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THE FICTIONAL GUIDE TO IMPOSSIBLE TRUTHS

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Abstract. In this paper, our main goal is to present a new account for contradictions and impossible truths. It is loosely based on both Austin's account of truth and the Logic of Impossible Truths (LIT), a formal semantics designed to address incomplete, inconsistent, and non-normal sets of sentences. Our main thesis is that some truths are impossible (in the sense that they accurately classify impossible situations), but no impossibility is real, no impossible truth is about real situations. We divide the paper into four sections. In section 1, we motivate the Riddle of Impossible Truths as a general observation based on paradoxes, counterpossibles and impossible fictions. In section 2, we present our account of impossible truths, drawing on LIT and Austin's views on truth and propositions. Section 3 is dedicated to reflecting on the artifactual nature of impossible situations. We also argue, based on Knuuttila's ideas about models, that impossible situations serve as epistemic tools. Finally, in section 4, we conclude the paper with some thoughts on future directions for research.

Keywords: impossible truths • dialetheism • paraconsistency • nonvacuism • situations • artifacts • epistemic tools

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1. The Riddle of Impossible Truths

"Impossible" is a predicate attributed to things that are not feasible, cannot occur, exist, or be real. A physicist would assert that plutonium-186 is impossible, a mathematician would posit that a round square object is impossible, and a logician would affirm that a contradiction is impossible. We encounter different modalities regarding the concept of impossibility, which depend on the restrictions we consider. Logic represents the weakest level of possibility, as anything that is possible is also logically possible. Plutonium-186 and round square objects are logically possible, but contradictions appear to be impossible at all levels.[1]

Some possibilities are unreal (nonactual) but nothing is both impossible and real (actual). The same holds for truth values: some sentences are false but possible while all impossible sentences are false. For example, the sentence "Isaac Asimov was born in the Netherlands" is false, but it could be true, while the sentence "Asimov is not Asimov" couldn't be true. Using the traditional worldly-based jargon, the first sentence is false in the actual world but it could be true in some worlds, while the second is false in all possible worlds.

In this paper, we argue that some impossible sentences are, in a certain sense, true. We sustain here a position concerning impossible truths that can be somehow linked with the doctrine of Plato's beard, as baptized by Quine:

> This is the old Platonic riddle of nonbeing. Nonbeing must in some sense be, otherwise what is it that there is not? This tangled doctrine might be nicknamed Plato's beard; historically it has proved tough, frequently dulling the edge of Occam's razor. (Quine 1963, p.1–2)

The riddle highlights a problem concerning nonexistent things. Some things do not exist. Thus, in *some sense*, nonexistent things do exist, as they *are* things that do not exist. Now, assume along with this tangled doctrine that some things (or objects) do not exist. What about making true statements about them? Does it also make sense that we can state truths about unreal (even impossible) things? Consider, for example, the round square cupola on Berkeley College. It is an impossible object, of course, but we can state certain truths about it: that it is both round and square, and that it is located near Berkeley College. We can, in some way, classify it accurately. Based on the Riddle of Nonbeing, we present and argue for what we call the *Riddle of Impossible Truths*.

> **The Riddle of Impossible Truths:** Some truths must, in a certain sense, be impossible; otherwise, how are we supposed to truly classify impossible things?

In what follows, we present some cases in favor of the Riddle of Impossible Truths, with no intention of being precise or convincing about how to make sense of impossible (unreal) truths. We postpone to the next section a framework in which we provide precise definitions that shed some light on this riddle.

Contradictions abound in natural language under naive (intuitive) representations of concepts like truth, set, and knowledge. Take, for example, the naive concept of truth, which is roughly guided by the following general schema:

> **Transparency:** We can replace α with $T(\ulcorner \alpha \urcorner)$ (and vice versa) in any context whatsoever.[2]

Since natural language is equipped with many different resources to produce self-reference, we can use Transparency to prove that some impossibilities (contradictions) are true. Such an argument is usually known as the **Paradox of the Liar**. Consider the following example among many versions of the Liar:

(1) Sentence (1) of this paper is not true.

Let sentence (1) be true. From Transparency, (1) is not true. Thus if sentence (1) is true, it is not true. Otherwise, if (1) is not true, again by Transparency, it is true. Therefore, (1) is true iff (1) is not true, from which we conclude that both (1) is true and (1) is not true, a contradiction.[3]

A similar, well-known argument can be made to prove a contradiction from the naive Comprehension Schema, which guides our intuitive concept of set-membership. This argument is known as Russell's Paradox. Another way to deliver contradictions is through the Paradox of the Knower, based on the intuitive concept of knowledge formalized by an epistemic modal operator that preserves both factivity and necessitation. There is a wide range of arguments based on paradoxes that support the idea that some contradictions are true. This is precisely what defines **Dialetheism**, the thesis according to which some contradictions are actually true. For more in-depth understanding of Dialetheism and the aforementioned paradoxes, readers are referred to works by Priest (2006) and Beall (2009), among others. We will further discuss the distinction between Dialetheism and the Riddle of Impossible Truths later in this paper.[4]

A second case for the Riddle of Impossible Truths can be made from fiction. It is undeniable that we can find impossible truths, including contradictions, in fiction. In Isaac Asimov's novel "The Gods Themselves", parallel universes with different physical laws exchange matter. This allows for tungsten-186 to transform into plutonium-186, an isotope that is naturally impossible in our universe. There is an infinite book with no beginning or end in "El libro de arena", a short story wrote by Borges. In one of the many alternative worlds explored in "Einstein's dreams",

> [...] the passage of time brings increasing order. Order is the law of nature, the universal trend, the cosmic direction. If time is an arrow, that arrow points toward order. The future is pattern, organization, union, intensification; the past, randomness, confusion, disintegration, dissipation. (Lightman 2011, p.67)

In Graham Priest's short story "Sylvan's Box", an empty box is found to also contain something inside. This is not an illusion but rather a true depiction of an impossible scene where a contradictory object exists.

