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THE POVERTY OF ANGLO-AMERICAN EMPIRICISM AND HUSSERL'S LONG SHADOW

GUILLERMO E. ROSADO HADDOCK
University of Puerto Rico, PUERTO RICO
gerosadohaddock@gmail.com
https://orcid.org/0009-0005-1063-8520

Abstract. Logic, Mathematics and Physical Science in the History of Philosophy. The Revolutions in the three Disciplines in the XIXth Century and of Physics in the Twentieth. The Demise of Logical Empiricism and Quine's Bug's Epistemology. Husserl's Transcendental Rationalism.

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1. Historical Background

Since the origins of philosophy in ancient Greece the problem of knowledge has played a central role in the discipline and, more particularly, both the nature of mathematics and our knowledge of mathematics already emerged so soon as fundamental issues of a probably more specific nature. Those problems already present in the two giants of ancient philosophy, Plato and Aristotle, were to flourish much later in the history of philosophy. The fundamental problems of our knowledge of the world, of us and of mathematics, and their respective certainties took center stage in the period of the history of philosophy usually called "modern philosophy". That period originated with the work of the great French mathematician and philosopher Réné Descartes and culminated with the work of another of the really great philosophers, Immanuel Kant. In fact, Descartes and Kant had the similarity of trying to obtain secure knowledge using the cognitive subject and its capabilities of knowing as point of departure. In this sense they can be considered as "transcendental" philosophers in a very strict sense that also applies to another philosopher with which we will be concerned later, namely, Edmund Husserl.

So-called modern philosophy began precisely with Descartes and ended with Kant. In this period, in which the theory of knowledge became the center of philosophical inquiry, two clearly delineated schools, already present since antiquity, flourished, namely, rationalism, represented by the continental Europeans Descartes, Spi-

noza and Leibniz, and empiricism, represented, very especially by British philosophers, beginning with the radical English empiricist Locke, and continued in a less radical fashion by Berkeley and in a still more nuanced sense by the Scottish David Hume. In fact, while the somewhat extreme empiricist Locke considered our minds a sort of *tabula rasa*, that is, a sort of blank table (or paper) devoid of any knowledge structure previous to our experiences, the much less radical Hume even coincided with rationalists like Leibniz in considering logic and mathematics as concerned purely with relations of ideas and based only on the analysis of such ideas, completely independent of experience.[1] Thus, the most distinguished of all empiricists, David Hume was by no means a pure empiricist, but tended to seek a place between empiricism and rationalism.

Precisely the conscious search for a middle point between rationalism and empiricism was the task that the great Kant set for himself in his mature philosophical writings on epistemological issues, specifically, in his *opus magnum Kritik der reinen Vernunft* (Kant 1990). He elaborated his views especially in the first two parts of that monumental work, namely, in the *Aesthetics* and in the *Analytic*. In fact, his views were as exactly in the middle between rationalism and empiricism that he propounded that our knowledge of mathematical truths and our knowledge of physical truths were both neither purely analytic nor empirical, but were what he labeled as *synthetic a priori*. He argued that the human subjects are endowed with cognitive faculties that allow us to obtain knowledge that is neither analytic, like our knowledge of logic —for Kant Aristotelian logic—, nor empirical, like our daily acquaintance with sensible objects, be it the computer on which I am presently writing or the chair on which I am presently sitting. That *synthetic a priori* knowledge was possible thanks to our cognitive faculties of sensibility, understanding and the mediating faculty of the imagination. According to Kant, we are endowed with two forms of sensibility, space and time, that allow us to have experience of physical objects as well as of mathematical objects, whereas our understanding is endowed with twelve *a priori* categories, divided in four groups of three categories corresponding approximately to the sorts of judgements in Aristotelian traditional logic.

It should be stressed that Kant's theoretical philosophy was bound to the physical and mathematical science considered valid during his lifetime. He accepted uncritically Euclidean geometry, Newtonian physics and even Aristotelian logic, although the more visionary Leibniz had already envisioned the possibility of a gigantic development of logic. In the case of Euclidean geometry, since according to Kant space and time are forms of our intuition, and spatial intuition was presumably Euclidean, he did not have the least doubt about its validity, and non-Euclidean geometries were basically inconceivable by human subjects that have Euclidean geometry ingrained in their forms of spatial intuition. Finally, Kant conceived his *Kritik der reinen Vernunft* as serving as a sort of mediate philosophical foundation of Newtonian mechanics. On

the basis of that fundamental work, Kant published five years later —in 1786— his *Metaphysische Anfangsgründe der Naturwissenschaft* (Kant 1996), a work that was to serve as a sort of more direct philosophical foundation of Newtonian mechanics.

But Kant was certainly no visionary. He died in 1804, hence, at the beginning of the nineteenth century, a century in which all the scientific theories whose truth he had taken for granted suffered intellectual earthquakes. Already at the beginning of the nineteenth century the extraordinary mathematician Carl Friedrich Gauss envisaged very seriously the possibility of non-Euclidean geometries. In fact, already in the time of Kant some mathematicians had begun to consider that possibility, though that remained mostly within closed doors. Gauss developed a non-Euclidean geometry, though he did not dare to publish his results because he was somewhat frightened by the possible reaction of Kant's followers. Nonetheless, he discussed his ideas in correspondence with an ex-fellow student and friend, the Hungarian mathematician Farkas Bolyai, who was also very interested in the parallels postulate, though in the opposite sense, namely, to try to prove it from the other axioms of Euclidean geometry, attempts that, of course, resulted in failures. But the son of Farkas, Janos Bolyai took seriously Gauss' ideas and not only developed but made known the first non-Euclidean geometry, a geometry not with 0 curvature like that of Euclid, but with negative curvature, that is, such that the sum of the angles of a triangle is less than 180 degrees. With complete independence both of the Bolyai and of Gauss, the Russian mathematician Nikolai Lobachevsky also developed a non-Euclidean geometry of negative curvature, for which a horse's saddle or the interior of a ball could serve as models.[2] But by the middle of the nineteenth century another exceptional German mathematician, Bernhard Riemann, one of Gauss' last students, developed a second sort of non-Euclidean geometry, one with a positive curvature —the sum of the angles of a triangle is more than 180 degrees—, of which the most common example is the surface of a ball. But the great Riemann went much farther and distinguished clearly between the mathematical —analytic and *a priori*— treatment and development of the two sorts of non-Euclidean geometries on the same foot as Euclidean geometry, and not only of two or three dimensions, but of any finite dimension, from the determination of the nature of our physical space, which had to be decided empirically.[3] Riemann's ideas were so revolutionary that before Einstein's work on special relativity and especially on general relativity (1915) only one important philosopher, Edmund Husserl, had accepted Riemann's views, as clearly seen from a letter of Husserl to his teacher Franz Brentano of 1892 and two letters to Paul Natorp of 1897 and 1901.[4]

The revolution in geometry was not the only mathematical revolution in the nineteenth century. A second one occurred in analysis. As is well known, during the seventeenth century Leibniz and Newton developed, with complete independence of each other, what is usually called the differential and the integral calculus, an area of mathematics that found immediate application in the classical mechanics that Newton also

developed. But up to the beginning of the nineteenth century that area of mathematics was studied and applied on a mere intuitive basis, without any clear and precise knowledge of its fundamental concepts. That changed already in the first half of the nineteenth century with the work of the very distinguished French mathematician Auguste Louis Cauchy and of the certainly important philosopher, mathematician and defender of a sort of socialism, the priest Bernard Bolzano.[5] Such a rigorization of analysis was continued in the second half of the same century, with the work, especially, of Karl Weierstrass, as well as Richard Dedekind and others. Finally, already in the first half of that prolific nineteenth century there was also a revolution in another area of mathematics, namely, in algebra with the development of algebraic systems by Galois, Abel and others. The development towards more general and fundamental mathematical theories continued during the whole nineteenth century and the first decades of the twentieth century, originating the idea, first in Edmund Husserl, then in Nicholas Bourbaki and his school and accepted nowadays, that mathematics is a theory of general formal structures, its combinations, its specializations and the combinations of such specializations.[6]

But physics also began to change after Kant's death. During Kant's lifetime and until the mid-nineteenth century there existed the belief that all physical phenomena were reducible to Newtonian classical mechanics. Already in the mid nineteenth century all attempts to reduce electromagnetic phenomena to mechanical ones seemed to fail and the foundation of all physical phenomena on classical mechanics began to be seriously questioned. At the beginning of the twentieth century three decisive revolutions in physics destroyed any hope of a mechanical foundation of physics. In 1900 Max Planck found the constant that bears his name, marking in some sense the prehistory of quantum mechanics. In 1905 Albert Einstein introduced his special theory of relativity, which clearly contradicted classical mechanics by asserting that the velocity of light was finite. In 1908 the mathematician Hermann Minkowski made precise the notion of space-time for special relativity. In 1915 Einstein introduced his theory of general relativity, just two weeks before the great mathematician David Hilbert also obtained the theory of general relativity. (Hilbert, by the way was not only a colleague of Minkowski at the University of Göttingen, but also his lifelong friend since the days they were together in school in their home city of Königsberg, which was also Kant's home city.)[7] A few years later began another revolution in physics, this time in microphysics, namely, quantum mechanics, thanks to the work of Niels Bohr, Werner Heisenberg, Erwin Schrödinger, Max Born, Wolfgang Pauli and many others.

