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OVERINTERPRETING LOGICS

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Abstract. Paraconsistent logics, minimally, are not explosive; that is, on these logics, not everything follows from a contradiction of the form '*A* and not-*A*'. Dialetheists, who argue that some contradictions are true (Priest 2006), have philosophically interpreted such logics as supporting the existence of true contradictions, since they allow for the possibility of true contradictions without triviality. In this paper, I provide basic criteria for philosophical interpretations of logics and consider the costs of violating such criteria, especially by engaging in logical overinterpretation. I argue that overinterpretation supports metaphysical or epistemological readings of logics that are not called for and that can be resisted on philosophically independent grounds. A more neutral, modalist-empiricist stance toward logics in general, and of paraconsistent logics in particular, is then advanced.

Keywords: paraconsistent logic • dialetheism • epistemic interpretation • modalism • empiricism

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1. Introduction

It is usual to take paraconsistent logics as those for which, minimally, the principle of explosion does not hold, that is, on these logics, given a contradiction '*A* and not-*A*', not everything follows from it (da Costa (1974) and da Costa, Krause and Bueno (2007)). But is it even possible that such a contradiction be true? If so, what does it take for this to be the case? To answer questions of this sort involves providing an interpretation of a logic — or, in this case, of a particular logical principle. An interpretation is an account of how the relevant principles of the logic can be true.

Deflationary interpretations offer minimal additions to a logical formalism, without requiring significant metaphysical or epistemological commitments. They invoke the weakest possible commitments — while granting that some attenuated metaphysics may be needed in the end. In contrast, substantive interpretations add significant philosophical glosses — or, as they can be called, reifications or burdens — going well beyond what is strictly required for interpretational purposes. For example,

positing the distinction between objects and properties with no additional metaphysical gloss specifying the nature of these categories is deflationary. It recognizes the need for distinguishing these categories — without them, it becomes unclear how one can make sense of predication. A substantive interpretation not only introduces the object/property distinction but also adds a particular metaphysical gloss characterizing objects as substances and properties as universals (or something along these lines), and insisting that, without such an addition, the distinction is not sufficiently drawn.

In the case of paraconsistent logics, Newton da Costa and the Brazilian school of paraconsistency (for a survey, see da Costa, Krause, and Bueno (2007)) gravitated toward a deflationary, minimal approach, and avoided metaphysical or epistemological commitments beyond what is strictly needed for the interpretative task at hand. In contrast, dialetheists adopted a substantive interpretation of paraconsistent logics, articulating a philosophical defense of the existence of true contradictions (Priest 2006). On their view, paraconsistent logics allow for the possibility of contradictions being true without the logical anarchy that would result if the underlying logic were classical and true contradictions, actual. This provides a substantive gloss on paraconsistent logics. The presence of true contradictions, such as the Liar sentence '$L = L$ is not true', leads to the violation of explosion (L and not-$L \models B$). If the Liar sentence L is both true and false, and if the designated truth-value of the semantics for the paraconsistent logic in question is 'true', then the premise of explosion is satisfied but the conclusion of the inference is not. Any false sentence 'B' will do the trick. In this way, the entailment of an arbitrary sentence of the language from a contradiction of the form 'A and not-A' is prevented by the interpretative addition of the existence of a true contradiction. This is a substantive addition.

In this paper, basic criteria for philosophical interpretations of a logic are advanced and the costs of violating such criteria are assessed. The resulting violations yield overinterpretations. Metaphysical and epistemological readings of logics leading to overinterpretations, it is argued, are not required. They can, and should, be resisted on philosophically independent grounds. As an alternative, a more neutral, modalist-empiricist stance toward logics, including paraconsistent logics, is proposed.

2. Topic Neutrality and the Interpretation of Logics

Logics can be naturally thought of as having no metaphysical commitments. After all, on the traditional understanding, logics are (taken to be) universal, topic neutral, necessary, and *a priori*. Logics are *universal* since their principles hold in general — in every domain (of inquiry, interpretation, or application). They are *topic neutral*

since the truth of their principles does not depend on any specific features of the domain under consideration. They are *necessary* since logical principles are true in every possible situation. They are *a priori* since knowledge of the truth of logical principles does not depend on experience — except perhaps for the experience required to acquire the relevant concepts.