These examples demonstrate impossible truths in fiction, depicting what is both impossible and, somehow, true. It is important to note that not everything can be considered true simply because it involves an impossibility (a true contradiction). For example, it is not true that Sylvan's box is also a time machine, and it is not true that Borges book is a round square. None of these scenes are real, but some sentences accurately represent fictional impossible truths. Therefore the examples above also testify that we can think, talk, reason, and make sense of the impossible in a nontrivial way. In the words of Priest about Sylvan's Box:

> There is a determinate plot: not everything happens in the story; and people act in intelligible ways, even when the inconsistent is involved. (Priest 2005, p.121)

A similar lesson seems to be available from the semantics of counterpossibles. A **counterpossible** is a subjunctive conditional (a counterfactual) in which the antecedent is not only false but impossible. To make the lesson clear, consider the following sentences:

(2) If Hobbes had squared the circle, the mathematicians of his time would not be surprised.

(3) If there were a recursive computer that could consistently prove any mathematical sentence that is true, then Gödel's incompleteness theorem would be true.

(4) If there was a real true contradiction, Dialetheism would be wrong.

We believe that sentences (2)-(4) above share an intuitive valuation; they all appear to be false since they suggest a wrong connection between the antecedent and consequent, despite the impossibility of the first. Nevertheless, according to the traditional account, they are all considered vacuously true. The traditional account for counterpossibles is derived from the worldly-based semantics proposed by Lewis (1973) and Stalnaker (1968). Roughly speaking, the traditional account for counterfactuals (TAC) states the following:

> **TAC:** A counterfactual $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ is true iff all the closest **possible** α-worlds are β-worlds.

Since there is no possible world at which the antecedents of (2)-(4) are true, the right hand side of TAC vacuously holds for each and (2)-(4) all turn out true. Notice that, when α is impossible, not only $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ takes the value true, whatever being said by β, but also $\alpha \mathrel{\Box\!\!\rightarrow} \neg\beta$ (the dual counterfactual). This is a kind of triviality result, one known as Vacuism.

Defenders of Vacuism (such as Lewis, Stalnaker, and Williamson 2018) argue against our previous statement regarding sentences (2)-(4) appearing to be intuitively false. Their argument is that anything follows from an impossibility, such as a contradiction, precisely because the impossible cannot happen. For instance, there is a logical proof demonstrating that the set of arithmetical truths in the standard interpretation is not recursive.[5] Consequently, the antecedent of (3) is logically impossible. As anything follows from what is logically impossible, it then follows that

Gödel's incompleteness theorem is true. This is perplexing because Gödel's theorem asserts (roughly) that for any sufficiently strong theory Σ, there are true arithmetical sentences that are not provable in Σ, contradicting the antecedent of (4).

Vacuism also presents a peculiar look when considering sentence (4). Since it is logically impossible for a contradiction to be actually true, anything could be the case whether it would. Thus (4) is also vacuously true. However, Dialetheism precisely states that some contradictions are actually true, so how can we say Dialetheism would be wrong as a consequence of some contradiction being actually true?

Contrary to Vacuism, our intuition suggests that sentences (2)-(4) are not true, but their dual counterfactuals are indeed true. We believe that some counterpossibles are nonvacuously true, while others are false. This perspective, known as Nonvacuism, has been advocated by Berto (2019), Bjerring (2013), Nolan (2016), and many others. Nonvacuism claims for an extension of TAC through the inclusion of the impossible. For a counterfactual $\alpha \mathrel{\Box\!\!\rightarrow} \beta$ to be false, we need to deny somehow the connection between α and β, we need, for example, a closest world in which α is true, and β is not, but this only happens outside of possible worlds. In the next section, we will present our own version of a semantics that is compatible with Nonvacuism.

Nonvacuism has a very close connection with Paraconsistency. A logic is called paraconsistent if the Principle of Explosion (or *Ex falso quodlibet*) doesn't hold in it, namely if the schema $\alpha \land \neg\alpha \vDash \beta$ is not universally true without restrictions. Explosion captures this aforementioned idea that anything follows from a contradiction. Thus an inconsistent set of sentences closed under classical consequence would also be a trivial set of sentences, one with all sentences in it. The main point behind paraconsistent logics is to allow contradictions without triviality, to allow an inconsistent set of sentences to be closed under logical consequence without triviality. We could make explicit the link between Nonvacuism and Paraconsistency by just considering the following counterpossible:

(5) If there was a real contradiction, everything would be true.

This is a counterfactual version of Explosion. Following the intuitions behind Nonvacuism, we could think of inconsistent nontrivial situations to make false this counterfactual, and this would also have to be a paraconsistent situation. Therefore, it seems, paraconsistent logics cannot agree with Vacuism.

Let's turn back now to the Paradox of the Liar. The reader can easily revisit the informal argument that establishes a contradiction from the Liar sentence (1) mentioned earlier. We could easily make the same argument from (1) to a contradiction, but using counterfactuals instead:

(6) If sentence (1) were true, (1) would not be true.

(7) If sentence (1) were not true, (1) would be true.

We employ Transparency to support both conditionals. The antecedents of both conditionals are not only false but also impossible since they lead to a contradiction in each case. Therefore, (6) and (7) are counterpossibles. According to Vacuism, we should not require an argument for them as they are considered vacuously true. Thus, this informal argument utilizing Transparency to establish a contradiction from the Liar sentence appears to presuppose a nonvacuous interpretation of counterpossibles. Now, as before, we could infer from (6) and (7) that "(1) would be true iff (1) would not be true". To conclude that (1) is both true and not true, we would siimply need *Modus Ponens*, conjunction rules, and a counterfactual version of *reductio* ($\alpha \mathrel{\Box\!\!\rightarrow} \neg\alpha \vDash \neg\alpha$). In fact, when restricted to possible worlds (as in TAC), $\alpha \mathrel{\leftarrow\!\Box\Box\!\!\rightarrow} \neg\alpha \vDash \alpha \wedge \neg\alpha$. The inclusion of impossible worlds would easily allow for useful countermodels in this context.

The last paragraph leaves open the way to an approach for the Paradox of the Liar based on a counterfactual reading of conditionals. This route is more deeply explored in Cardoso 2024, here we are solely concerned with the Riddle of Impossible Truths, including Liar sentences as examples. Things become considerably more challenging with a **Counterpossible Curry** sentence:

(8) If sentence (8) were true, everything would be true.