Finally, in logic, which had been basically stagnant for more than twenty-two centuries, a revolution also occurred in the nineteenth century. Already in the first half of that century the British George Boole and Augustus de Morgan began to expand logic with their work in what has been usually called the "algebra of logic". And in

the second half of that century the Anglo-American Charles Sanders Peirce and the German Ernst Schröder also contributed to this expansion of the domain of logic. However, without any doubt the greatest contributor to this fundamental and gigantic development of logic was the German mathematician, logician and philosopher Gottlob Frege, who began this revolution in logic with his small book *Begriffschrift* in 1879 and further elaborated his revolutionary contributions in his *opus magnum*, the two-volume book *Grundgesetze der Arithmetik* of 1893 and 1903. The revolution in logic continued a decade later with the monumental three volume work of Alfred North Whitehead and Bertrand Russell, *Principia Mathematica*, and experienced another decisive small revolution beginning in the early 1930s with the work of Gödel, Tarski and other distinguished logicians that transformed logic in a way that Frege, Russell and Whitehead probably could not have suspected.

Returning to philosophy after this succinct *excursus* into the three fundamental sciences, the main reaction to Kant's philosophy in Germany was a big jump into the void of metaphysics by Fichte, Schelling and Hegel, who dominated German philosophy during the first half of the nineteenth century. That does not mean that more grounded philosophy was not done in the German language during that period. In fact, the very important philosopher, Bernard Bolzano, a rationalist that can be seen both as continuing the work of the great Leibniz and preparing the way both for Frege's philosophical views and for Husserl, was a contemporary of Hegel, but during his lifetime did not receive the deserved recognition. And in the second half of the nineteenth century Hermann Lotze followed in the footsteps of Bolzano and influenced Frege in a more direct way. Of course, there were German-speaking philosophers of other tendencies, like the so-called positivists Ernst Mach and Richard Avenarius, and, on the other hand, the Austrian Franz Brentano, who did not belong to any of the extreme tendencies. In England, by the way, in the first half of the nineteenth century another sort of extreme empiricist made his appearance, namely, John Stuart Mill, whose views on logic were severely criticized both by Frege and Husserl.

2. The Surge and Demise of so-called Logical Empiricism

The revolutions in physics and in logic impressed sufficiently a few young German speaking philosophers with some background in the sciences, especially in physics, who in the 1920s originated a sort of philosophical school that has been known as "logical empiricism" or "logical positivism". The school developed around Moritz Schlick in Vienna and in a lesser degree around Hans Reichenbach in Berlin, and was influenced both by the work of the positivists like Ernst Mach and the revolution in logic. In fact, they tried to use the tools developed by Frege and later by Bertrand Russell and Alfred North Whitehead in the philosophical study of the empirical sciences,

especially of physics. It should be clearly underscored that since they considered logic and mathematics as analytic and *a priori* sciences, their militant empiricism was after all a moderate form of empiricism much nearer to David Hume's empiricism than to that propounded by the more radicals Locke and Stuart Mill. Nonetheless, their attempt to give an empirical foundation to physical laws was a complete failure. There is no way in which one can verify a physical law of higher level, as Husserl had already pointed out in the last chapter of the first volume of his *opus magnum*, *Logische Untersuchungen*. Moreover, as Husserl also stresses, there exist always more than one physical law of higher level compatible with the relevant empirical data.[8]

In any case, the Vienna Circle around Moritz Schlick, a student of Max Planck in physics, later turned philosopher, included the economist Otto Neurath, the mathematician Hans Hahn, the philosopher Edgar Zilzel, and the best known of them all, the philosopher Rudolf Carnap. Carnap had studied physics and then philosophy in Jena, and had even attended two of Gottlob Frege's lecture courses at the mathematics department. For that reason, and in complete ignorance of Germany's university system, Anglo-American scholars have anointed Carnap as a student of Frege. However, in the lecture courses in German universities there was absolutely no contact between the professor and the students hearing him —in fact, not even any questions were asked. Thus, Carnap never spoke to Frege nor had any sort of personal contact with him. (The personal contact occurred only in the more specialized seminars.) Hence, he was a student of Frege only in a very wide non-German sense.

The Berlin group was probably smaller and orbited around Einstein's student Hans Reichenbach, also a physicist turned philosopher, and the philosopher Kurt Grelling, but also included the younger Carl G. Hempel, who was going to be one of the best-known members of this school of thought usually called "logical empiricism" or "logical positivism". The logical empiricists separated clearly logic and mathematics from the empirical sciences. Logical and mathematical statements were considered as analytic and *a priori*, and required no empirical examination. On the contrary, statements of the empirical sciences needed to be grounded on experience. In fact, they attempted to show that the laws of the empirical sciences and, particularly, those of physics could be justified by experience. They first tried to show that such laws are verifiable, but did not succeed, since —as already mentioned two paragraphs back— the universal physical laws of higher level cannot be verified by experience. Then, following a suggestion of Karl Popper, who had been Moritz Schlick's student, they tried to falsify them, but also did not succeed, since existential physical statements cannot be falsified; and, finally, they tried to show that they are confirmable, that is, either verifiable or refutable by experience, and once more did not succeed.[9] In the meantime, and due to the surge of Nazism, some of the most prominent logical empiricists, like Carnap, Reichenbach and Neurath, emigrated to the United States of America, a land in the English tradition of empiricism.

As already mentioned, those difficulties had been pointed out by Husserl in 1900, and a few years later also by the French historian and philosopher of physics Pierre Duhem, better known for another different but related observation, namely, that contrary to what happens in other natural sciences like chemistry or biology, in physics one cannot isolate hypotheses to examine whether they are true or false, since in the design of the instruments used in the experiment the physicist has to presuppose the truth of other physical laws —for example of optics in the case of telescopes—, and, thus, if the experimental result is negative, what is refuted is not an isolated hypothesis but the conjunction of the hypothesis under discussion and other accepted physical laws.[10]

3. The Empiricist Response to the Demise of Logical Empiricism

The failure of this mild sort of empiricism to do justice to the physical laws of higher level should have been sufficient reason to abandon all forms of empiricism and once more attempt to find a middle point between empiricism and rationalism. But the reaction in Anglo-American philosophical circles to the demise of logical empiricism went precisely in the opposite direction. Under the "guidance" of Willard van Orman Quine, the response to the difficulties encountered by logical empiricism was a move towards a radical empiricism much nearer to Locke's and Stuart Mill's than to Hume's. By the way, Quine concentrated his discussions on Carnap and the logical empiricists, as if other philosophical schools did not exist and had never existed, and as if Carnap were a much more important philosopher than he really was. Thus, let us begin with the problem of analyticity.

The problem of analyticity is an old one and has been linked from the very beginning to logical and mathematical truths. Logical and mathematical truths had been acknowledged by many in the philosophical tradition to be analytic and *a priori*. As already mentioned, even Hume considered logical and mathematical truths as mere relations of ideas, coinciding on this point with the rationalist tradition. Kant, however, considered mathematical truths as synthetic, though at the same time *a priori*, but considered logical truths as analytic. However, what is interesting in Kant with respect to analytic truths is that Kant gave in the *Kritik der reinen Vernunft* two very different definitions of analyticity. In the Introduction[11] to his *opus magnum* he stated that a statement —in his terminology: judgement— is analytic if its truth can be obtained by the mere analysis of the concepts contained in it. Much later[12] in his *opus magnum*, he introduces the main principle of all analytic judgements as consisting of their derivability from the principle of non-contradiction. But from the principle of non-contradiction, even if supplemented by the also traditional principles of iden-

tity and of the excluded third —*tertium non datur*— almost nothing can be derived. Other important philosophers have ventured other definitions of analyticity certainly different from Kant's. Thus, for example, Gottlob Frege also considered analytic all statements that could be derived from the principles of logic[13], but of contemporary logic, which is incommensurably more powerful than Kant's Aristotelian logic and would include also all of mathematics with the exception of geometry. With regard to geometry, Frege had views similar to those of Kant, namely, considered that the only true geometry is Euclidean geometry, and that its statements were synthetic *a priori*.[14] Finally, Husserl considered a statement as analytic if it is true and remains true when it is completely formalized.[15] As already pointed out, contrary to Frege and Kant, Husserl considered that the different rival geometries — of 0, negative or positive curvature and of any finite number of dimensions — are equally justified from a mathematical standpoint and can be treated analytically, while the spatial structure of the physical world can only be determined empirically .[16]

