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A New Problem for Naturalizing Logico-mathematical Knowledge?

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Abstract. This article advances a categorial challenge to physicalistic versions of naturalized epistemology as an account of logico-mathematical knowledge and practice. It argues that, *qua* physically constituted, any putative rule-governed system is logico-mathematically indeterminate: physical facts are systematically insufficient to fix a unique logico-mathematical (L–M) profile to the exclusion of incompossible alternatives. Building on a Kripkean line of thought concerning the truthmakers of rule attributions and rule applications, the paper develops the point through a Stabler-style mechanism allegedly computing a function f over the natural numbers, and argues that the same finite physical profile equally supports an incompossible alternative function g that coincides with f on all physically realizable cases. The result is that the physical facts fail to determine whether the mechanism determinately and objectively implements any unique or definite L–M rule at all. Crucially, the indeterminacy-generating features exploited by the Stabler-style case are structural: they are characteristic of any finite physical mechanism. Attempts to restore determinacy via counterfactual robustness, normal-operation conditions, or supervenience are argued to be question-begging, since they covertly presuppose precisely the rule whose objective implementation is at issue. The upshot is a dilemma: either determinate L–M rule-following is possible, in which case physicalistic NE is false; or physicalistic NE is true, in which case determinate L–M rule-following (and knowledge) is impossible.

Keywords: Kripke • philosophy of mind • rule-following • materialism • physicalism • naturalism

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Introduction

The project of naturalizing logico-mathematical knowledge is, in general, an instance of the more general program of naturalizing epistemology. Naturalized epistemology (NE) is, broadly speaking, the attempt to allocate the philosophical discipline known as 'epistemology' within the rubric of empirical science. Likewise, NE considers the subject matter of epistemology (knowledge, epistemic status and activity, and so forth) to be natural (and, therefore, empirically workable) phenomena. Yet, beyond this vague and somewhat misleading characterization,[1] there is no consensus, let alone a single or standard definition, about what naturalism amounts to or what a naturalized epistemology consists of. Rather than bicker about definitions, I aim to tackle a metaphysical position that has been the predominant framework within analytic philosophy for the past 70 years or so: physicalism. Physicalism is the naturalistic view that everything is physical.[2] Physicalism and naturalism may or may not be equivalent terms, depending on which philosopher is answering the question.[3] Since most of the physicalists I wish to refute consider themselves naturalists, I shall address my argument against this type of naturalism. So, by NE, I shall mean physicalistic NE.

Here's the dilemma: either there is such a thing as following or applying, determinately, a logico–mathematical rule, in which case NE is false; or NE is true, in which case there is no such thing as following or applying, determinately, a logico–mathematical rule. But if there is no such thing as following or applying, determinately, a logico–mathematical rule, then there is no such thing as determinate logico–mathematical knowledge and practice. Yet there evidently is such knowledge and practice. Therefore, NE is false.[4] In other words: either we accept that logico–mathematical rules can be determinately followed or applied, and thus reject naturalism; or we accept naturalism, and must therefore deny the very possibility of logico–mathematical, rule-governed activity (knowledge and practices) in any objective and determinate sense.[5] Again, in what follows I frame the argument in relation to the naturalistic view known as physicalism.

The foundations of the argument I shall defend date back to the early 1980s, when the late American philosopher Saul Kripke developed his argument against the computational theory of the mind.[6] The argument I will develop, however, is my own. My concern is not primarily with Kripke's refutation of computationalism as such, but with the type of argument that his refutation makes possible. I contend that this argument type undermines physicalist NE as a viable philosophical framework for explaining the nature of logico–mathematical knowledge and practice. What I propose, in effect, is an intuitive proof of what logico–mathematical (L–M) knowledge and activity cannot be.

I will frame the argument in terms of the concept of a logical or mathematical (L–M) *rule* and will argue that:

1. No physical thing, in virtue of physical properties alone, can objectively follow an L–M rule.
2. NE, however, requires that whatever objectively follows an L–M rule must do so in virtue of physical properties alone.
3. Hence, NE fails as an account of L–M rules, since it presupposes a condition that cannot obtain.

Subsidiary considerations to be developed include the claim that L–M rules (or objective L–M rule following) are constitutive of logico–mathematical knowledge and practice and that, consequently, NE fails as an account of L–M knowledge.

The logico-mathematical indeterminacy of the physical

Kripke's argument against computational theories of mind turns on a contrast between the ideal character of a mathematical structure (e.g., an infinitary function, a finite-state automaton (FSA), and so on) and the physical character of the concrete mechanisms that purport to implement such a structure.[7] What is interesting about Kripke's argument is not merely its challenge to a particular view in the philosophy of computation or the philosophy of mind but the breath and generality of its implications: it calls into question the very intelligibility of "physical implementation/realization" of ideal structures as an objective notion. The argument exposes a categorical and ontological inconsistency in the assumption that an ideal mathematical rule or function can be determinately realized by a physical mechanism *solely in virtue of its physical properties*.