Topic neutrality together with universality become central to support the putative lack of metaphysical commitment of a logic. After all, in light of them, there is no dependence on particular situations or features of a domain to constrain the application of logical principles, as topic neutrality would have it. Moreover, such logical principles hold in general: they can be properly applied to any domain whatsoever, given universality. A metaphysical commitment typically characterizes, in full generality, the most fundamental features of a domain. This requires the specification of a domain described by the relevant metaphysical principles — depending on the metaphysical view, the domain may be the entire universe. Without the specification of a domain, the principles in question will lack content, as it would be indeterminate what, if anything, the principles rule out. A principle has content provided that it rules out something. If a logical principle applies in general and independently of what goes on in any domain, that is, if the principle is universal and topic neutral, no metaphysical constraints seem to be specified, since nothing seems to have been ruled out. In contrast, if there are such constraints on the application of a logical principle, some content is involved as something has indeed been excluded.

For instance, if objects in a given domain have no sharp boundaries — consider clouds or mountains — it may be indeterminate precisely where the object begins and the background ends. If excluded middle — the principle according to which '$\forall x(Px \vee \neg Px)$' — holds in general, it would apply to clouds and mountains. But if the boundaries of such objects are indeterminate, it is also indeterminate whether the objects are precisely located at a certain region or not. This seems to lead to a violation of excluded middle, since for a given mountain (or a certain cloud), consider a region in its border, it is not the case that the mountain is definitely located on that region nor is it the case that it definitely is not.

A classical logician may insist that vagueness offers no counterexample to excluded middle since there is a sharp boundary precisely specifying where the mountain begins: we just do not know where it is (see Williamson 1994). This offers a substantive interpretation of excluded middle by adding the existence of sharp boundaries where none seems to be present or required. There is no need, nor is it possible, to determine the precise location of a mountain in order to refer to it. We refer to Mount Everest without difficulty even though its precise borders have never been determined — nor could they be. Adding the existence of an unknowable sharp boundary provides a philosophical gloss designed only to avoid the violation of excluded middle in the presence of a vague object.

The epistemic account of vagueness also fails to preserve the most salient aspect of the phenomenon at hand: the fact that vagueness cannot be made precise without losing its significance. It is the fact that the precise location of Mount Everest is indeterminate and that it does not matter precisely where it begins that allows us to apply the predicate 'mountain' so effectively. Taken together, the indeterminacy of the mountain's precise location and the irrelevance of its precise determination allow speakers to use vague predicates without settling on any precise location of the mountain (or of any vague object, for that matter). As long as the predicate's use respects the vague boundaries of its scope of application, no difficulty emerges. Within these boundaries, it is indeterminate whether the predicate applies or not — it is indeterminate, for instance, whether the mountain is in a specific region of the penumbra or not. And it does not matter how the issue is resolved — one way or another.

The result is that excluded middle is then violated: for each point in the region of indeterminacy (the penumbra), it is not the case that the mountain is there — since it might just as well be a bit further away — nor is it the case that it is not — since it might just as well be there, after all. There is no fact of the matter one way or another. This is what the indeterminacy amounts to.

By recognizing the failure of excluded middle in vague contexts, one can make perfectly good sense of vague discourse and avoid the need for attempting to tame it artificially. Furthermore, one can also thereby avoid the aim of dissolving vague predicates altogether by transforming them into something precise — either in terms of artificially stipulating an unknown sharp boundary, as the epistemicist does (Williamson 1994), or by considering all possible precisifications, as the supervaluation theorist insists on (Keefe 2000). The violation of excluded middle in the presence of vagueness suggests that (a) the principle does have content (it rules out vague objects), and (b) it does matter what the principle is applied to — topic neutrality does not seem to hold in general.