(8) can be true in no possible world, otherwise (by *Modus Ponens*) it would be a trivial (impossible) world. Thus (by Transparency) the antecedent of (8) is impossible. From TAC, it follows that (8) is vacuously true in all possible worlds, and its antecedent is (by Transparency) also true in all possible worlds. Therefore (by *Modus Ponens* again), everything is true in all possible worlds, but then, there are no possible worlds, all worlds closed under classical logic, TAC, and Transparency are trivial (impossible) worlds.

Counterpossible Curry presents a significant triviality issue for TAC, an independent triviality result that warrants serious consideration here.[6] It is important to emphasize that Nonvacuism aligns with the Riddle of Impossible Truths. To challenge the triviality result of counterpossibles, we must create space where the antecedent is true while the consequent is not true. This requires accommodating impossible truths. By expanding TAC to include impossible worlds, we reject the notion that counterpossibles are vacuously true and instead find that some closest (impossible) world renders the antecedent of the counterpossible true, while its consequent is not true.

To summarize the point we have been trying to make in this section: some impossibilities are, in a certain sense, true. The existence of plutonium-186, Sylvan's box, and the infinite book with no beginning or end are impossible truths. Hobbes squaring the circle is an impossible truth. The Liar sentence is a true contradiction, and the Counterpossible Curry sentence is an impossible trivializing truth. In the next section, we provide a framework with an important distinction concerning truth that allows for impossible truths, as required by the Riddle of Impossible Truths, without taking the same steps as Dialetheism.

2. Situating Impossible Truths

As we have argued thus far, some impossibilities must, in a certain sense, be true. In what follows, we present a view on propositions according to which propositions always concern (are about) situations. We maintain that some impossibilities, including contradictions, are true. However, our perspective confines the impossible to non-actual (unreal) situations, while allowing true propositions to say the impossible. Yet, no impossible truth concerns actual situations.

Before that, we present a simplified version of the **Logic of Impossible Truths** (LIT), a framework developed to handle situations (as sets of sentences) of any kind, including inconsistent, incomplete, trivial, non-normal, and empty ones.[7] We also make an important remark concerning truth based on Austin's distinction between demonstrative and descriptive conventions.[8] As a result, we provide an account that allows for impossible truths without positing any impossible truth to be about real situations.

LIT is constructed based on the notion of situation developed in the works of Barwise and Perry (1989), Perry (1986), Barwise and Etchemendy 91987), and Barwise (1989). However, we make some significant changes. Therefore, it is convenient to begin with some conceptual clarifications regarding the intended meanings of such primitive terms in the semantics of LIT.

Situations are ways things could or could not be. Thus, situations include not only actual situations but also nonactual (possible and impossible) ones. An **actual situation** is a part of reality, determined by the insertion of an informational agent in a specific space-time location; it is what the agent actually sees. Consider the scene in which Dudu and Dani are face to face, playing cards, in front of each other. Dudu can see his own face cards, but he can't see Dani's face cards nor the backside of his own cards. Dani has a dual perspective, seeing her own face cards but not seeing Dudu's face cards nor the backside of her own cards. Thus, Dudu sees that he has the four clubs, but only Dani sees that the backside of this card is stained with coffee. Of course, situations are not only determined by perspectives; different agents can see differently in the same location, but they share an environment, they cut different actual situations as distinct slices of the same reality.

A **non-actual (unreal) situation** is an alternative way, a fiction. Fiction is always relative to an actual situation; it is an alternative (possible or impossible) relative to what is actually the case. For example, it is actually false that the Axis Powers won

World War II, but in alternative possible situations (as depicted in the famous novel by Philip K. Dick, "The Man in the High Castle"), it is true. It is actually false that Hobbes squared the circle, but in some fictional alternative impossible situations, he did. Alternative ways are fictions in the sense that they are relative to real ways. In the words of Eco (1995, p.82), "[t]his means that fictional worlds are parasites of the real world".

A **normal situation** is one that is closed under logical consequence, as we are going to define next. An **inconsistent situation** is one that satisfies a contradiction, satisfying both α and $\neg\alpha$ (for some sentence α). A **world** is a **complete situation**, one that satisfies at least one of the set $\{\alpha, \neg\alpha\}$ for each sentence α. A world is a situation that is about everything. To preserve the weakness feature of logical possibility, we understand that all possible situations are both normal and consistent. Since logical consequence, as we define it in LIT, is both paracomplete and paraconsistent, not all situations are worlds, and not all inconsistent situations are trivial. We can, of course, have a completely full, normal, and trivial situation, just as we can have a completely empty, nonnormal situation.[9]

We introduce later an important constraint, stating that all actual situations are consistent with one another (and with themselves). An essential result in LIT, which we won't delve into here, is that, given Transparency, no actual situation qualifies as a world since there is no possible (consistent) world. Therefore, we should consider worlds as fictions. This should not be interpreted as antirealist but only as regarding reality as a proper class, an incomplete universe, in the sense proposed by Grim (1991).[10]

For simplicity, consider a propositional language $\mathscr{L}$ based on a denumerable set of atoms $\mathscr{A} = \{p_0, p_1, p_2, ...\}$ and logical symbols $\{\neg, \wedge, \vee, \rightarrow, \leftrightarrow\}$, using the standard recursive rules to define the set of sentences (well-formed formulas, WFFs). To establish a logical consequence relation on sets of sentences in this language, we introduce the notion of LIT-models.