Quine, however, opted to concentrate on Carnap's definition of analyticity as those statements whose truth can be known by the mere analysis of the concepts involved[17] a definition certainly inspired in Kant's first definition of analyticity, but also easier to refute than the others. In fact, though that definition would make all logical statements and presumably all non-geometric mathematical statements analytic, it also presumably included statements like "All bachelors are not married", which are clearly non-analytic and *a posteriori*, because dependent on the historical —and, thus, empirical— evolution of natural language. Of course, Quine concentrated his attack on those easily refutable cases of presumed analyticity and then generalized his "refutation" to all other purported analytic statements. This is a clear case of *non*-sequitur, but Anglo-American philosophers with empiricism in their veins gladly accepted Quine's "refutation" and concluded that there are no analytic statements. The fact that they so easily accepted Quine's argumentation as definitive is a pristine example of the poverty of Anglo-American empiricist philosophy, or better, empiricist ideology, in the sense of a set of internalized dogmas immune to any sort of possible refutation, be it empirical, logical or whatever.

But Quine went much further with his superficialities and sustained that all statements we consider true are connected in a sort of web of belief with *equal rights* (Quine 1953, pp.42–43) —for example, (i) the Principle of Non-Contradiction or (ii) the equation that '$2 + 2 = 4$" and the statements expressing that (iii) Quine had a terrible headache when he wrote "Two Dogmas of Empiricism" or that (iv) Newton had a toothache when he discovered the Law of Gravitation— and we can pragmatically decide to protect some of them while abandoning others. Hence, we could very well decide to abandon (i) or (ii) while protecting (iii) or (iv) from any refutation. It is difficult not to consider Quine's contention as a joke, were it not for its many followers in Anglo-American philosophy. This incredible assertion has been baptized

by empiricist priests as the "Duhem-Quine thesis" doing an incredible favor to Quine and offending the memory of Pierre Duhem.

Finally, it should be mentioned here that in his *Word and Object* and in many other later writings Quine tried to develop what he called a "naturalized epistemology" —and we have baptized it as "bugs epistemology"[18]— in order to explain from his extreme version of empiricism how we obtain knowledge of physical objects. He bases his naturalized epistemology on the discredited psychological school of behaviorism, which he considers as scientific and based on physical theory. It seems pertinent to quote Quine on this issue: "Impacts of molecules and light rays upon our nerve endings set off neural impulses that travel to the brain, where they get processed in **extravagantly complex and unexpected ways**.... Stimulations and neural impulses build up speech habits and other habits in us that continue to be brought into play by subsequent sensory stimulation, through **elaborate processes of association and reinforcement**."[19] Moreover, Quine adds: "Among **the myriad manifestations of these largely unfathomed processes**, one outcome is a neat correlation between certain ranges of sensory stimulation and certain sentences.... We can limit our considerations of neural events to the superficial ones that answer directly to the impacts and input from the external world."[20]

Certainly, phrases like "they get processed in extravagantly complex and unexpected ways", "stimulations and neural impulses built up speech habits", "among the myriad manifestations of these largely unfathomed processes" and "one outcome is a neat correlation between certain ranges of sensory stimulation and certain sentences", are vague and seem —to say the least— to require a lot of explanation. (To say the most: those phrases seem like the utterings of some comedians trying to make us laugh a little with some verbal nonsense.) In any case, naturalized epistemologists would need first to convince us that epistemology should be based on some sort of physiological-psychological foundation and, on the other hand, they need to clearly distinguish our epistemological capacities from those of other animals, no matter how developed, since we are the ones concerned with mathematics, logic, physical science and other sciences, as well as with poetry, music and art, while even the most intelligent animals, for example, dolphins and orangutans do not deal with such sophisticated enterprises, although they have similar sensory impressions to ours and in their brains probably similar processes occur as those that Quine presumably attempted so vaguely to describe. I can already guarantee that naturalized epistemology is not apt for such a task. But, in any case, let Quine rest in peace —even if just a little— and move to other related issues.

4. A Brief Excursus into the Philosophy of Mathematics

Anglo-American philosophy of mathematics has not done better than the more general concerns about our knowledge of the physical world. Certainly, Platonism was banned from the very beginning as a philosophy of mathematics from that extreme empiricist world, since most philosophers in that tradition felt terrorized by Platonic entities. Thus, Anglo-American philosophers of mathematics have flirted with the traditional nominalism and constructivism, while adding some new varieties like fictionalism and more recently indispensabilism. However, all such variants of anti-Platonism face the most varied insurmountable difficulties and none is capable of doing justice to present-day mathematics.

Let us begin with some brief observations on constructivism. The first objection against constructivism is that there are a variety of non-equivalent constructivisms, beginning with Kant's, Brouwer's intuitionism, Bishop's constructivism, Markov's constructivism, Griss' negationless mathematics and possibly others.[21] But leaving aside the problem of which of the many constructivisms is the true or most acceptable one —Griss' constructivism, by the way, is a non-starter—, there is no constructivism that could do justice to the whole of present-day mathematics. Basically, most of set theory is out of limits for constructivism, and even central parts of mathematical analysis. In the very best of cases, constructivism can acknowledge the first two infinite cardinals, namely, that of the natural numbers and that of the real numbers, which is the first uncountable cardinal. But already the set of functions on real numbers has a next higher cardinality, that is, the third infinite and the second uncountable cardinality. Hence, if constructivism, namely, the view that mathematical entities are constructed by the mathematician were true, we would have to reject fundamental parts of current mathematics.[22]

Fictionalism, namely, that mathematical entities are fictions, like literary works, and, thus, that it is a matter of subjective preference whether $1+1=2$ or $1+1=5$, as it is to prefer Goethe's *Faust* or Cervantes' *Don Quijote de la Mancha*, is simply preposterous and does not deserve much commentary. Thus, let us consider its more popular cousin nominalism. According to nominalism, mathematical entities are not even literary fictions; they simply do not exist. But if mathematical entities do not exist, then all existential mathematical statements are false, while all mathematical universal statements are vacuously true, as has been acknowledged by present-day's best-known nominalist, Hartry Field. However, if one accepts the totality of present mathematics and logic, one can prove that nominalism is false. Specifically, if one accepts model theory, one of the most important and fascinating areas of contemporary logic, one can show that nominalism is false.

Let us consider a first-order theory and its semantics, classical model theory. In model theory —from now on without the adjective "classical", since we are not deal-

ing with extensions of model theory to other logical languages— there are theories T that are "model-complete", that is, that if a model M of T is a substructure of another model M^* of T, then M is an elementary substructure of M^*.[23] Moreover, one can show that model-complete theories exist. But there is a theorem in model theory, namely, the "Model-Completeness Test" that states that a theory T is model-complete if, and only if —among other things— for any existential statement "S_e" in the language of T, there is a logically equivalent universal statement "S_u". Since the existential statement "S_e" is logically equivalent to the universal statement "S_u", they have the same truth value and, thus, it cannot be the case that all existential statements are false and all universal statements (vacuously) true. Hence, there are true existential mathematical statements and nominalism has been refuted.

Finally, let us consider the last attempt by Quine and others to avoid Platonism, namely, indispensabilism.[24] According to that view, some mathematical entities exist and others do not exist. Those that exist are the ones that have applications to physics —I suppose also to other empirical sciences. But their existence is a sort of physical existence, that is, similar to that of physical objects, not of Platonic entities. Such a view, like other of Quine's audacious ideas, presents great difficulties, to say the least. If the existence of mathematical entities is not only dependent on their applications but similar to that of physical objects, I wonder why in my long life and the many countries I have visited I have still not encountered the physical entity '1', or the physical entity 'Cartesian product', or the physical entity 'differential equation', or the physical entity 'Hilbert space'. (In the case of the latter, but not in the other cases, a disciple of Quine could argue that since Hilbert spaces are applied in the domain of quantum mechanics, they are also subatomic entities, even indistinguishable from other subatomic entities.) In any case, those who believe that the existence of mathematical entities is similar to that of physical entities have to respond where in nature can we find the number '5' or any similar entity.