Topic neutrality has been a salient feature of any interpretation of logic. This is expressed in Tarski's (1936) formulation of logical consequence via the *substitutional requirement*, which is introduced as a necessary (but not sufficient) condition on logical consequence. Let K be a class of sentences and X be a given sentence. According to the substitutional requirement:

> If, in the sentences of the class K and in the sentence X, the constants — apart from purely logical constants — are replaced by any other constants (like signs being everywhere replaced by like signs), and if we denote the class of sentences thus obtained from K by K', and the sentence obtained from X by X', then the sentence X' must be true provided only that all sentences of the class K' are true. (Tarski 1936, p.415; emphasis omitted).

Central to Tarski's account is the fact that logical constants are *not* subject to replace-

ment: they are not given a *semantic* interpretation. Otherwise, validity may not be preserved: a sentence of the form *A* or *not-A* could be replaced by one of the form *A* and *not-A*, thus transforming a classical tautology into a contradiction.

The fact that the consequence relation is preserved under variations of the extension of the non-logical vocabulary expresses the fact that *it does not matter* which objects are under consideration — anything that follows logically from the original class K will similarly follow from the replaced class K'. This ensures the *topic neutrality* of a logic, given the irrelevance of any particular objects to the outcome of what is meant to be a purely *formal* consequence relation. Tarski makes this point explicitly:

> Since we are concerned here with the concept of logical, i.e. *formal*, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, *this relation cannot be influenced in any way by empirical knowledge*, and in particular by *knowledge of the objects of which the sentence X or the sentences of the class K refer*. The consequence relation cannot be affected by replacing the designations of the objects referred to in the sentences by the designations of any other objects (Tarski 1936, pp.414–415; emphasis added, except for the italics in 'formal').

Since the properties of objects of any domain should *not* affect the outcome of what follows from what, logics are topic neutral. Topic neutrality is, thus, tied with formality.

Two conclusions can be drawn from these remarks: (a) Epistemological considerations about the objects in the domain should not affect the consequence relation: "this relation cannot be influenced in any way by empirical knowledge" (Tarski 1936, p.415). (b) Similarly, if the nature of the objects in the domain does not matter, metaphysical considerations are also excluded. As Tarski emphasizes, the relation of consequence cannot be influenced by "the objects [to] which the sentence X or the sentences of the class K refer" (Tarski 1936, p.415). Strictly speaking, Tarski talks about one's "*knowledge* of the objects" in the domain under consideration as being irrelevant. But since knowledge is factive (that is, if one knows that P, then P), the *objects themselves* become irrelevant to the determination of logical consequence. This point is highlighted by the substitutional requirement: the fact that the replacement of objects does not affect the consequence relation expresses the irrelevance of these objects and their nature to what follows from what. Topic neutrality, in this way, seems to ensure epistemological and metaphysical neutrality as well.

It turns out, however, that topic neutrality does not seem to hold in general. After all, as noted, there are some contexts in which certain logical principles do not seem to go through. One may wonder how is even possible that logical principles are violated given the substitution requirement. If the permutation of objects in the domain of interpretation is not meant to affect what follows from what, then no such violations *can* emerge. Otherwise, the inference in question would not be a

logical inference — one that depends only on the features of the logical connectives in question — but would also depend on specific features of the domain to which the principles are applied to. The trouble with this requirement is that it assumes by fiat that features of negation, conjunction, disjunction, and quantification can be isolated from the domains of objects to which the principles that use these logical connectives apply. And this is done by ruling out potential violations as impossible, and by blaming any alleged violations on some mistaken non-logical assumption that has been introduced.

For instance, in classical logic, no statement and its negation can hold together. This rules out by fiat the very possibility of studying properties of inconsistent objects, such as the Russell set — the set of all non-self-membered sets, that is, $R = \{x : x \notin x\}$ (for discussion and references, see da Costa, Krause, and Bueno 2007). After all, the principle of explosion, according to which everything follows from a contradiction (of the form '*A* and not-*A*'), immediately turns any investigation of inconsistent objects into an impossibility, given the resulting identification of inconsistency and triviality. In the presence of explosion, every statement of the language becomes a theorem, which effectively blocks any attempt to differentiate the properties an inconsistent object has from those it does not. Yet, it makes perfectly good sense to inquire about properties of inconsistent objects. To achieve that, a possibility that is foreclosed by classical logic needs to be recognized, namely, the possibility of distinguishing inconsistency and triviality.