Definition 1. A LIT-model for $\mathscr{L}$ is a tuple $\mathscr{M} = \langle S, N, P, @, \{R_\alpha \mid \alpha \in WFF\}\rangle$, where:

1. $S \subseteq \wp WFF$ is the set of all situations.
2. $N \subset S$ is the set of normal situations.[11]
3. $P \subseteq N \cap C$ is the set of possible situations.[12]
4. $@ \subseteq P$ is the set of actual situations, such that, $\bigcup @ \in P$, and $@ \neq \emptyset$.
5. $R_\alpha \subseteq S \times S$, and $f_\alpha(s) = \{s' \in S \mid sR_\alpha s'\}$.
6. $f_\alpha(s) \subseteq S \cap \{\alpha\}$, for all $s \in N$.
7. If $\alpha \in s$, and $s \in N$, then $s \in f_\alpha(s)$, for all $s \in S$.
8. For all $s \in N$, s is closed under the following rules:

1. $\alpha \wedge \beta \in s$ iff $\alpha \in s$ and $\beta \in s$.
2. $\neg(\alpha \wedge \beta) \in s$ iff $\neg\alpha \in s$ or $\neg\beta \in s$.
3. $\alpha \vee \beta \in s$ iff $\alpha \in s$ or $\beta \in s$.
4. $\neg(\alpha \vee \beta) \in s$ iff $\neg\alpha \in s$ and $\neg\beta \in s$.
5. $\alpha \in s$ iff $\neg\neg\alpha \in s$.
6. $\alpha \to \beta \in s$ iff $f_\alpha(s) \subseteq S \cap \{\beta\}$.
7. $\neg(\alpha \to \beta) \in s$ iff $f_\alpha(s) \cap \{\neg\beta\} \neq \emptyset$.

Thus, we present the notion of an LIT-model, based on constraints over situations represented as sets of sentences. Normal situations are constrained by logical rules for logical symbols, as outlined in item 8 of definition 1. From 8.1 to 8.5, LIT-models coincide with propositional FDE rules (First Degree Entailment). It's important to note that we can have both inconsistent normal situations and incomplete normal situations, but due to item 4, no actual situation is inconsistent.

8.6 and 8.7 of Definition 1 deliver a nonvacuist *ceteris paribus* interpretation for conditionals, without making any semantic distinctions between indicative and subjunctive moods. We can interpret $f_\alpha(s)$ as "the set of all situations in which, with everything relevant being the same as in s, and α being true" or as "all *ceteris paribus* situations with s in which α is true". Notice that a counterpossible $\alpha \to \beta$ might end up false in a normal (or even possible or actual) situation s if there is an impossible situation s' that is *ceteris paribus* s under α, and $\neg\beta \in s'$. For some counterpossibles to be untrue in normal situations, we need more than just impossible *ceteris paribus* situations where the consequent is false; we need non-normal situations. Take, for example, the Counterpossible Curry (8) mentioned earlier. Due to transparency, all normal situations in which (8) is true are also situations where the antecedent of (8) is true, and by MP, the consequent is therefore true as well. Consequently, the antecedent of (8) can hold in only one normal situation — the one where everything is true. Worse, if we do not include non-normal situations in the model, (8) is true in all normal situations, making everything true in all of them.

Constraints provided by items 6 and 7 are intended to preserve conditional identity and MP, but they also explicitly articulate the intuition behind the *ceteris paribus* relation in the definition of LIT-models. Item 6 states that all situations which are *ceteris paribus* with respect to s under α are situations in which α is true. Item 7 states that s is *ceteris paribus* itself under α when α is true in s.

Alternatively, we might conceptualize the set of unreal situations $S \backslash @$ of each model $\mathscr{M}$ as furnished by @ along with a fictionalizing operator $\Phi : @ \longrightarrow \wp\wp WFF$. For each real situation $s_1 \in @$, Φ yields $\Phi(s_1) \in \wp WFF$, a collection of alternative situations (relative to s_1). Thus S is the image of Φ, while $N \subset S$ once again is the set of situations in S that are closed under LIT logical consequence. The nature of these alternative situations, relative to s_1, hinges on various facets of the scenario at hand. Consider, for instance, the scenario where a reader is engrossed in the novel "Flatland" by Anthony Abbott or the scenario where the reader contemplates becoming a thrash metal drummer. In the latter instance, we introduce a few specific modifications to grasp the intended meaning (maybe some scary monster tattoos on her arms), whereas in the former, substantial alterations to the actual physical reality are necessary, such as rendering it two-dimensional and imbuing geometric entities with intentions and emotions. Thus, Φ undeniably operates within a contextual framework; while we may lack an effective method to ascertain S based on @, we can still regard it as the image of Φ. Φ mirrors, in a way, the function of imagination in actual scenarios by generating alternative outcomes rooted in reality.

Using the notion of LIT-models, we can now define LIT logical consequence:

Definition 2. Let $\Gamma \subseteq WFF$ and $\alpha \in WFF$. Thus $\Gamma \vDash \alpha$ iff there is no LIT-model $\mathscr{M}$ with $s \in N$, such that, $\Gamma \subseteq s$ and $\alpha \notin s$.

Logical consequence in LIT is thus truth preservation in every normal situation of all LIT-models. In this paper, we are not concerned with the logical consequence delivered by LIT-models. It suffices to say that it is paracomplete, paraconsistent, and preserves Conditional Identity and MP, but it invalidates Contraction ($\alpha \rightarrow (\alpha \rightarrow \beta) \nvDash \alpha \rightarrow \beta$), and Conditional Reduction ($\alpha \rightarrow \neg\alpha \nvDash \neg\alpha$).[13]

Up to this point, we have mostly argued for the Riddle of Impossible Truths and presented a logical consequence relation based on accommodating impossible truths. However, we have not yet provided a framework for understanding impossible truths. How are we supposed to make sense of impossible truths? How is it possible for something to be both impossible and true? And could such an account be distinguished from Dialetheism? Next, we explain how Austin's (1950) conception of truth aligns with this idea, thus supporting a situated view of impossible truths.

