, and thought processes.

Eduardo Barrio, Edson Bezerra, and Bruno da Re, in *'Philosophical interpretations matter'*, consider the problem of providing a philosophical interpretation to paraconsistent logics itself. As we have briefly presented before, one may raise some doubts as to whether the very idea of having formal systems first, and laying some philosophical content on the top of them afterwards is a productive one. Arenhart (2022) argued that one may dispense with the very idea of a philosophical interpretation and make sense of logical theories only by appealing to the usual distinction between pure and applied logics. After carefully revising the different approaches to paraconsistent logics and the claims that such logics may need no interpretation, Barrio, Bezerra, and da Re develop three major arguments for the relevance of the philosophical interpretations of logics, in addition to the idea that a logic may have an application. In a nutshell, the arguments invoke the use of such interpretations to discussing the idea of a 'right logic', the importance of interpretations in making sense of the role of distinct logical concepts, and finally, the idea that one and the same application may have distinct interpretations.

The idea that philosophical interpretations are important is also playing a role in the contribution by Otávio Bueno. In his paper, *'Overinterpreting logics'*, Bueno relates philosophical interpretations to the claims that logic is universal and topic neutral. As he argues, giving up topic neutrality allows one to investigate distinct domains by using distinct systems of logic. What such systems investigate are different kinds of possibilities in the space of possibilities. Bueno advances a modalist approach to account for this claim. Also, he classifies distinct kinds of interpretations of paraconsistent logic into metaphysical (which involves dialetheism), epistemic (which accounts for the epistemic approach by Carnielli and Rodrigues) and a semantic interpretation, which is the most deflationary one, and the only one needed in logic. As Bueno sees it, the distinct kinds of interpretations, although being dispensable, enrich our understanding by adding possibilities to the field of investigation.

Hitoshi Omori investigates a question concerning the relation between negation and modality in his contribution, *'Is **S5** paraconsistent?'*. Some recent suggestions that one may define a paraconsistent negation in modal systems such as **S5** by combining the operator of possibility with classical negation. Omori raises the worry that this strategy also produces a paracomplete negation, when one combines necessity with negation. Why should one way be preferred over the other? Besides suggesting that without further argument it is arbitrary to say that **S5** is paraconsistent, and not paracomplete, Omori investigates weaker modal systems that are both paraconsistent

and paracomplete in the mentioned sense, while still having interesting properties.

Guilherme Araújo and Francisco Augusto Lages develop a detailed and philosophically motivated view which could be seen as a development of the strong paraconsistentism view as suggested by Beall. In their *'A fictional guide to impossible truths'* they claim that some truths, which include some contradictions, correctly describe some impossible situations. In that sense, one may have impossible truths, truths that are understood as not actual, but that must still be available in some model of our logical theory. The paper also develops a logical system to account for such a view — the Logic of Impossible Truths —, discussing also the nature of logic according to such a view. For the latter purpose, the authors borrow from the philosophy of science an account of scientific models where models are understood as having an artifactual nature, being better seen as epistemic tools that serve some aim. The authors also offer some open problems for future developments.

Ederson Safra Melo and Jonas R. Becker Arenhart explore, in *'Newton da Costa on true contradictions: from aporias to reality'*, the historical accuracy of attributing to da Costa some radical anti-dialetheism. By investigating claims advanced by da Costa in his *'Ensaio Sobre os Fundamentos da Lógica'*, the authors argue that da Costa may be seen as advancing dialetheistic theses that, in some sense, may even go beyond the scope of Graham Priest's defense of dialtheism. More precisely, it seems that da Costa offered not only arguments for the existence of true contradictions such as the Liar and Russell's set, which live in an abstract domain, but also envisaged the possibility that the concrete world may hide contradictions. A parallel between da Costa's arguments and the usual distinction between semantic and metaphysical versions of dialetheism is pursued, as well as some similarities between the treatment given by Priest and da Costa to some paradoxes motivating dialetheism.

Décio Krause discusses the consequences of using paraconsistent logics in the philosophy of empirical sciences in his paper *'Some remarks about going towards inconsistencies'*. By analyzing some sample cases of inconsistencies in the sciences, such as the early calculus, complementarity in quantum mechanics, and the famous Azande case, Krause argues that one may deal with them, or live well in their presence, without necessarily changing to a paraconsistent logic. By adhering to a Wittegnsteinean motto that once a contradiction is spotted, one needs not to 'go there', Krause suggests that in practice people avoid logical triviality by not reasoning further from the contradiction. This strategy may remind one of the distinction between the differences introduced by Gilbert Harman (1988) between the statics and the dynamics of reasoning. Clearly, this is one available strategy to deal with inconsistencies and still avoid even paraconsistency.

References

Arenhart, J. R. B. 2021. The evidence approach to paraconsistency versus the paraconsistent approach to evidence. *Synthese* **198**: 11537–11559.

Arenhart, J. R. B. 2022. Interpreting philosophical interpretations of paraconsistency. *Synthese* **200**: 1–21.

Barrio, E.; da Re, B. 2018. Paraconsistency and its Philosophical Interpretations. *Australasian Journal of Logic* **15**(2): 151–170.

Beall, Jc. 2004. Introduction: At the intersection of truth and falsity. In: G. Priest; Jc Beall; B. Armour-Garb (eds) *The law of non-contradiction. New philosophical essays*, pp.1–22. Clarendon Press: Oxford.

Carnielli, W.; Rodrigues, A. 2019. An Epistemic Approach to Paraconsistency: a Logic of Evidence and Truth. *Synthese* **196**: 3789–3813.

Harman, G. 1988. *Change in view. Principles of reasoning*. Bradford Book: Cambridge.

Szmuc, D.; Pailos, F.; Barrio, E. 2018. What is a paraconsistent logic? In: W. Carnielli; J. Malinowski (eds.) *Contradictions, from Consistency to Inconsistency*, pp.89–108. Cham, Switzerland: Springer.

Tanaka, K. 2003. Three schools of paraconsistency. *Australasian Journal of Logic* **1**: 28–42.

Notes

[1]Nota bene: this is not the same as saying that contradictions in the actual world need to concern *concrete* entities; the contradictions only need to be found true in the actual world. The contradiction resulting from the Liar paradox is one such example, if one takes it to be true.

[2]Here, specifically, dialetheism is identified by Carnielli and Rodrigues with this very specific understanding of the position — namely, that contradictions must be true about *concrete reality*.

[3]See https://sites.google.com/ufma.br/iiwilp for details concerning the second edition. The third edition of the WILP took place in Florianópolis, in December 2024.