Once this possibility is identified in logical space — that is, the space of all possibilities (see Bueno 2021) — it is then codified in a logic in which explosion in effect does not hold, a paraconsistent logic (see da Costa 1974, Priest 2006, and da Costa, Krause, and Bueno 2007). Interestingly, in a suitable paraconsistent set theory, the Russell set is an inconsistent but nontrivial object, which has some properties but not others (da Costa, Krause, and Bueno 2007). And this fact — that the Russell set both is and is not a member of itself but does not have every property — provides a clear counterexample to explosion, illustrating a particular case in which from a contradiction not everything follows.

Since the Russell set was excluded from consideration given the way classical logic has been set up, the appearance emerged that negation does not depend on specific features of the domain to which explosion is applied. Appearances, however, can be deceptive. The recognition of the Russell set identifies a blind spot in classical logic. As a result of blind spots of this sort, topic neutrality does not hold in general. Properties of negation do depend on whether inconsistent domains are under consideration. Such domains include possibilities that classical logic excludes, but once acknowledged, these possibilities enrich the properties that can, and should, be investigated, such as those of inconsistent objects.

This case illustrates how a logical principle depends on specific features of its

domain. As noted, this is not the only case, though. The principles of identity and of distributivity arguably do not hold in quantum mechanics (French and Krause 2006, Rédei 1998, and da Costa and Bueno 2001). Excluded middle is arguably violated in contexts involving constructive mathematics (Dummett 2000) or vague objects (Bueno 2022). And existential introduction is arguably undermined when applied to nonexistent entities (Lambert 2004). None of these cases should even be possible had logics been topic neutral.

The violation of topic neutrality indicates that logical principles do have content: there is something they are about, or, at least, there is something that constrains their adequacy — especially when applied to specific domains. In other words, they do rule out certain possibilities: the possibility of inconsistent but nontrivial objects in the case of explosion; the possibility of objects for which identity does not apply in the case of the principle of identity; the possibility of indeterminate or vague objects in the case of excluded middle; the possibility of existentially quantifying over nonexistent objects in the case of existential introduction.

Furthermore, if logical principles lacked all content, they would be applicable across the board with no exception. The mere possibility of such violations, let alone their actuality, is enough to question the alleged contentless nature of logical principles. After all, as noted, not only were logical principles supposed to hold universally but they were also assumed to necessarily hold, that is, to be the case in every possible situation. Given the strong modal content such principles display, the mere possibility of their violation questions their alleged necessity, for the necessity of a logical principle is undermined by the mere possibility of its negation. And if a principle is not necessary, it may fail in some context or another — some of these contexts may even be actual.

In light of these considerations, the issue then immediately emerges: in the contexts to which logical principles apply, how should they be interpreted?

3. Kinds of Interpretation

Three kinds of interpretations of logics — and their principles and rules — should be distinguished: semantic, epistemological, and metaphysical. They will be considered in turn.

(i) A semantic interpretation provides truth conditions for logical principles. These interpretations do not specify the meaning of logical constants. The meaning is already presupposed in the metalanguage in which the corresponding constants are used (Azzouni 2006). Consider: '$A \wedge B$' is true if, and only if, 'A' is true *and* 'B' is true. As the truth conditions make clear, one cannot understand the meaning of the logical connective of conjunction unless one already understands the meaning of 'and'

in natural language — in terms of which conjunction is characterized. Nonetheless, a semantic interpretation makes no demand on the epistemology or the metaphysics of the logical principles.

The framework, however, in which the semantics is implemented does impose some constraints. For example, if a classical set theory, such as ZFC (Zermelo–Fraenkel set theory with the axiom of choice; Jech 2003), is used as the framework for the semantics, certain possibilities may not be expressible given the framework. There is no set of all sets in ZFC, and so the domain of interpretation of set theory cannot be such a set, even though this is the obvious candidate for the domain in this case (Field 1989). A truncated semantics results, one that does not validate the objects the theory is supposed to be about. If Quine's NF (New Foundations; Quine 1937) is adopted instead, given its incompatibility with the axiom of choice (Specker 1953 and Forster 1995), some proofs of the full version of the completeness theorem for first-order logic may not go through, given their reliance on that axiom. As these examples illustrate, semantic notions are not absolute but crucially depend on the background framework in which they are formulated (da Costa 2008).