Moreover, the set of mathematical theories and entities applied in physical theories cannot be rigidly determined once and for all, since new physical theories could probably require the application of mathematical theories never applied before, and then it is a consequence of indispensabilism that some mathematical entities can be born. For example, before general relativity theory Riemannian spaces did not exist. They were born in the moment in which Albert Einstein introduced the general theory of relativity. And if Einstein had not done it, then they would have been born some two weeks later, with David Hilbert, instead of Einstein, playing together with general relativity the role of midwife. But if the physical theories requiring the existence of given mathematical theories and entities are abandoned as false, then the mathematical entities applied in those theories would die. Hence, mathematical entities would have similar properties as those of human beings and all animals: they can be born and can also die. And, of course, there are some mathematical entities,

like uncountable cardinal numbers or the beautiful logical notion of ultraproducts that seem to have no chance of being born. Those are the absurd consequences of indispensabilism.

5. Carnap the Philosopher and its Interpretation

For scholars as blind as Quine and his followers, who believed that Carnap was a born empiricist, and rendered Carnap's *Der logische Aufbau der* Welt (briefly: Aufbau) as an empiricist book, and even for those somewhat more enlightened Anglo-American scholars, like Michael Friedman (1999), Alan Richardson (1998) and their friends, who believe that the Aufbau was written under Kant's influence, it is important to learn a little about Carnap's *True Intellectual Biography*. Carnap certainly studied in Jena, with the neo-Kantian Bruno Bauch as director of his doctoral thesis, and heard two lecture courses of Frege. But he wrote his dissertation while living in Buchenbach, a small town in the outskirts of Freiburg, where Husserl was professor. There is no evidence of Carnap having met Husserl during that period (nor evidence against it), only a great suspicion that they met; but the fact of the matter is that in his dissertation, *Der Raum* (Carnap 1991), Carnap shows no interest in the work of Frege, nor on that of his *Doktorvater*, that is, thesis director and neo-Kantian, Bruno Bauch. On the contrary, the influence of Husserl is so evident and decisive, that one can very well think that it was written by a disciple of Husserl. Moreover, after obtaining his doctor title in 1922, Carnap returned to Buchenbach and Freiburg, and there is evidence that he then visited Husserl's seminars. In fact, after the first session of the first seminar of Husserl after Carnap's return, the great philosopher invited a visiting professor who attended the seminar and Rudolf Carnap to his home. It would certainly be very strange that Husserl had invited Carnap to his home, if that were the first time that Husserl and Carnap met. Moreover, in those years in Freiburg —until 1926— Carnap befriended Husserl's assistant, the ten years younger Ludwig Landgrebe, who was then working on a possible publication of the second volume of Husserl's *Ideen zu einer reinen Phänomenologie und einer phänomenologischen Philosophie*, while Carnap was writing his professorship's thesis, the already mentioned *Aufbau*, and certainly hoping that it would be sponsored by Husserl.[25]

Even a superficial reading of the *Aufbau* should have served to convince anyone —but staunch fanatics of religious empiricism of the Quinean sort— that the book is not written by an empiricist. The constitutional basis of the *Aufbau* points without much doubt to some sort of transcendental philosophy, thus, either Descartes, Kant and the neo-Kantians like Bauch, or Husserl; though the term "constitution" seems to point more to Husserl than to his fellow transcendental philosophers. But as we have shown elsewhere, the decisive point that makes clear that Carnap's transcen-

dental philosophical standpoint is Husserlian and not Kantian is the thematization of what Husserl calls 'the problem of intersubjectivity' and Carnap 'the constitution of the heteropsychological'. That problem, which Descartes touches but does not seriously discuss, is almost completely absent from Kant's transcendental philosophy. Only in the third paralogism of Kant's 'Dialectic', that is, in the third part of his *Kritik der reinen Vernunft*, does Kant briefly touch the problem of intersubjectivity. Interestingly enough, that is a central issue of Husserl's second volume of *Ideen*, on whose grounding manuscripts Landgrebe was working in those days with a publication in sight. In fact, Husserl's reaction not accepting to sponsor the *Aufbau* as Carnap's professorship's thesis seems to point to Husserl's dissatisfaction with Carnap's appropriating too much from his unpublished manuscripts. After losing any hope of Husserl sponsoring his projected professorship's thesis, Carnap decided to abandon Freiburg for Vienna but continue with the *Aufbau* project, while making some hurried changes —probably eliminating some of the more important references to Husserl—, in order not to irritate his new sponsor, Moritz Schlick, who had criticized Husserl and had been scolded by Husserl's student and friend Hermann Weyl. The result of those changes made to the *Aufbau* is probably the reason why Carnap ended plagiarizing Husserl in *Aufbau*. While the second volume of his *Ideen* was published only posthumously, in his lectures in Paris of 1928, published a year later, the *Cartesianische Meditationen* (Husserl 1950), Husserl already presents his constitution of intersubjectivity.

But once a person fails to his moral principles, it is very improbable that he does not fail once again, and both in his 1932 paper 'Überwindung der Metaphysik durch logische Analyse der Sprache' (Carnap 2004) and in his famous 1934 book *Logische Syntax der Sprache* (Carnap 1937) Carnap very consciously plagiarized Husserl. On this issue, it is especially interesting that in his dissertation, *Der Raum* and in *Aufbau* Carnap included Husserl's *opus magnum*, *Logische Untersuchungen*, in the bibliography, even though in *Aufbau* there is no significant reference to that work. But in the writings of 1932 and 1934 mentioned above Carnap does not include *Logische Untersuchunge*n in the references, while taking some important insights from that work and consciously presenting them as his. In 'Überwindung der Metaphysik durch logische Analyse der Sprache' he takes Husserl's distinction between two sorts of nonsense and presents it as his, and in his *Logische Syntax der Sprache* Carnap takes Husserl's distinction in *Logische Untersuchungen* between laws that protect against nonsense and laws of consequence and that protect against countersense —which had appeared also in Husserl's 1928 *Formale und transzendentale Logik* (Husserl 1974)[26], a work that Carnap also ignores—, and rewrites them as formation rules and transformation rules.[27] Hence, this last fundamental distinction, usually made in many logic textbooks, is not a contribution of Carnap to logic but of Husserl.

6. Frege versus Husserl

Gottlob Frege is not only one of the greatest logicians ever, but presumably[28] an important and certainly an influential philosopher. Although he never taught philosophy —having written both his dissertation and his professorships' thesis in mathematics— , besides his two revolutionary logic books, *Begriffsschrift* (Frege 1964[1879]) and *Grundgesetze der Arithmetik* (Frege 1962[1893 & 1903]), most of his remaining work was in philosophy, including his book *Die Grundlagen der Arithmetik* (Frege 1986) of 1884, and his now classic essays of 1891 'Funktion und Begriff' (Frege 1990, pp.125–142), of 1892 'Über Sinn und Bedeutung' (Frege 1990, pp.143–162) and 'Über Begriff und Gegenstand' (Frege 1990, pp.163–178), and 'Der Gedanke' (Frege 1990, pp.342–362) of 1918. Frege was a staunch rationalist —though with respect to geometry he adopted a Kantian stance— and it seems somewhat strange that he has had such a great acceptance among the British and the Anglo-American philosophers, that use to be staunch anti-rationalists, though without doubt Bertrand Russell and Rudolf Carnap helped to make him known and palatable even to philosophers allergic to rationalism.[29]

His critique of psychologism in logic both in his 1884 book and in the introduction to the first volume of his two-volume foundational logic book played a very important role in the demise of the very influential psychologism in logic of the nineteenth century. Nonetheless, his critique of psychologism in logic, though very important, was not completely satisfactory, since he tended to mix specific psychologism (or psychologism relative to the species of humans) with individual psychologism, a conception that certainly none of the targets of his criticism ever committed to. The decisive annihilation of psychologism in logic came with the publication in 1900 of the first volume of Husserl's *Logische Untersuchungen*, a detailed examination and criticism of psychologism in the first nine of the eleven chapters of that volume. The almost eleven years younger Husserl was also a mathematician, a student of Kronecker and especially student and later assistant of Karl Weierstrass, one of the greatest mathematicians of the nineteenth century. But Husserl then turned to philosophy under the influence of another of his teachers, Franz Brentano, and wrote his professorship's thesis, *Über den Begriff der Zahl* in philosophy, though clearly in the philosophy of mathematics in 1887, and under the sponsorship of Franz Brentano's former student Carl Stumpf and with Georg Cantor, another student of Weierstrass and Kronecker, as a second reader. Husserl expanded that work and published the expanded version four years later under the title *Philosophie der Arithmetik* (Husserl 1970). In that work Husserl defended a sort of mild Brentanian psychologism, which he abandoned almost immediately after the publication of the book. Three year later, in 1894, Frege reviewed Husserl's first book and criticized its Brentanian psychologism, though Husserl had already abandoned it and was working precisely in his definitive and devastating

critique of psychologism in the first volume of *Logische Untersuchungen*.[30]