The key feature of Austin's version of correspondence, the one that allows for our understanding of (unreal) fictional truths, is that situations are understood to attain truth in two different aspects. Roughly, truth is a form of correspondence between a historical situation (as a token) and a type of situation. In his words:

> [...] there must be two sets of conventions:-
>
> *Descriptive* conventions correlating the words (=sentences) with the *types* of situation, thing, event, etc., to be found in the world.
>
> *Demonstrative* conventions correlating the words (=statements) with the *historic* situations, etc., to be found in the world.
>
> A statement is said to be true when the historic state of affairs to which it is correlated by the demonstrative conventions (the one to which it "refers") is of a type with which the sentence used in making it is correlated by the descriptive conventions. (Austin 1950, p.115–116)

The same point is made by Barwise:

> **Thesis 2:** *Facts are relative to the focus situation s of concern to a situated, cognitive agent. Basic propositions classify the agent's classification of the focus situation. Hence a basic proposition has two components: the focus situation s it concerns and the state of affairs σ it uses to classify s. Such a proposition is true if σ correctly classifies s; otherwise it is false.* (Barwise 1989, p.228)

When presenting LIT, we considered situations as represented by sets of sentences, implicitly suggesting that the sentences in the set are made true by the represented situation. Now, it is clear from the remarks made by Austin and Barwise that truth relates to both types of situations and tokens of situations. Truth cannot be isolated from the concerned situation; it is an attribute of propositions, not sentences, and a proposition is always concerned with a situation.

Consider the card game example between Dudu and Dani as before, with the addition of a TV in the corner playing a soccer game. At some point, Dudu says, "the game is over". He might be talking about the soccer game, but he might also be talking about the card game. Truth is the correct classification of the concerned (focused) situation, determined in one case by the referee's whistle and in the other by a winning hand of cards.

In our own words, we propose the following definition:

Definition 3. Let $\alpha \in WFF$ and $\mathscr{M}$ a LIT-model with $s \in S$. Thus:

1. $\{s, \alpha\}$ is a proposition.
2. Let $p = \{s, \alpha\}$. Thus p is true in $\mathscr{M}$ iff $\alpha \in s$.[14]
3. Let $p = \{s, \alpha\}$. Thus p is false in $\mathscr{M}$ iff $\neg\alpha \in s$ and $s \in @$.
4. Let $p = \{s, \alpha\}$. Thus p is really true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in @$.
5. Let $p = \{s, \alpha\}$. Thus p is unreally true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in S \backslash @$.
6. Let $p = \{s, \alpha\}$. Thus p is possibly true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in P$.
7. Let $p = \{s, \alpha\}$. Thus p is impossibly true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in S \backslash P$.
8. Let $p = \{s, \alpha\}$. Thus p is normally true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in N$.
9. Let $p = \{s, \alpha\}$. Thus p is unormally true in $\mathscr{M}$ iff $\alpha \in s$ and $s \in S \backslash N$.

We can now use Definition 3 to provide a meaningful answer for the questions raised by the Riddle of Impossible Truths.

Proposition 1. *There is a model $\mathscr{M}$, such that;*

1. *Some contradiction is normally true in $\mathscr{M}$, but no contradiction is possibly true in $\mathscr{M}$.*

2. *Some counterpossible is really false in $\mathscr{M}$.*
3. *Some proposition truly says that everything is true in $\mathscr{M}$, but everything is not possibly true in $\mathscr{M}$.*

Proof. Let $s_1 \in N$, $s_2 \in @$, and $s_3 = WFF$, such that, $\alpha \wedge \neg\alpha \in s_1$, $\neg\bot \in s_1$ ($\bot$ is the sentence "everything is true"), and $s_1 \in f_{\alpha\wedge\neg\alpha}(s_2)$. Thus for 1, $\{s_1, \alpha\wedge\neg\alpha\}$ is normally true in $\mathscr{M}$ (by def. 3.8), but, by items 3 and 4 of definition 1, $s_1 \notin P$. For 2, $\{s_2, (\alpha \wedge \neg\alpha) \rightarrow \bot\}$ is really false in $\mathscr{M}$, by definitions 1.8.7, 3, because $f_{\alpha\wedge\neg\alpha}(s_2) \cap \{\neg\bot\} \neq \emptyset$ and $s_2 \in @$. For 3, $\{s_3, \bot\}$ is normally true (by def. 3.8), but since $\bot \vDash \alpha \wedge \neg\alpha$, if $\{s', \bot\}$ and $s' \in N$, then $s' \notin P$. □

We can roughly interpret Proposition 1 as saying that any impossibility truly characterizes unreal situations or fictions, or that impossibilities can be true propositions associated with fictions. In this sense, we allow impossibilities to be true, including contradictions, without taking the Dialetheist route, since no true contradiction concerns actual situations.

The reader might be wondering at this point: isn't this just like Dialetheism in sheep's clothing? We believe it is not. According to Beall:

> Paraconsistentists, those who construct or use or rely on some paraconsistent logic, usually divide into (at least) three classes:
>
> » Weak Paraconsistentist: a paraconsistentist who rejects that there are 'real possibilities' in which a contradiction is true; paraconsistent models are merely mathematical tools that prove to be useful but, in the end, not representative of real possibility.
>
> » Strong Paraconsistentist: a paraconsistentist who accepts that there are 'real possibilities' in which contradictions are true, and more than one such 'real possibility' (and, so, not only the trivial one); however, no contradiction is in fact true.
>
> » Dialetheic Paraconsistentist: a paraconsistentist who accepts that there are true contradictions—and, so, that there could be (since our world is a 'real possibility' in which there are some). (Priest 2004, p.6)

We can conclude from this that our view on contradictions as impossible truths cannot be classified as Dialetheic Paraconsistentist or Strong Paraconsistentist. Our view on contradictions is at most weak paraconsistent. Even if some contradictions are indeed true, they are never about actual or even possible situations, contradictions only about impossible situations.

3. Impossible Truths as Epistemic Tools

> In the human predicament, for use in which our language is designed, we may wish to speak about states of affairs which have not been observed or are not currently under observation (the future, for example). And although we can state anything 'as a fact' (which statement will then be true or false) we need not do so: we need only say 'The cat may be on the mat'. (Austin 1950, p.160)

It is our *wish to speak about states of affairs which have not been observed or are not currently under observation*. Thus, we will try to do so using a double axis structure. Firstly, we introduced a formal semantics that can logically support speaking about such states of affairs — impossible situations, in our case. We also wish to provide a further view on what these situations turn out to be, hence the reason why we talk about abstract artifacts. This dual arrangement is the main underlying motivation we had in presenting this link between impossible situations and epistemic artifacts.