(ii) Epistemological interpretations provide readings of logical principles that emphasize their epistemological significance. They highlight, for instance, the close connections between proofs and the content of a logical principle or emphasize ways in which such a principle expresses the evidential content of certain situations. Consider, as an illustration, the case of the principle of excluded middle, that is, $P \vee \neg P$, under the Brouwer-Heyting-Kolmogorov interpretation of intuitionistic logic (see Heyting 1983 [1931]; see also Linnebo 2017, Chapter 5). On this interpretation, a proof of '$P \vee \neg P$' is either proof of P or a proof of not-P. In other words, the principle is interpreted as conveying the presumption that, given a certain statement, a mathematical construction can be identified that either proves the statement or its negation. Of course, since the availability of such proofs depends on what goes on in the specific mathematical context under consideration, the logical principle depends on that context and the relevant features of the mathematical objects in question. Given that the required constructions may not be available, the principle does not hold in general.

As Arend Heyting notes regarding excluded middle:

> One can assert this law for a particular proposition p only if p either has been proved or reduced to a contradiction. Thus, a proof that the law of excluded middle is a general law must consist in giving a method by which, when given an arbitrary proposition, one could always prove either the proposition itself or its negation. Thus the formula '$p \vee \neg p$' signifies the expectation of a mathematical construction (method of proof) which satisfies the aforementioned requirement. Or, in other words, this formula is a mathematical proposition; the question of its validity is a mathematical problem which, when the law

> is stated generally, is unsolvable by mathematical means. In this sense, logic is dependent on mathematics (Heyting 1983 [1931], p.59).

Similarly, with regard to the principle of non-contradiction, that is, $\neg(P \wedge \neg P)$, one would be led to the derivation of a contradiction from the assumption that the negation of the principle, i.e., $\neg\neg(P \wedge \neg P)$, holds. Given that a proof of $\neg\neg Q$ is not in general a proof of Q, one falls short of providing a proof of the contradiction in question. Not surprisingly, on this interpretation, proofs by *reductio ad absurdum* are not generally valid, for precisely the same reason. The constraints imposed by these interpretations are clearly epistemic: they concern the availability of suitable evidence — in this case, proper proofs. The lack of evidence for $P \wedge \neg P$ in general is an expression of this kind of epistemic constraint.

(iii) Metaphysical interpretations, in contrast, articulate metaphysical conditions that underlie logical principles. These interpretations aim to specify the ontological content that logical principles ultimately have. If excluded middle is considered, a metaphysical interpretation specifies that only two options hold: either P is the case or its negation is (and not both). If every object has the property of being P or has the property of being not-P, this rules out the possibility of objects that are indeterminate or are vague, since vagueness or indeterminacy would undermine the sharpness of the property in question. As noted above, vague predicates typically allow for borderline cases. There are cases to which they definitely apply; cases to which they definitely do not apply, and cases to which it is indeterminate whether they apply or not. Before shaving his head, Otávio was a borderline case of being bald. It was not definitely true that Otávio was bald (for he had some hair on his head) nor definitely true that he was not bald (for his head did lack some hair too). Otávio was a vague object, who clearly offered a violation of excluded middle.

As also noted above, one could insist that no violation is involved here as there is a sharp boundary between being bald and non-bald. We just do not know what that boundary is (Williamson 1994). Yet, the costs of epistemicism about vagueness are significant. In particular, the view introduces the existence of sharp boundaries where none is ultimately needed for the application of vague predicates. It does not matter exactly where in the borderline area a vague predicate eventually fails to apply. As long as it does — as it eventually does — that is enough. Interestingly, despite the role of epistemological considerations in epistemicism — a certain lack of knowledge claim — the view provides, in fact, a metaphysical conception about excluded middle, requiring the existence of sharp boundaries, despite our lack of epistemic capacity to identify them. A metaphysical view that introduces the existence of items we have no way of knowing faces a significant concern. The issue is not that the items in question do not exist. Clearly, things may exist even if we are unable to know them. But to posit items that allegedly explain some phenomena and then insist that there is no

way for anyone to know them is to invite suspicion about the ontological status of these items.