Indetermination

Consider the following thought experiment, developed first by computer scientist Edward Stabler in an early attempt to refute Kripke's argument.[8] Let P be a simple physical mechanism (PM, for short) that, on Stabler's interpretation, applies the infinitary rule of identity over the natural numbers, such that for any $n \in \aleph$ that P computes, $n = n$. Here's the diagram of P:

$$\text{Input} \;\rule[0.5ex]{12em}{0.4pt}\; \text{Output}$$

This is a circuit diagram of a PM that takes in sequences of n voltage pulses at an entry point, labelled "Input", and transmits through a conductor that very sequence of n voltage pulses to an exit point, called "Output". Physical laws guarantee that, if nothing intervenes and the conductor works as it should, the device will transmit any sequence of n voltage pulses from its input point to its output point without loss of voltage pulses. Using unary notation and adding some counterfactual constraints grounded in physical laws and conditions of normal operation, Stabler proclaims that P applies (or computes) the identity rule.[9]

Stabler rightly notes that the simplicity of this thought experiment is a virtue, for it directs our attention to those aspects of a physical mechanism that are relevant for the application of an L–M rule, thereby offering a blueprint for how a physical process can objectively and mind-independently carry out L–M operations. As long as physical laws guarantee that, given certain input–output specifications, internal and transition states, and conditions of normal operation, P will output through its conductor precisely the number of voltage pulses received at its input, then P will (or *would*, barring interference) operate in a way that preserves the structure of the ideal L–M structure or rule in question. In this sense, P can be said to follow L–M rules objectively and mind-independently *in virtue of its physical properties alone*.[10] Unfortunately for Staler, however, his illuminating example points to the opposite conclusion: namely, that whatever L–M rule a physical mechanism follows or applies, it cannot do so solely in virtue of its physical properties.[11]

Suppose *P's* physical structure limits it to producing at most a sequence of 10 voltage pulses, such that no output occurs when a sequence of more than 10 voltage pulses are input.[12] One might describe P as a very restricted identity computing device (aren't all computing physical devices, relative to infinitary L–M functions, very restricted?), yet an identity-computing device nonetheless. But on what basis? What is it about P that makes it the case that it applies the identity rule? What, in other words, are the truth-makers of judgments of the sort "P is an identity-computing device", where P is a purely physical mechanism. Consider, for contrast, the following non-standard rule, call it *quidentity*:

Quidentity: for any $n \in \aleph$, if $n < 10$, $n = n$; 0 otherwise.[13]

This rule, defined over the natural numbers, coincides with the identity rule for all inputs lees than 10 but diverges from it for inputs greater than 10. Thus, we have two distinct (incompossible) L–M rules, yet one single physical device. What fact about P rules out judgments such as "P is a quidentity-computing device"? Why should we regard P as an identity-computing device rather than a quidentity-computing one?[14]

Let us assume, in addressing such questions, that only the physical facts of P are admissible. This is precisely the constraint imposed by the naturalistic view known as physicalism. As noted previously, physicalism is the thesis that everything is physical. This claim is articulated in different ways: either as an identity thesis (L–M properties just are physical properties), or as a realization/supervenience thesis (L–M properties are realized by, or supervene on, physical properties)—with the supervenience version being the most common.[15] According to supervenience, a set of properties or facts A is said to supervene on a set of properties or facts B (supervenience physicalism) if there can be no change in A without a change in B.[16] Applied to our case, the idea would be that the L–M properties of a PM supervene on the PM's physical properties just in case there can be no difference in L–M properties without a difference in PM's physical properties. So, if Stabler is correct in his judgment that P is an identity computing device, the physical facts of P should physically determine (i.e., guarantee) a specific set of L–M properties—namely, the identity rule.

Suppose that it is known that *P* does not transmit sequences of more than 10 voltage pulses. Stabler and others would classify *P* as an identity-computing device, claiming either that *P* malfunctions after a sequence of more than 10 voltage pulses or that *P* is a very limited identity computing device that fails to apply the rule of identity past a certain point.[17] Now imagine that *P* is transported to Twin Earth (TE), suffering no damage or internal physical change whatsoever, and is examined by the inhabitants of TE.[18] It so happens that people$_{\mathrm{TE}}$ judge *P* to be a *quidentity-computing device*—and an excellent one at that! (Perhaps quidentity is the standard L–M rule in TE, while identity is regarded as non-standard; perhaps humans$_{\mathrm{TE}}$ do *quathematics* rather than *mathematics*, and so forth.) Is their judgment *wrong*? If so (or if not so), on what grounds? The physical facts of *P* remain unchanged, yet the L–M profile ascribed to *P* is different.[19] Humans$_{\mathrm{TE}}$ and humans$_{\mathrm{E}}$ attribute to *P* different L–M rules on the basis of the very same physical facts, yet nothing in those physical facts seems to determine either judgment as objectively true (or false). The physical profile of P may provide grounds for reasonable rule-attributions, but it does not appear to constitute the truth-makers of those attributions. Why not?