One must observe at the outset two usually concurring schemas concerning the fictional character of impossible situations and the operation that allows for their generation, i.e., a function held in LIT by Φ. We have opted to think of our proposal in light of the current model-fiction discussion, that is, the scientific theoretical models debate. However, let us note that it isn't our intention to analyze nor give any insight into those accounts, we're merely referring to that particular framework because it illustrates the issue at hand.

Traditionally, philosophers of science will argue in favor of one of two approaches, a fictionalist — inspired by Kendall Walton's "Mimesis as Make-Believe" (1990), and an artifactual one — based on Saul Kripke's works on artifactualism, such as "Vacuous Names and Fictional Entities" (Kripke 2011) and "Reference and Existence: The John Locke Lectures" (Kripke 2013). The latter were further developed by Amie Thomasson in "Fiction and Metaphysics" (Thomasson 1999).

The proponents of Waltonian-like arguments will usually subscribe to some version of the following line of thought, a model is an element of a game of make-believe in which scientists pick a set of restrictions and from there work a way of modelling something to work with *vis a vis* certain world's phenomenon.

Fiora Salis (2016, 2019) suggests that there are two main types of Waltonian theses to models, the direct and the indirect fictionalist views. The first one is that in which model descriptions are representations about a real system; whereas the latter argues that model descriptions prescribe imagining about a model system acting as a mediator towards the real physical system, hence the *indirectness*.

Salis takes Martin Thomson-Jones's (2010) characterization of face-value practice to be a fundamental requirement for attempts to link modelling and fiction. "Because the practice of talking and thinking this way involves taking descriptions of missing systems at face value in a certain respect (or at least seeming to do so), I will call it the face value practice" (Thomson-Jones 2010, p.285). A theoretical model of an ideal pendulum apparently incorporates characteristics that only concrete objects can have, and even though such a model doesn't really exist, scientists still think and talk as if there was such an object representing a physical system.

> Every real pendulum encounters air resistance, and frictional forces at the point of suspension; no real rod or piece of string is perfectly rigid; no real pendulum moves through a perfectly uniform gravitational field; and so on. Competent physicists, of course, know all of this. A passage we are wont to call a 'description of the simple pendulum' is thus a description of a missing system. (Thomson-Jones 2010, p.284)

Thus, in line with Walton's notion of make-believe, all the sentences being used are taken at face value with regards to the fictional world[15] created from those restrictions and whichever activities it prompted and are either true or false against that fictional world according to a direct fictionalist view. However, our main point still lies ahead and we won't further discuss the aforementioned types of fictionalist views.

> The Waltonian implementations of the fiction approach to missing-systems modeling are, in many respects, very attractive, but they reject the indirect picture of targeted missing-systems modeling. Targeted missing-systems modeling then becomes a matter of purely linguistic representation. (Thomson-Jones 2020, p.84)

We take impossible situations in the spirit of the second sort of schema in the model-fiction debate. Defendants of artifactualism will typically try to anchor fictional practices and fictional objects to real-world activities. An example of that is Amie Thomasson's (1999) case for construing juridical laws as cultural abstract artifacts, i.e., "(...) a law of state might exist only where it is enacted by a legitimate legislative power" (1999, p.41). The specific point of this quotation is to highlight the importance of human activity and creation in dealing with abstractions and its representations. Thomasson (2020) develops further theses advocating for the use of artifactual theories in dealing with models, however, her focus appears to be more on the reference problem, and internal and external discourse distinction, all of which we aren't concerned with here.

Thomson-Jones also argues in favor of an artifactual account of missing-systems.

> The account of missing-systems modeling we get by adapting this account of ordinary fiction is the view I will call the abstract artifacts account. On the abstract artifacts account of targeted missing-systems modeling, then, missing systems such as simple pendula are abstract artifacts, created by physicists at a certain point (or over a certain period) in the history of classical mechanics. (Thomson-Jones 2020, p.86)

He advances an indirect realist account in which the missing systems are abstract artifacts created by agents, scientists in the case of models, that can be studied and manipulated. Here, we see yet again the importance of human agency and intention. According to Thomson-Jones (2020, p.88), what scientists do when they engage in activities such as studying or manipulating these abstract artifacts and discover certain features about the missing system "(...) they are discovering that the abstract artifacts in question are such that, according to the simple pendulum fiction, they have certain features".

It is worth mentioning that Thomson-Jones (2020) suggests that a more holistic approach to the various fringes of this debate is needed. He offers a handful of insights regarding the main qualms on the matter, to use his own terminology, those are questions on the activities, ontology, language and epistemology of abstract artifacts. Following Thomson-Jones' corollary, in the remainder of this paper, we will be taking a closer look into what he labels as activities and what we can do with them.

> *Activities*: The initial imagining and describing of a missing system in the sciences is an instance of fiction-making (an activity, that is, of the same sort as any standard instance of the construction of a work of ordinary fiction) and subsequent episodes in which scientists think about the missing system in question are episodes of imaginative engagement with a work of fiction (just like, say, a reader's interaction with a novel). (Thomson-Jones 2020, p.79)

Our view, however, is inspired and more in line with another artifactualist strand that suggests a possible conciliation between fictionalists and artifactualists[16], a task that we don't pretend to tackle in this paper. Tarja Knuuttila argues in line with traditional artifactualism that theoretical models should be treated as abstract artifacts "(...) the artifactual account focuses on how models as purposefully designed artifacts provide access to the empirical and theoretical questions scientists are interested in" (Knuuttila 2022, p.10).

> That a model is an epistemic artifact implies, firstly, that *human agency*, or rather traces of it, are more or less manifestly present in it. Secondly, it implies that models are somehow *materialized* inhabitants of the intersubjective field of human activity. Thirdly, it implies that models can function also as knowledge objects. (Knuuttila 2003, p.1487)

So here we are, intentionally taking an impossible situation to be an output of LIT's fictionalizing operator Φ construed as an epistemic tool that functions similarly to abstract artifacts. We can say that it is also anchored in the real-world, since it has been materialized through what Knuuttila calls a media, which turns out to be this paper, utilizing both mathematical, formal and natural language mediums. This media-specific characterisation functions as an external scaffolding as it enables us to tinker with and think of those situations, otherwise impossible. Think of written numbers and mathematical signs on a piece of paper, the fact that most of us rely on the use of these apparatuses to make our way around mathematical equations is an example of real-world media anchoring abstractions.