In the case of the principle of non-contradiction, a metaphysical interpretation insists on the nonexistence of true contradictions in the world: no object has the property *P* and not-*P* in reality. This is not an unreasonable interpretation, given that objects we routinely interact with — especially observable ones — all seem to satisfy it. It is unclear, however, that such a metaphysical interpretation is required by the principle. After all, a far less ontologically committing account, provided by a semantic interpretation, is readily available (Mares 2004). Dialetheism offers a correspondingly strong metaphysical interpretation of the negation of the principle of non-contradiction, insisting that there are true contradictions in the world (Priest 2006). This adds an additional layer of philosophical gloss to the logical principle, and it is unclear that it is called for, as the principle can be characterized and applied without the extra metaphysical burden (Bueno and Colyvan 2004).

A semantic interpretation of a logical principle does not add significantly more than the truth conditions for the principle under consideration, given the presuppositions of the underlying framework in use. In contrast, epistemological and ontological interpretations add something that goes beyond the logical principles at hand: they add epistemological and metaphysical assumptions in an attempt to highlight the nature of the principles in question. This is problematic. The characterization of a logic — as a pure logic — does not require metaphysical or epistemological interpretations, nor does the application of a logic — as an applied logic — make such demands (da Costa and Bueno 2001). The semantic interpretation, together with proof-theoretic devices as needed, is more than enough for these purposes. Epistemological and metaphysical interpretations are colorful but ultimately dispensable traits for the characterization and application of logical principles. After all, the principles at stake can be formulated and applied quite independently of any specific philosophical glosses that are added to them.

For example, in the presence of inconsistencies, in order to avoid triviality without information loss — at least while one investigates the source of the conflicting information (Bueno 2006) — all one needs are the inferential resources offered by a paraconsistent logic. Additional interpretations stating that the inconsistency reflects a salient trait of the world or an epistemic feature of our information system are unnecessary for this task and can be dispensed with.

The violation of logical principles, given the failure of their topical neutrality, does not depend on offering metaphysical or epistemological interpretations of these principles either. It is enough to identify possibilities that have been ruled out by fiat by the introduction of the principles in question. (What I referred to above as a 'blind spot' in connection with the revision of classical logic prompted by the Russell set.) Sensitivity to the expressive and inferential power of the principles them-

selves is enough to uncover hidden possibilities that have been prevented from being considered given the available logical principles. Guided by these possibilities, counterexamples to logical principles can be formulated and new principles and rules that explore such possibilities come to light. The new rules and principles, in turn, need not be interpreted in philosophically substantive ways — although they obviously can and, very likely, will be. (Philosophical glosses are so irresistible!)

4. Modalist Empiricism and the Interpretations of Logics

Uncovering and exploring possibilities is one of the central tasks of logic. Modalists insist that the modalities in question — necessities or possibilities — are primitive and cannot be defined in more basic, nonmodal terms (Bueno and Shalkowski 2009, 2015, 2020, Bueno 2021). After all, attempts to define the modal *via* the nonmodal, such as by invoking possible worlds (Lewis 1986), ultimately fail since they end up presupposing modality (Shalkowski 1994). For the very structure of the logical space encodes possibilities about what can or cannot be the case in that space. These possibilities, not being world bound — since they are possibilities *about* worlds — cannot be expressed in terms of worlds. Attempts to define them — given that they themselves express what is possible — presuppose the very possibilities one aims to characterize. Not surprisingly, taking the possibilities in question as primitive makes good methodological and philosophical sense. Hence, modalism.