Recall that, *per* Stabler, physical laws guarantee that (ceteris paribus) *P* conducts a sequence of *n* voltage pulses from its input point to its output point. Introducing, for example, a sequence of 3 voltage pulses at the input results in 3 voltage pulses being emitted at the output, and so on. *Ex hypothesi*, *P* cannot conduct sequences of more than 10 voltage pulses; inputting more than 10 therefore results in no output at all. Physical laws and facts *may* explain why *P* fails to conduct beyond this point (perhaps due to thermodynamic effects or the decay of copper, if its conductor is made of copper), but such explanations say nothing about whether *P* is a properly functioning quidentity-computing device or, instead, a malfunctioning (or merely very limited) identity-computing device. Similarly, the physical structure of a PM cannot tell us whether it is following or applying some L–M rule *R* and therefore computing correctly, or whether it is instead following or applying some other incompossible L–M rule *R** and therefore computing incorrectly. Physical laws and facts cannot by themselves supply the L–M profile of *P*, because that profile is not *instrinsically* and *objetivively* in the physical structure of *P*. *P* can be equally regarded as either an identity-computing device or a quidentity-computing device, precisely because, intrinsically, it implements neither.

Two principles play a central role in the L–M indeterminacy of physical structures:

1. **The finitude principle:** physical mechanisms possess only finite capacities and finite lifespans. As such, their physical profile can always conform to incompossible infinitary L–M rules, structures, or profiles.
2. **The normativity principle:** whether a given set of physical facts counts as a correct or incorrect application of an L–M rule depends on the particular

rule it is taken to be following—a rule that is not determinately or objectively present in the physical properties of the system. The notion of correct or incorrect application of an L–M rule cannot be derived from the notions of proper or improper *physical* functioning, nor from what physical (or empirical) laws guarantee.

Consider, for example, the following two interpretations:

- *P* under an identity interpretation (e.g., "*P* is a finite identity computing device that caps out at 10", "*P* is malfunctioning after 10", etc.);
- *P* under a quidentity interpretation (e.g., "*P* is a finite quidentity computing device that does not cap out at 10", "*P* is properly functioning after 10", etc.).

The physical explanation of *P's* internal states, processes and behavior tell, under certain physical conditions, what *P* will do, but it will not tell whether *P* is applying correctly or incorrectly an L–M rule (i.e., whether what *P* is doing is mathematically or logically *correct*). From what has been said so far, it follows that the physical facts of *P* cannot exclude incompossible L–M rules from being realized at the same time and in the same respect. At any given moment, the very same physical facts that are said to realize the identity function in *P* (and to ground its computational malfunction, deficiency, or limits) are also said to realize the quidentity function in *P* (and to ground its computational aptness, adequacy, or proper functioning). No physical fact, it seems, can decide which rule is being implemented. But this shows only that physical properties, taken by themselves, are insufficient to determine an L–M profile.

At this point, several objections may seem pressing. Before addressing them, however, let us briefly review the considerations developed so far. We began with a simple device, chosen to minimize conceptual noise and to highlight a more general point. The device was designed with the identity rule in mind. As physically constituted, it can indeed be regarded as a very limited identity-computing device. But it can just as well be regarded as a very apt quidentity-computing device. According to physicalism, physical facts should suffice to determine, metaphysically, which rule a physical mechanism (PM) realizes. But the distinction between identity and quidentity is not captured—or preserved—by the physical facts. Those facts accord equally well with the description "very limited identity-computing device" and "very apt quidentity-computing device."[20] If we say that *P* is both, then we are committed to holding that *P* is, at the same time and in the same respect, both an identity-computing device and not an identity-computing device. Interpreted as an objective and intrinsic feature of *P*, this leads to absurdity, since it entails contradictory counterfactuals. (We will return to this issue later.)