> As materialized things models have their own construction and thus their distinctive ways of functioning. They are not open to all possible interpretations and uses, which simplifies or modifies the cognitive task scientists face in their work. In scientific work one typically tries to turn into affordances the limitations of the models or the constraints built into them; one devises the model in such a way that one can learn from using or 'manipulating' it. (Knuuttila 2004, p.1267)

At this point, the notion of artifactual manipulability adds explanatory value to our discussion. It is precisely because we can manipulate those situations via Φ that we should not take it for granted that LIT helps us deal with them. Manipulating a model, or a semantic operator for that matter, is a feature of this kind of abstract artifact, or epistemic tools. It allows us to learn while we make use of it. Hence, the artifactual use comes into place, i.e., now one can better understand why manipulating an abstract artifact could help us make sense of and learn about something, even if that is an impossible situation. That is the importance of Φ in LIT, as it acts as an operation — restricted by LIT — capturing human creative acts and developing them further into situations.

> What is the cognitive point of constructing artificial hypothetical systems? How are they supposed to give us knowledge if not by means of representing more or less accurately some real target system? The suggestion already implicit in the results-orientedness and systemicity of models is that their cognitive value is largely based on manipulation. A theoretical model could be seen as a system of interdependencies, whose various features can be studied by manipulating it in the light of its results. That this way of proceeding should give us knowledge is dependent on the theoretical information built into the model and the way it facilitates the study of various hypothetical possibilities. This points to the modal nature of modelling: Modellers are interested in studying also different non-actualized and inexistent systems in an effort to thus understand some basic relationships and interactions that might explain the phenomena we encounter. (Knuuttila 2011, p.268–69)

Just looking at the set of definitions that have been outlined in the previous sections, and all the restrictions that follow it, we have reason to believe that this scheme enables us in its way to deal with impossible situations. In this regard, impossible situations are token outputs of an operator Φ. This fictionalizing operator is informed by human creative practices and provides us with a situation. On the one hand, because actual situations are always partial, it may be the case that Φ will give us an actual situation if fed, for example, with fiction embedded in a real-world framework. On the other, it also means that the output could be an impossible situation detached from a real-world scenario. In this case, the situation itself is impossible precisely because it can't be actualized. Hence, it doesn't classify any real situation.

Now, assuming we want to talk about a non-actual situation within a formal framework such as LIT, how can we do it? Austin argues in favor of statements being either true or false in historic situations, i.e., events or states of affairs in the real world, but that is precisely what we don't have at our disposal when dealing with impossible situations. So why not make an impossible situation historical by producing it ourselves?

In an Austinian vein, to label something an 'impossible situation' is to say that there is a token-like scenario of a somewhat impossible scene captured by said scheme. Ultimately, we take impossible situations to be outputs of Φ in LIT as products of an epistemic artifact, namely Φ. It is because LIT offers us a model of dealing with non-actualizable situations via Φ, that we can construct, informed by human creative practices as input, impossible situations.

Φ makes it possible for us to construct and deal with impossible situations, meaning that we have at hand an epistemic tool, Φ, that generates other abstract objects, that is, impossible situations. One should think of Φ as $\sqrt{}$, and the input for impossible situations as negative numbers, i.e., impossible situations are on par with abstract and complex numbers. Say we have a scientific paradox input to Φ, then we would proceed and use the operator to create an impossible situation, or a complex number, based on the information first provided. This operation also works for positive numbers, although the outcome would be a real number, or a partial actualizable situation.

Assume we have $\sqrt{-25}$. We would go ahead and do $\sqrt{25} \times \sqrt{-1} = 5i$. It is worth mentioning that the contextual framework aspect of Φ can be observed in $\sqrt{25} \times \sqrt{-1}$. That means that $\sqrt{25}$ is the real-world furniture for a situation and $\sqrt{-1}$ is its impossible character.[17] Think of $\sqrt{25}$ to inform, for example, (England, Victorian Era), and $\sqrt{-1}$ to inform something impossible, like (John Watson, had sustained only one battle wound on only one limb — one leg and the left arm). Watson's wound is often considered to be impossible since the same wound was described to be in two different limbs. According to Conan Doyle in A Study in Scarlet, the character sustained a wound on his left arm during his time in the Army. However, in The Sign of Four, Watson also sustained the 'same' battle wound, but this time said to be on one of his legs, the reason why he limps. That demonstrates Φ's ability to give us either an actualizable or a non-actualizable situation depending on the context. In other words, an impossible situation is the outcome of an operation by Φ that is restricted by a set of definitions in LIT.

> In model construction and use, representational mode and media are often closely coupled, yet it is analytically useful to distinguish between them. The distinction enables a more unified treatment of different kinds of models. For example, the material embodiment of mathematical models is crucial for their manipulation, yet the concrete media plays a different, more prominent role in physical three-dimensional models than in mathematical modeling. (Knuuttila 2021a, p.10)

While discussing the fact that "(...) there can be several epistemically possible models of a certain actual target system", Knuuttila also admits, in contrast to the epistemically possible, the objective possibility of "unactualized state of the world" and she even writes on a footnote that "(...) there can also be models of impossible targets" (Knuuttila 2021b, p.65).

In this sense, the Φ operator should be understood as an artifact generating artifacts while informed by the activities of an intentional agent that can give us the necessary input to generate an impossible situation through LIT representing a non-actual situation. Once processed by LIT, it generates particular token-like of non-actual situations.

Say I can see the squaring of the circle happening in the very room in which I am writing these words and that there are dishes in the sink. Now, say that I see the same thing happening in the same room but the dishes in the sink are not there anymore. Those situations are both impossible, not because dishes are impossible things, but because of the squaring of the circle. However, they are not the same situations in the sense that something is different from one another. Hence, we can talk about the fictionalizing function as an epistemic artifact in the same sense as an abstract model, and, at the same time, preserve the Austinian type-token intuition.