For some strange reason scholars in the Anglo-American empiricist tradition are fond of Frege, while either do not like or, most probably, do not understand Husserl. They have not only argued that Frege's late criticism of Husserl played a decisive role in the latter's abandonment of psychologism, but probably still believe that Husserl learnt from Frege what they consider one of Frege's greatest philosophical achievements, the distinction between sense and referent. Such a false view was propounded by Dagfinn Føllesdal[31], E. W. Beth (1965, p.353) and others and accepted without critique by members of the Anglo-American empiricist church. On this issue, a few remarks seem pertinent. First of all, the distinction between sense and referent is not such a deep discovery. In fact, both Bernard Bolzano[32] and Georg Friedrich Hegel[33] had basically made it many years before Frege and Husserl. Secondly, Husserl made the distinction in a paper written in 1890 under the title 'Zur Logik der Zeichen' (Husserl 1970, pp.289–339), thus, before the publication of any of Frege's writings that mention the distinction. Thirdly, both Frege and Husserl made the distinction publicly in writings published in 1891, Frege in 'Funktion und Begriff', published in January of 1891, and Husserl in his review of Ernst Schröders *Vorlesungen über die Algebra der Logik* (Husserl 1979, pp.3–43), published in March of 1891, but which was already in press in January of 1891. Moreover, in a letter of Frege to Husserl of that same year the former acknowledged that the latter had arrived at the distinction independently, and examines a small difference between their views, namely, that for Frege the concept was the referent of the conceptual word, whereas for Husserl the concept was the sense of the conceptual word and the extension its referent.[34] Hence, there is no ground for any further discussion of this issue, nor for continuing to take as true the myth of the influence, since Frege himself refutes the myth.

A much more important difference between Frege's and Husserl's versions of the sense-referent distinction concerns statements. As is well known, Frege considered that the sense of a (declarative) sentence was a thought and its referent a truth-value. On this issue, Husserl's final version of the distinction —in the second volume of his *Logische Untersuchungen*— is much more nuanced than Frege's.[35] Husserl coincides with Frege that the sense of a (declarative) sentence is a proposition (in his preferred terminology) or thought, but the referent of that sentence is a state of affairs. Moreover, two or more states of affairs —which are for Husserl categorial objects— are related to a single "situation of affairs", a sort of referential basis of states of affairs[36], while, of course, the most varied situations of affairs relate to the truth-value the True, while others relate to the truth-value the False. Thus, Husserl's theory of sense and referent is by far more sophisticated than the simple-minded one of Frege, and that is no small detail. Frege's semantics really functions only for propositional logic[37], where everything that counts is the truth-value, whereas Husserl's is not only theoretically richer but immensely more applicable.[38]

In fact, Frege's philosophical views are by far much simpler than Husserl's. He was clearly a rationalist, a Platonist and a logicist, and there is not much to argue about those issues. Husserl, who wrote immensely more about philosophy than Frege, indeed, most surely more than any other philosopher, is much more complex to evaluate. Since a few years after writing his *Logische Untersuchungen*, Husserl adopted the methodological stance of transcendental idealism —and did it more seriously than his two great predecessors Descartes and Kant—, many of his followers have tended to associate him with a sort of transcendental subjectivism. The fact of the matter is that his version of transcendental idealism, which he baptized as "transcendental phenomenology" is essentially a methodological stance that does not deter from his objectivism and rationalism. That becomes clear when one reads his *Formale und transzendentale Logik* —and also his *Erfahrung und Urteil* (Husserl 1974)— written certainly after his "transcendental turn" but expounding basically the same conception of logic and mathematics as the final chapter of the first volume of his *Logische Untersuchungen*. His Platonism in mathematics and his rationalism were in no way affected by his methodological stance of transcendental phenomenology. In fact, as he put it in his last book published during his lifetime, *Die Krise der europäischen Wissenschaften und die Phänomenologie* (Husserl 1962), genuine philosophy and genuine science are nothing else than pure rationalism.[39] What Husserl never accepted was logicism, that is, that mathematics is derivable from logic, being this one a sort of mother of mathematics. For Husserl, logic and mathematics are sister disciplines, not mother and daughter, and while logic is syntactical-semantical, mathematics, which he called a "formal ontology", is a theory of structures, logic's ontologically fat sister discipline. Moreover, both logical and mathematical knowledge —including geometry— is for Husserl both analytic and *a priori*.

Nonetheless, Husserl maintained that there exists some synthetic *a priori* knowledge, though not in mathematics, like Kant believed, but in a "material" realm, that of essences. He developed a method to try to obtain such a knowledge of essences, a method probably inspired in the process of going to the limit, for example, of a convergent sequence, used in mathematical analysis. That method of eidetic variation, usually called the "intuition of essences", tends to make crazy more than one phenomenologist. On this issue we just want to make two brief comments. Firstly, though we do not deny the possibility of obtaining synthetic *a priori* knowledge by the method of eidetic variation, we are not convinced that by such means one can obtain especially profound knowledge.[40] Secondly, Husserl developed in the Sixth Logical Investigation another sort of non-sensible intuition, namely, categorial intuition, that we consider more fruitful and important than the so-called intuition of essences. To put it as briefly as possible, what we human beings perceive are not mere objects, but states of affairs. We do not perceive the paper separately, as well as the pen and the desk. We perceive the pen and the paper side by side on the desk.

And we do not perceive the door, Charles and Peter separately, but Charles and Peter together at the door. And we perceive that Charles is taller than Peter. But there is no sensible correlate to "on", "at the side of", "and", "at" or "is taller than". All those are categorial components of our perceptions without any sensible correlation. Moreover, what Husserl calls "formal abstraction" and we would call "formalization", applied to categorial intuition originates for Husserl mathematical intuition, and that is certainly no small issue.[41]

7. Appendix: Frege and Husserl: a very Odd Couple

We will finish our discussion of Frege —and the comparison with Husserl— talking about a pair of issues certainly marginal but, in any case, almost completely ignored by mainstream philosophy. Firstly, we will only mention a serious accusation levelled against Frege by a distinguished scholar specialized in ancient philosophy and especially in the Stoics, an accusation that if true, would make Frege a sort of "teacher" of Carnap after all, though certainly not in a positive sense. Just recently, Suzanne Bobzien in a paper titled 'Frege plagiarized the Stoics' (Bobzien 2021)[42] accused Frege of plagiarism. Her contention is based on several sources, among others on Prantl's book on the Stoics[43], which presumably Frege read, and especially on his personal interaction with the philology professor, but very well-schooled in ancient philosophy, especially in the stoics, Rudolf Hirzel, and with the philosopher Rudolf Eucken, who were not only his colleagues, but lived very near Frege.[44] Gabriel *et alia*[45] were already conscious of Frege's indebtedness to the Stoics more than a decade before Bobzien's paper, but tried to minimize it by pointing out that the replacement of the pair ⟨subject-predicate⟩ by ⟨function-argument⟩ dates from Frege's 1879 *Begriffschrift*, and the (trivial) distinction between sign and what the sign stands for predates Frege's acquaintance with Hirzel, though they acknowledge that many of Frege's philosophical distinctions and conceptualizations had precedents in the Stoics. But Gabriel and his co-authors' anticipatory response to future accusations of plagiarism against Frege could serve only as an answer to a small part of Bobzien's contentions. Interestingly enough, it should be pointed out that when Gabriel and his co-authors wrote their paper they were all linked to the University of Jena, that is, precisely to the university where Frege taught 100 years before them.

Leaving that thorny issue of plagiarism aside, the second probably even less known fact about Frege are his far to the right political views expounded in his *Tagebuch* (Frege 1994)[46], and probably unknown to many Fregean scholars.[47] In that diary of his last years —1923-1924— Frege manifests very strong anti-Jewish feelings —and also anti-French— and shows some sympathy for the views of Ludendorff, who was Hitler's collaborator in their failed attempt at a putsch in 1923. We do not

know whether Frege knew that Husserl was a Jew, and not even of German —but of Czech-Austrian— origin, but if he did somehow learn about it, any sort of dialogue between the two philosophers would have been almost impossible, due to Frege's racial prejudices.

But the disparities between Frege and Husserl were even more radical. Though we suspected that Husserl's solution to Descartes' problem of intersubjectivity — Carnap's constitution of the heteropsychological— could serve as the starting point and the grounding for an ethical worldview like that of the primitive Christians or the libertarian socialists, only recently did we learn that in fact in the last years of his life Husserl had established the grounding of a socialist libertarian ethics. It seems appropriate to end this discussion with some quotations from Husserl's, the first three on his rationalism —which even some so-called Husserl specialists have still not discovered- and the last four on his socialist-libertarian ethics.