As noted above, logics encode possibilities about certain domains, and in some cases, they *foreclose* some possibilities. Whenever this happens, changing the logic is called for, as the logic in use does not allow for the expression of the very issues — the very possibilities — that need to be expressed as a condition to investigate a certain portion of the logical space. If classical logic prevents the possibility of investigating inconsistent domains without triviality, due to explosion (da Costa, Krause and Bueno 2007), or blocks the possibility of inquiring into quantum objects that lack identity conditions, due to principles of identity (French and Krause 2006), these are sensible, modal grounds to revise such a logic.

Modalist empiricism emphasizes the empiricist constraints involved in this way of approaching inquiry and revising logics — constraints that shape logical theorizing as well. The stance is modalist given the central role played by modality as part of the determination of the unfolding structure of the logical space (Bueno 2021). It is empiricist since modality is not reified. Modality is not characterized in terms of possible worlds, abstract objects, propositions, or any such substantive metaphysical posits. Instead, the modal is explored, in good empiricist fashion, as part of the exploration of the world.

Modalist empiricism also recognizes that pure logics, similarly to pure mathe-

matics, are conducted *a priori*. This feature does not pose a challenge to empiricism since neither pure mathematical theories nor logical systems alone describe parts of the physical world. They only express possibilities — of a logical or mathematical kind. Applied logics, similarly to applied mathematics, are different as they do depend, for reasons indicated above, on the domains to which the logical or mathematical principles in question are applied (da Costa and Bueno 2001 and Bueno and French 2018).

The application of logics — a thoroughly empiricist enterprise — also depends on particular (semantic) interpretations. The recognition of logical form relies on specific interpretations of logical principles, so that it becomes clear what counts as an instance, or as a counterexample, of each principle. For example, consider the argument below that includes an embedded conditional in the consequent of the first premise of *modus ponens* (the example updates to the 2016 US presidential election an argument originally given in McGee 1985; see Bueno 2018):

(P_1) If a Republican wins the election, then it if is not Donald Trump who wins, it will be Ted Cruz.

(P_2) A Republican will win the election.

(C) If it is not Donald Trump who wins, it will be Ted Cruz.

Does the argument provide an instance of *modus ponens*? Or, because the consequent of (P_1) contains additional structure, does this entail that no such instance is present? The resolution of this issue requires an interpretation of *modus ponens* and a determination of the requirements for exhibiting its logical form. Interestingly, neither epistemological nor metaphysical interpretations are needed for that — a semantic interpretation should be enough. The point here is not to resolve the matter and determine whether *modus ponens* has or has not been violated in this instance, but rather to indicate that substantive interpretations are not required for this task.

The various ways in which classical logic forecloses certain possibilities also provide clear illustrations of modalist empiricism at work. If constructive features of mathematical reasoning cannot be expressed given classical logic's nonconstructive traits, a revision of logic that allows for these features to be investigated needs to be implemented. If incomplete objects are ruled out by fiat by the underlying logic and a study of these objects is precisely what one aims to achieve, a logical revision is similarly called for. In all these cases uncovering the structure of the logical space and the underlying possibilities is crucial.

Changing logical principles should not be taken lightly, though. One does not revise a logic simply to avoid a difficulty. *Ad hoc* maneuvers are typically problematic whether they involve logical, mathematical, or empirical principles. Changing a logic needs to be properly motivated — and expressive limitations of a given logic re-

garding what is possible (or not) provide clear instances of good motivation. Simply blocking a counterexample does not.

Once again, the expression of these possibilities does not require the adoption of metaphysical or epistemological interpretations of the logics in question: a semantic interpretation, plus perhaps proof-theoretic devices as needed, is enough. Metaphysical and epistemological interpretations yield philosophical understanding about the world associated with a logic. Since the same logic is compatible with multiple interpretations, some metaphysical other epistemological, one can separate the logic from its philosophical interpretations. The Logic of Paradox (LP) can be interpreted as positing true contradictions (Priest 2006), but also as positing gaps in reality (Priest 2019). These are fundamentally different and ultimately incompatible interpretations. The fact that they are all possible ways of making sense of LP indicates that this logic does not dictate its own interpretation. One can also advance epistemological interpretations of LP — similarly to other paraconsistent logics (Carnielli and Rodrigues 2019a, 2019b and 2020) — as embodying an incompleteness in our information and evidence about various aspects of the world.