To sum it up, we believe that we have good reasons to think of the LIT framework for fictionalization as an epistemic tool that generates impossible situations, i.e., abstract artifacts. Once we admit that and put to use the semantics presented in earlier sections, we should be able to use this mechanism and understand how we can use it to deliver some outcome *vis a vis* the input we offered. We also believe that there's a lot more work that can be done in characterizing impossible situations, but that it requires a dedicated discussion. Here, however, we tried to present our own view of how to generate impossible situations. Hopefully, that will lead to more profound research.

4. Final Remarks

As final remarks, allow us to summarize the main points we have been trying to make in this paper, as well as suggest some possible extensions for future works.

First, we motivated an observation (supported by numerous examples in the literature) that we term the "Riddle of Impossible Truths". According to this observation, certain truths must, in a certain sense, be impossible. We illustrated this concept with examples from paradoxes, counterpossibles, and impossible fictions. This observation has largely contributed to the development of Dialetheism.

Secondly, we propose an alternative to Dialetheism by offering a different account of impossible truths, based on LIT and Austin's notion of proposition. Essentially, we represent the impossible in different situations through inconsistent, and/or non-normal sets of sentences. However, a proposition always relates (concerns, is about) a specific situation, such that, no impossible truth concerns actual situations; instead, some truths relate to impossible situations.

Finally, we propose an artifactualist perspective on impossible situations, drawing from the works of Kripke, Thomasson, and Knuuttila. We argue that impossible situations can be considered artifacts. Unlike historic slices of reality, impossible situations may be regarded as slices of alternative stories or scenarios produced by imagination. As artifacts, we contend that they offer us epistemic benefits, serving as tools for understanding, akin to the role of models in Knuuttila's framework.

The purpose of this paper was to present a preliminary exploration of impossible truths. However, many important topics discussed herein would benefit from more thorough and dedicated investigation. Therefore, we defer separate papers on themes such as impossible truths in fiction, counterpossibles, impossible situations as artifacts, and impossible situations as epistemic tools to future works. For the present purposes, our focus was on arguing the main points outlined in this paper.

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Notes

[1]There are various ways to define a contradiction. A more syntactic and restrictive definition considers a contradiction to be a conjunction of a sentence and its negation. In contrast, a more semantic and inclusive definition presents a contradiction as any sentence that can be true in no interpretation. For the purposes of this paper, we will not delve into the intricacies of these different definitions. A more complete discussion can be found in Grim (2004).

[2]Where $\ulcorner\alpha\urcorner$ is a term naming sentence α and T is a truth predicate of the same language. Notice that we are not restricting the schema here with either logical or material equivalence, as it is usually done (e.g. Tarski 1935, and Kripke 1975). As pointed out by an anonymous referee, there might be intensional contexts where transparency fails, such as one in which an agent does not have the concept of truth but knows that 2+2=4, thus not knowing that '2+2=4' is true. We did not restrict transparency to extensional contexts because we aim to allow it to hold within the context of conditional formulas, as we propose a hyperintensional semantics for them. Epistemic operators are not the primary focus of this paper, so we postpone a detailed discussion of them. For now, we restrict ourselves to contexts involving formulas that could be constructed in a basic propositional language, like the one in the next section, potentially enhanced with a truth predicate and terms naming sentences.

[3]To obtain $T(1) \wedge \neg T(1)$ from $T(1) \leftrightarrow \neg T(1)$, we need *Modus Ponens*, conjunction rules, and the following version of *reductio*: $\alpha \rightarrow \neg\alpha \vDash \neg\alpha$.

[4]We could also consider omniparadoxes (paradoxes related to the special properties associated with the concept of God, such as omniscience, omnipotence, etc.). Thus, it is also an impossible truth that God knows all truths. For a more detailed discussion on that, see Cardoso and Miranda 2021.

[5]In Boolos et al. 2007, p.222–223, this result is named as the "Undecidability of arithmetic", and it is used in the proof of Gödel's first incompleteness theorem.

[6]Note that a defender of TAC could restrict TAC to subjunctive conditionals in a way that allows for resolving the extensional paradoxes (such as the Liar and Curry paradoxes) independently of addressing the non-extensional ones (like Counterpossible Curry). For example, consider adding a subjunctive conditional to a modal version of the Logic of Paradox (LP) and restricting TAC to it, while allowing possible worlds to be inconsistent. We could also modify TAC to handle nonvacuous counterpossibles without significant consequences for extensional paradoxes.

[7]For a more detailed presentation of LIT, readers are referred to Cardoso 2024.

[8]Austin 1950.

[9]Impossible truths might prove useful in addressing the Paradox of Nothingness — the challenge of making truthful statements about nothingness. The problem is based on the impossibility of an absolute empty world, as argued, for example, by Lewis (1986, p.73) and Conee & Sider (2014, p.102).

[10]In this paper, we do not delve into Transparency. The outcome of Transparency in LIT is discussed in Cardoso 2024.

[11]We require that every model includes at least one non-normal situation, because sentence (8) cannot be true in any normal situation within any model.

[12]Where $C \subset \wp WFF$ is the set of consistent sets of sentences. Notice that $P \subseteq N$ and $P \subseteq C$, but we are nota llowed to say that $N \subseteq C$ nor that $C \subseteq N$, since $\{p_1\} \in C \backslash N$ and some inconsistent nontrivial sets are closed under logical consequence.

[13]These results about LIT are essentially important for a nontrivial approach of Truth Transparency, as it is more carefully explained in Cardoso 2024.

[14]Notice that being true is a subcase of the satisfaction relation between situations and propositions. If a proposition $p = \{s, \alpha\}$ is true, there is $s' \in @$, such that, the sentence $\alpha \in s$ is in s'. We leave satisfaction relation for future works.

[15]See Walton 1990, p.57–58.

[16]This conciliation remounts to the exact same point we are dealing with, but would require a lot more space than we now have. The *use* of an artifact and the affordances it provides us with could be construed as a *prop* in the waltonian sense, however artifactually manipulated.

[17]Of course, complex numbers are neither logically nor mathematically impossible. However, they extend beyond the realm of real numbers (no real number can be the output of $\sqrt{-1}$), which initially led to the perception that they are merely imaginary constructs created solely for solving equations.