> ".... true and genuine philosophy, respectively, science and true and genuine rationalism are the same thing."[48]
>
> "If we call "rationalism" the conviction, yes, the auto-evidently generated (rational) and repeatedly established conviction that all true opinions have rational groundings, and that one, without asking for other sorts (extra-rational groundings or motivations for judgements), can put the task of a universal and purely rational science, then are philosophy and science in the Platonic-Aristotelian sense and in that of the whole modern philosophy the birth of a universal rationalism.[49]
>
> "Universal science in its infinity is perfect rationalism."[50]
>
> "The rational I finds himself ... with other Is in the context of a community and it belongs to the development of humankind the motivation, that also directs to the idea of the genuine humankind .... To this belongs the development of humankind to personalities of higher order and to the idea of a total personality, that extends to the whole humankind on earth (the complete possibly included humankind in the unity of a reasonably changing community).[51]
>
> "The human being finds himself in front of other human beings as his "closest", that is, as members of a narrower community. He himself belongs to it and takes himself subjectively as its zero member; [but] from the point of view of the members, that he internally replaces as corresponding zero members, he is then a member. The human being takes himself with regard to his nearest community, and also his nearest [human beings], as unities of this community, and has this nearest community as central member of more embracing communities, that also can be central members of [still] more embracing [communities]."[52]
>
> "True happiness in this sense is the joy of obtaining the best under consideration, and to obtain the best for everyone you love, [and] ultimately for all

> human beings. Individual happiness is not sufficient, if other human beings are not happy .... The greatness and purity of joy is dependent on the joy of others."[53]
>
> "But they are for me co-subjects, as subjects in my ethical field of foresight; their joy and suffering concern me.... There originates the idea of an intersubjective personal community praxis, directed to the world not to 'dominate" it, not to use it, but to humankind, to educate it —thus, we all in the community— [to be] an absolute humankind ....[54]

Since contrary to physical space, political space is most surely Euclidean, Frege's and Husserl's views on what human society should be were in the opposite extremes.[55]

References

Abbot, S. 2001. *Understanding Analysis*. New York: Springer.

Bell, J. L.; Slomson, A. B. 1969. *Models and Ultraproducts*. Amsterdam: North Holland.

Beth, E. W. 1965. *The Foundations of Mathematics*. Revised edition. Amsterdam: North Holland.

Bobzien, S. 2021. Frege plagiarized the Stoics. In: F. Leigh (ed.), *Themes in Plato, Aristotle, and Hellenistic Philosophy*, pp.149–206. London: University of London Press.

Bolzano, B. 1963[1978]. *Wissenschaftslehre*. Hamburg: Felix Meiner, revised edition.

Bolzano, B. 1978. *Grundlegung der Logik*. Hamburg: Felix Meiner.

Bourbaki, N. 1949. Foundations of Mathematics for the Working Mathematician. *Journal of Symbolic Logic* **14**(4): 1–8.

Bourbaki, N. 1950. The Architecture of Mathematics. *American Mathematical Monthly* **57**: 221–252.

Bourbaki, N. 1966. *Éléments des Mathématiques: Théorie des Ensembles*. Paris: Hermann.

Carnap, R. 1991. *Der Raum*. Vaduz: Topos Verlag.

Carnap, R. 1928. *Der logische Aufbau der Welt*. Hamburg: Felix Meiner.

Carnap, R. 2004. Überwindung der Metaphysik durch logische Analyse der Sprache. In: *Scheinprobleme in der Philosophie und andere metaphysikkritische Aufsätze*, pp.81–108. Hamburg: Felix Meiner.

Carnap, R. 1937. *Logische Syntax der Sprache*. Expanded English edition. London: Routledge.

Carnap, R. 1956. *Meaning and Necessity*. Enlarged edition. Chicago: University of Chicago Press.

Carnap, R. 1956. Meaning Postulates. In: *Meaning and Necessity*, Appendix B. Chicago: University of Chicago Press.

Carnap, R. 2016. A Letter of Rudolf Carnap to Jonas Cohn of 26 September 1925. In: G.E. Rosado Haddock (ed.), *Husserl and Analytic Philosophy*, pp.331–332. Berlin: De Gruyter.

Cederberg, J. 2001. *A Course in Modern Geometries*. New York: Springer.

Chang, C. C.; Keisler, H. J. 1990. *Model Theory*. Revised edition. Amsterdam: North Holland.

Duhem, P. 1996. Quelques réflexions au sujet de la physique expérimentale. Translated. In: *Essays in the History and Philosophy of Science*, pp.75–111. Edited by Roger Ariew and Peter Baker. Indianapolis: Hackett.

Duhem, P. 1996. Examen logique de la théorie physique. Translated. In: *Essays in the History and Philosophy of Science*, pp.232–238. Edited by Roger Ariew and Peter Baker. Indianapolis: Hackett.

Duhem, P. 1991. *La Théorie Physique, son Objet, sa Structure*. Translation, *The Aim and Structure of Physical Theory*. Princeton: Princeton University Press.

Duhem, P. 1996. *Essays in the History and Philosophy of Science*. Edited by Roger Ariew and Peter Baker. Indianapolis: Hackett.

Føllesdal, D. 1994. *Ein Beitrag zur Beleuchtung der phänomenologischen Philosophie*. Translated. In: L. Haaparanta (ed.), *Mind, Meaning and Mathematics*, pp.3–47. Dordrecht: Kluwer.

Frege, G. 1964. *Begriffschrift*. Hildesheim: Georg Olms.

Frege, G. 1986. *Die Grundlagen der Arithmetik*. Centenary edition, edited and with an Introduction by Christian Thiel. Hamburg: Felix Meiner.

Frege, G. 1990. Funktion und Begriff. In: *Kleine Schriften*, pp.125–142. Hildesheim: Georg Olms.

Frege, G. 1990. Über Sinn und Bedeutung. In: *Kleine Schriften*, pp.143–162. Hildesheim: Georg Olms.

Frege, G. 1990. Über Begriff und Gegenstand. In: *Kleine Schriften*, pp.167–178. Hildesheim: Georg Olms.

Frege, G. 1990. Der Gedanke. In: *Kleine Schriften*, pp.342–362. Hildesheim: Georg Olms.

Frege, G. 1962. *Grundgesetze der Arithmetik*. 2 volumes, 1893 & 1903, reprinted in one volume. Hildesheim: Georg Olms.

Frege, G. 1974. Brief an Husserl vom 24ten Mai 1891. In: *Wissenschaftlicher Briefwechsel*, pp.94–98. Hamburg: Felix Meiner.

Frege, G. 1983. Über Euklidische Geometrie. In: *Nachgelassene Schriften*, pp.182–186. Revised edition. Hamburg: Felix Meiner.

Frege, G. 1990. *Kleine Schriften*. Revised edition. Hildesheim: Georg Olms.

Frege, G. 1994. Tagebuch. *Deutsche Zeitschrift für Philosophie* **6**: 1067–1098.

Friedman, M. 1999. *Reconsidering Logical Positivism*. Cambridge: Cambridge University Press.

Gabriel, G.; Kienzler, W. 1994. Frege's Politisches Tagebuch. *Deutsche Zeitschrift für Philosophie* **6**: 1057–1066.

Gabriel, G.; Hülser, K.; Schlotter, S. 2009. Zur Miete bei Frege: Rudolf Hirzel und die Rezeption der stoischen Logik und Semantik in Jena. *History and Philosophy of Logic* **30**: 369–388.

Greenberg, M. J. 1973. *Euclidean and Non-Euclidean Geometries: Development and History*. San Francisco: W. H. Freeman.

Hegel, G. 1970. *Vorlesungen über die Philosophie der Religion*. Vol. 16 of Hegel's *Werke in zwanzig Bänden*. Frankfurt am Main: Suhrkampf.

Hume, D. 1975. *An Enquiry Concerning Human Understanding*. Third revised edition. Oxford: Oxford University Press.

Husserl, E. 1970. *Philosophie der Arithmetik*. Husserliana XII. Den Haag: Martinus Nijhoff.

Husserl, E. 1979. Besprechung von E. Schröders *Vorlesungen über die Algebra der Logik I*. In: *Aufsätze und Rezensionen (1890-1910)*, pp.3–43. Husserliana XXII. Den Haag: Martinus Nijhoff.