Philosophical interpretations, since they entail going beyond the semantics, can result in overinterpretations of a logic, requiring more from the world or one's evidence about it than the logic demands. This is clearly the case of dialetheism (Priest 2006), given the difficulty of making sense of how a contradiction could be true in reality. The epistemic interpretation of paraconsistent logic (Carnielli and Rodrigues 2019a, 2019b and 2020) offers a less committed alternative, rejecting the existence of real contradictions. Yet, it ends up adding controversial epistemological requirements when the inconsistencies in question are understood epistemically, as a matter of evidence. After all, arguably if one has evidence for a claim and its negation, one would be inclined to conclude that the evidence is misleading and needs to be revised rather than taken literally. Note that the difficulty does not arise if instead of evidence one adopts a nonfactive account of information. For information lacks the epistemic standing that evidence demands. This illustrates where epistemological assumptions are built into this interpretation of a logic.

Jonas Arenhart (2022) offers a different picture. He argues that the distinction between pure and applied logics gives all that one needs in this debate. On his view:

> *Dialetheism is not an interpretation of paraconsistent logics*, but rather a view on truth and negation which requires *use* of paraconsistent logics. Something similar may be said about the epistemic approach itself [by Carnielli and Rodrigues]. The result is that there is nothing distinctive to be called *a philosophical interpretation* of paraconsistent logics, but rather what we have are distinct *applications* of paraconsistent logics (Arenhart 2022, p.1).

Arenhart's claims about dialetheism and the epistemic approach to paraconsistency are correct. In each case, a particular conception of truth and negation is of-

fered. The dialetheist advances a conception according to which some contradictions are true and, thus, objects in the world may both have and do not have some properties. In contrast, the epistemic view advanced by Carnielli and Rodrigues denies the existence of true contradictions and insists that any such inconsistency has at best to do with our evidence about the objects in question rather than about the objects themselves.

Nevertheless, it does not follow from these considerations that "there is nothing distinctive to be called *a philosophical interpretation* of paraconsistent logics, but rather what we have are distinct *applications* of paraconsistent logics" (Arenhart 2022, p.1). After all, even though we do have here different applications of paraconsistent logics, there is something distinctive about each interpretation: they indicate how the world could be if paraconsistent logics were true, that is, if they offered an account of logical consequence that appropriately captured the inferential relations among the objects in the relevant domain. In other words, they allow for the relevant possibilities to be properly preserved. (For a corresponding view regarding the interpretation of quantum mechanics, minus the modalist components, see van Fraassen 1991.)

On the metaphysical, dialetheist interpretation of paraconsistent logics, the world would be one in which true contradictions occur. In contrast, on the epistemic interpretation, the world would be one in which our evidence about the information regarding the world is inconsistent. In each case, possibilities are highlighted by each interpretation that go beyond what paraconsistent logics alone require — including for their applications. These logics do not demand the existence of true contradictions, nor do they require the evidence to support inconsistencies. Paraconsistent logics only *allow for these possibilities*, without ever demanding that either be the case.

In other words, both dialetheism and the epistemic approach offer, respectively, metaphysical and epistemological interpretations of paraconsistent logics. The approaches illuminate these logics by indicating how the world could be given the possibilities in question. In this way, the interpretations yield philosophical understanding — independently of any of them being true. By offering overinterpretations of the logics and going beyond what is strictly required by them, these approaches do provide something philosophically distinctive and significant in the end, even if what they deliver is not required by the phenomena.

5. Conclusion

Philosophical interpretations of logics, including paraconsistent logics, are not needed for the characterization or the application of such logics — a semantic interpre-

tation (together with proof-theoretic devices) is more than enough for that. Where philosophical interpretations are helpful is in giving us understanding of the possibilities that a given logic allows for and how the world could be given such logics. This is significant and can be appreciated without any commitment to the truth of such interpretations, given that the same logic is compatible with radically different interpretations. At least, this is a modalist empiricist approach to the topic.

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