Husserl, E. 1975-1984. *Logische Untersuchungen*. Husserliana XVIII. Den Haag: Martinus Nijhoff.
Husserl, E. 1976. *Ideen zu einer reinen Phänomenologie und einer phänomenologischen Philosophie I*. Husserliana III. Revised edition. Den Haag: Martinus Nijhoff.
Husserl, E. 1974. *Formale und transzendentale Logik*. Husserliana XVII. Den Haag: Martinus Nijhoff.
Husserl, E. 1950. *Cartesianische Meditationen*. Husserliana I. Den Haag: Martinus Nijhoff.
Husserl, E. 1962. *Die Krise der europäischen Wissenschften und die Phänomenologie*. Husserliana VI. Second edition. Den Haag: Martinus Nijhoff.
Husserl, E. 1985. *Erfahrung und Urteil*. Sixth edition. Hamburg: Felix Meiner.
Husserl, E. 1952. *Ideen zu einer reinen Phänomenologie und einer phänomenologischen Philosophie II*. Husserliana IV. Den Haag: Martinus Nijhoff.
Husserl, E. 1970. Zur Logik der Zeichen, Appendix B. In: *Philosophie der Arithmetik*, pp.289–339. Husserliana XII. Den Haag: Martinus Nijhoff.
Husserl, E. 1983. *Studien zur Arithmetik und Geometrie*. Husserliana XXI. Den Haag: Martinus Nijhoff.
Husserl, E. 2014. *Grenzprobleme der Phänomenologie*. Husserliana XLII. Dordrecht: Springer.
Kant, I. 1990. *Kritik der reinen Vernunft*. Third edition. Hamburg: Felix Meiner.
Kant, I. 1996. *Metaphysische Anfangsgründe der Naturwissenschaft*. Hamburg: Felix Meiner.
Mac Lane, S. 1971. *Category Theory for the Working Mathematician*. New York: Springer.
Mayer, V. 1991. Die Konstruktion der Erfahrungswelt: Carnap und Husserl. In: W. Spohn (ed.), *Erkenntnis Orientated*, pp.287–303. Dordrecht: Kluwer.
Mayer, V. 1992. Carnap und Husserl. In: D. Bell; W. Vossenkuhl (eds.), *Wissenschaft und Subjektivität*, pp.181–205. Berlin: Akademie Verlag.
Mayer, V. 2016. Der *Logische Aufbau* als Plagiat. In: G.E. Rosado Haddock (ed.), *Husserl and Analytic Philosophy*, pp.175–260. Berlin: De Gruyter.
Quine, W. v. O. 1953. Two dogmas of empiricism. In: *From a Logical Point of View*, pp.20–46. Cambridge, MA: Harvard University Press.
Quine, W. v. O. 1953. *From a Logical Point of View*. Cambridge, MA: Harvard University Press.
Quine, W. v. O. 1960. *Word and Object*. Cambridge, MA: The MIT Press.
Quine, W. v. O. 1995. *From Stimulus to Science*. Cambridge, MA: Harvard University Press.
Quine, W. v. O. 2008. *Confessions of a Confirmed Empiricist and other Essays*. Cambridge, MA: Harvard University Press.
Quine, W. v. O. 2008. The Sensory Support of Science. In: *Confessions of a Confirmed Empiricist*, pp.327–337. Cambridge, MA: Harvard University Press.
Richardson, A. 1998. *Carnap's Construction of the World*. Cambridge: Cambridge University Press.
Riemann, B. 1960. *Über die Hypothesen, welche der Geometrie zugrunde liegen*. Third edition. New York: Springer.
Robinson, A. 1963. *Introduction to Model Theory and to the Metamathematics of Algebra*. Amsterdam: North Holland.
Rosado Haddock, G. E. 1982. Remarks on Sense and Reference in Frege and Husserl. *Kant Studien* **73**(4): 23–40.
Rosado Haddock, G. E. 1987. Husserl's Epistemology of Mathematics and the Foundation of Platonism in Mathematics. *Husserl Studies* **4**(2): 81–102.

Rosado Haddock, G. E. 1991. On Husserl's Distinction between State of Affairs (Sachverhalt) and Situation of Affairs (Sachlage). In: D. Føllesdal; J.N. Mohanty; T.M. Seebohm (eds.), *Phenomenology and the Formal Sciences*, pp.35–48. Dordrecht: Kluwer.

Rosado Haddock, G. E. 2006. *A Critical Introduction to the Philosophy of Gottlob Frege*. Aldershot: Ashgate.

Rosado Haddock, G. E. 2008. *The Young Carnap's Unknown Master*. Aldershot: Ashgate.

Rosado Haddock, G. E. 2021. On Analytic *a posteriori* Statements: are they Possible?. *Logique et Analyse* **229**: 25–33. Spanish Translation with an Appendix reprinted in *Iconoclastic Papers in Rigorous Philosophy*, pp.13–29. Moldova: Lambert Academic Publishing.

Rosado Haddock, G. E. (ed.). 2016. *Husserl and Analytic Philosophy*. Berlin: De Gruyter.

Rosado Haddock, G. E. 2018. *Unorthodox Analytic Philosophy*. London: College Publications.

Rosado Haddock, G. E. 2018. The Fine Structure of Sense-Referent Semantics. In: *Unorthodox Analytic Philosophy*, pp.31–56. London: College Publications.

Rosado Haddock, G. E. 2018. Husserl and Kant: voilà la différence'. In: *Unorthodox Analytic Philosophy*, pp.115–141. London: College Publications.

Rosado Haddock, G. E. 2019. Why Unorthodox Analytic Philosophy?. *Disputatio* **8**(11): 61–98.

Rosado Haddock, G. E. 2021. *Iconoclastic Papers in Rigorous Philosophy*. Moldova: Lambert Academic Publishing.

Rosenfeld, B. A. 1988. *A History of Non-Euclidean Geometries*. New York: Springer.

Russell, B.; Whitehead, A. N. 1969. *Principia Mathematica* (3 vols.). London: Allen and Unwin.

Stadler, F. 1993. Wien-Berlin-Prag: Zum Aufstieg der Wissenschaftlichen Philosophie. In: R. Haller; F. Stadler (eds.), *Der Aufstieg der Wissenschaftlichen Philosophie*. Vienna: Hölder-Pichler-Tempsky.

Tappenden, J. 2006. The Riemannian Background of Frege's Philosophy. In: J. Ferreirós; J. Gray (eds.), *The Architecture of Modern Mathematics*, pp.97–132. Oxford: Oxford University Press.

Vongehr, T. 2016. Jonas Cohn und Edmund Husserl: eine Skizze ihrer Beziehung. In: G. E. Rosado Haddock (ed.), *Husserl and Analytic Philosophy*, pp.323–329. Berlin: De Gruyter.

von Prantl, K. 1997. *Geschichte der Logik im Abendlande* (4 vols., 1855-1885). Hildesheim: Georg Olms.

Notes

[1]See Hume (1975).

[2]For non-Euclidean geometries, see, e.g., Judith Cederberg (2001) or Marvin J. Greenberg (1973).

[3]See Riemann (1960).

[4]See Edmund Husserl, *Briefwechsel*, Vol I, p. 10 for the letter of 29 December 1892 to Brentano, as well as Vol. 5 for Husserl's letters to Natorp of 29 March 1897 and 7 September 1901.

[5]See on this issue any good textbook on analysis, for example, Abbot (2001).

[6]See Chapter XI of the first volume of Husserl's (1975-1984), as well as his (1974), and Nicholas Bourbaki (1950), as well as his (1966). By the way, we consider categorial foundations of mathematics — as well as Bourbaki's set-theoretic foundations — as a mere variant of structural foundations, not needing here a separate treatment. In any case, see, for example, Saunders Mac Lane (1971).

[7]By the way, Hilbert was also a colleague in Göttingen of Husserl for more than a decade and his life-long friend.

[8]See, precisely, Chapter XI of the first volume of Husserl (1975-1984).

[9]What they did not understand —but should have also learnt from Husserl's *Logische Untersuchungen* (see the preceding footnote)— is that physics is not a mere empirical science, but a sort of double-headed science. It is, on the one hand, an explanatory rational science like logic and mathematics, but contrary to these last two sciences, physical theories need to account for experience.

[10]See Pierre Duhem (1991), as well as Duhem (1996).

[11]See Kant (1990), Introduction, Section IV, pp.45–48.

[12]Kant (1990), p. 207 (A 151, B 190), under the heading 'Von dem obersten Grundsatze aller analytischen Urteile'.

[13]See already the Introduction to *Die Grundlagen der Arithmetik*.

[14]See Frege (1986), §14. See also Frege's posthumously published paper 'Über euklidische Geometrie', in Frege (1990), pp.182–184, where Frege compares on p.184 non-Euclidean geometry to alchemy and to astrology. Those comparisons made by Frege notwithstanding, the Anglo-American Jamie Tappenden dared to write a paper on the presumed influence of Riemann's views on Frege, namely, Tappenden (2006). It is difficult to find in the annals of philosophy a greater blunder than Tappenden's —though see footnote 49 below for a possible rival. In any case, Riemann had as much influence on Frege's philosophical views on geometry as Santa Claus or Fidel Castro.

[15]See Husserl (1975-1984), Untersuchung III, §12.

[16]See footnote 6 above. By the way, the definition of analyticity proposed by the present author is also completely immune to Quine's criticisms. See on this issue Rosado Haddock (2015).

[17]See Carnap (1956) as well as his paper 'Meaning Postulates', in Carnap (1956), and Quine (1953).

[18]See our 'Husserl and Kant: voilà la différence' in Rosado Haddock (2018), pp.115–141.

[19]See Quine's 'The Sensory Support of Science', p.329, in Quine (2008). The bold letters in this paragraph are ours, not Quine's, though the vague and somewhat mysterious expressions are his.

[20]Quine 'The Sensory Support of Science', p.329, in Quine (2008).

[21]On the variety of constructivisms, see, for example, Beth (1965). For the exotic Griss' constructivism, see pp.437–439.

[22]Hence, we would have to renounce to Cantor's paradise and much more, e.g. to the set of all functions on real numbers and to the set of all functions on complex numbers.

[23]See any good textbook on model theory for the definition of model-completeness and the Model-Completeness Test, for example, the classic textbooks of Robinson (1963), Bell and Slomson (1969), and Chang and Keisler (1990).

[24]For a more extensive examination of indispensabilism, see our 'Why and How Platonism?' as well as the appendix to our 'The Fine Structure of Sense-Referent Semantics' in Rosado Haddock (2018).

[25]See on this issue Carnap's letter to Jonas Cohn of 26 September 1925, reprinted in Rosado Haddock (2016), pp.321–322, as well as Vongehr (2016).

[26]In this book Husserl already distinguishes between a syntactic (apophantic) and a semantic side of logic, the latter obtained by incorporating the notion of truth and related semantic notions to the apophantic side, thus, anticipating the now common distinction — since Tarski's and Gödel's epoch-making writings in the early 1930s— between the syntactic and the semantic components of logic.

[27]See Carnap (1937) §§46, 47, pp.120–125 for the general formulation of formation and transformation rules.

[28]See, however, §7 below and the reference to Bobzien's recent paper.

[29]Empiricists make miracles to defend their chosen ones. Thus, it is no surprise that they have a partial amnesia when considering Frege, that makes them forget that he was a staunch rationalist. Another blatant example of this sort of amnesia occurred to Friedrich Stadler in a book he co-edited, in which he stated that Leibniz and Bolzano, **of all people**, were empiricists. See Stadler (1993), p.13.

[30]For the evolution of Husserl's thought in those years, it seems pertinent to examine Husserl (1983).

[31]See Føllesdal (1964). Contrary to Føllesdal's intentions, the word "*Verdunkelung*" would be much more adequate than *Beleuchtung* in the title of that work, since trying to make Husserl palatable to Anglo-American empiricists, he distorted the fact of Husserl's independent discovery of the sense-referent distinction.

[32]See Bolzano 1978, selected sections from volumes I and II of Bolzano 1964[1978].

[33]See Hegel (1970). We are indebted for this information on the anticipation to our lifelong friend and former colleague Miguel A. Badía Cabrera, and for the bibliographical information to our other lifelong friend and former colleague Eliseo Cruz Vergara, who is a Hegel specialist.

[34]See Frege (1974).

[35]See Husserl (1975-1984), Sixth Investigation, Second Part: Sensibility and Understanding.

[36]The states of affairs '5<7' and '7>5' have as basis the same situation of affairs, namely, that the number 5 comes first than the number 7 in the natural number sequence.

[37]The semantics for first-order logic is the immensely rich and beautiful classical model theory originated by Alfred Tarski, Abraham Robinson and their collaborators, for which Frege's poor and primitive semantics could not be more inadequate.

[38]See on this important issue Rosado Haddock (2018), pp.31–56.

[39]See footnote 47 below for the exact quotation.

[40]It seems pertinent to stress here that we are not transcendental phenomenologists, like most Husserlian scholars, and have never consciously applied their beloved method of eidetic variation. We accept, however, Husserl's transcendental turn as a methodological-epistemological instrument to free the philosopher of all prejudices. But as happened with Husserl, the transcendental turn does not deter us from rationalism. In fact, our philosophical views, which we have called 'unorthodox (or heterodox) analytic philosophy', but would have pre-

ferred to call 'critical rationalism' —if Popper had not already used that name for his philosophy—, are nearer to Husserl's views than to those of any other philosopher.

[41]For categorial intuition and, in particular, mathematical intuition, besides Husserl's Sixth Logical Investigation, see Rosado Haddock (1987).

[42]We thank our old friend, the linguist William Greenberg for the acquaintance with Bobzien's paper.

[43]Carl Prantl, *Geschichte der Logik im Abendland* (4 vols.) 1855-1885. See also Gabriel et al. 2009, pp.369–388, especially §§5-6.

[44]Hirzel lived in the second floor of Frege's house and Eucken at the other side of the street.

[45]See the paper by Gabriel, Hülser and Schlotter mentioned in footnote 69.

[46]We are indebted for this reference to our former professor, and later colleague and friend, the late Manfred Kerkhoff.

[47]By the way, Frege's *Tagebuch* is preceded by an introductory paper written also by Gottfried Gabriel, this time with Wolfgang Kienzler as co-author. See Frege (1994), pp.1057–1066.

[48]"...wahre und echte Philosophie bzw. Wissenschaft und wahrer und echter Rationalismus ist einerlei.", Husserl (1962), p.200.

[49]"Nennt man „Rationalismus" die Überzeugung, ja die selbst einsichtig (rational) geschöpfte und sich fortbewährende Überzeugung, dass alle wahren Meinungen ihre rationalen Gründe haben und dass man, ohne nach anderweitigen (außerrationalen „Gründen", Urteilsmotiven) zu fragen, die Aufgabe einer universalen und rein rationalen Wissenschaft stellen kann, dann ist Philosophie und Wissenschaft im platonisch-aristotelischen Sinn und im ganzen neuzeitlichen Sinn Geburt eines universalen Rationalismus.", Husserl (2014), p.185.

[50]"Universale Wissenschaft in ihrer Unendlichkeit ist vollkommener Rationalismus.", Husserl (2014), p.490.

[51]"Das Vernunft-Ich findet sich ... im Zusammenhang der Gemeinschaft mit anderen Ich und es gehört zur Entwicklung der Menschheit die Motivation, die für sie ebenso zur Idee der wahren Menschheit führt .... Dazu gehört die Entwicklung der Menschheit zu Personalitäten höherer Ordnung und zur Idee einer Allpersonalität, die die gesamte Erdenmenschheit (die gesamte in die Einheit einer sich wechselverständigen Gemeinschaft möglicherweise eintretende Menschheit) umspannt. Husserl (2014), p.284.

[52]"Der Mensch findet sich gegenüber andere Menschen als seine "Nächsten", nämlich als Mitglieder einer engeren Gemeinschaft. Er selbst gehört ihr zu und fasst sich subjektiv als ihr Nullglied; vom Standpunkt der Mitglieder, die er in Hineinversetzung in sie als jeweilige Nullglieder auffasst, ist er selbst dann Mit-glied. Der Mensch hat sich gegenüber seine nächste Gemeinschaft und zugleich seine Nächsten als Einheiten dieser Gemeinschaft und hat diese nächste Gemeinschaft als Zentralglied für umfassendere Gemeinschaften, die selbst wieder Zentralglieder für umfassende sein können. Husserl (2014), p.297.

[53]"Wahres Glück in diesem Sinn ist die Freudigkeit am Gelingen des Besten, das man vorhat, und Gelingen des Besten für alle, die man liebt, schließlich für alle Menschen. Eigenes Glück genügt nicht, wo nicht andere Menschen im guten Glück haben .... Die Größe und Reinheit dieser Freude hängt ab von den Freuden der Anderen.... Husserl (2014), p.330.

[54]"Aber sie sind auch für mich Mitsubjekte, als Subjekte in meinem ethischen Fürsorgefeld; ihr wahres Wohl und Wehe geht mich selbst an .... Von da aus erwächst die Idee einer intersubjektiven personalen Gemeinschaftspraxis, nicht auf die Welt gerichtet, sie zu „be-

herrschen“, sie zu nutzen, sondern auf die Menschheit, sie —also uns alle in Gemeinschaft— zu einer absoluten Menschheit zu erziehen .... Husserl (2014), p.438.

[55]We are indebted to Prof. Miguel A. Badía Cabrera —see footnote 33— for a pair of linguistic and, more importantly, some stylistic suggestions that helped soften our sometimes rude wording.