The point here is that if the physical properties of *P* are all there is, then *P* cannot exclude incompossible L–M rules from being realized at the same time and in the same respect. But this state of affairs, if taken as an objective property of P, leads to absurdity. Consequently, whatever fails to exclude L–M incompossibles from being equally realized cannot, in any objective sense, follow or apply L–M rules. Since physical facts fail in precisely this way, they cannot constitute objective L–M rule-following.[21] Here's a formulation of the general argument,

P1. PMs realize L–M rules. *(Assumption)*

P2. If a PM realizes an L–M rule, then it realizes that rule solely in virtue of physical properties. *(Physical sufficiency/supremacy. Physicalism/NE requires this)*[22]

P3. On pain of contradiction, nothing can realize incompossible L–M rules at the same time and in the same respect. *(Exclusion)*

P4. Physical properties, taken solely in virtue of themselves, cannot exclude incompossible L–M rules from being realized at the same time and in the same respect. *(Kripkean indeterminacy)*

C1. From P2 and P4, if PMs realize L–M rules, then they realize incompossible rules at the same time and in the same respect.

C2. From P3 and C1, contradiction.

C3. Therefore, PMs do not realize L–M rules solely in virtue of physical properties.[23]

It is important to note that the primary issue here is not epistemological. The problem is not that we cannot know which rule-attribution is correct. Rather, the problem is that we can know that *no rule-attribution is correct or incorrect if the only admissible facts are physical facts*. If we assume, *per* physicalism, that only physical facts are admissible, then incompossible L–M rules would be, in an objective sense, realized at the same time and in the same respect, which is absurd.[24] Physical facts, therefore, do not determine the L–M properties attributed to them, whether by supervenience or by any other physicalist relation.

Some further observations. L–M properties require that their realization, if it is to be *de re*, exclude incompossible L–M properties. Otherwise, the purported realizer would leave the L–M properties indeterminate, making it the case that it is, at the same time and in the same respect, both an R-realizer and a not-R-realizer. If we assume the truth of physicalism, we will arrive at the consequence that nothing has, objectively or *de re*, a definite L–M profile, since physical properties are inherently unable to rule out incompossible L–M profiles from being equally realized, or to determine which one, among the incompossibles, is true of it. Physical determination and L–M determination are not merely non-identical (contra physicalist identity theories); the relation of their realization or implementation (whether described in terms of supervenience or any other physicalist dependence criterion) cannot be an entirely mind-independent fact. A physically determinate thing is, *in se*, L–M indeterminate. If so, exemplified L–M properties cannot be grounded in, or accounted for by, physical properties alone. The argument is, first and foremost, a *categorial* one: there is an ideal specificity and determinacy proper to these types of rules that is categorically different, and therefore escapes, the type of specificity and determinacy proper to physical facts.

Kripke (cf. Stabler 1987; Buechner 2011; Burgess 2016) argued that for a PM to have a definite L–M profile (that is, a determine Lto the exclusion of any incompossible), the intentions of its human designers must play a constitutive role. Thus, in order to know which L–M profile P instantiates, we must appeal to the designer's intentions—explicitly, by consulting the design manual, or implicitly, by being part of a community in which those intentions form part of the shared background conditions of interpretation. But, as should now be clear, the L–M indeterminacy considerations generalize to all physical systems and processes *qua*physically or materially constituted—whether artificial or natural (e.g., organic). Whether something realizes or follows, objectively and intrinsically, some L–M operation or rule is not solely a matter of matter. And if we reduce it to nothing but matter, we risk eliminating the very possibility of objective L–M activity and practice, making it wholly relative to the intentions of some interpreter (in the case of artifacts, a designer). On this view, physicalistic naturalized epistemology (pNE) would entail that we never reason logically or mathematically in any *de re* sense, but only relative to external interpretations. Yet since we *do* reason in a genuinely logical and mathematical way—unlike physical computing mechanisms—the only reasonable conclusion is that human beings possess cognitive capacities (specifically, the capacity for logic and mathematics) that transcend the potentialities of matter or of the merely physical.[25]

The insight I take from Kripke's reflections is that pNE is not a correct framework for explaining the objective ability of human beings to follow L–M rules, nor for explaining the sense in which it can be intelligibly said that physical processes realize or follow such rules (a sense that differs from how it is said of *us*). The L–M profile of a physical mechanism requires, in addition to the appropriate material structure and causal relations, an extrinsic mental or anthropological determinant to resolve its L–M indeterminacy. In other words, which L–M rule a given PM implements is not an entirely mind-independent fact.

The problem for the NE advocate at this stage is that the physical constitution of any agent that purportedly applies an L–M rule is inherently insufficient for the application of such a rule. And if my Kripkean argument is sound, then no mind-independent, physically anchored fact can determine a unique or definite L–M rule among the many incompossible ones. Yet L–M knowledge and practice are partly constituted by the knowledge and application of determinate L–M rules. If this is so, then NE cannot account for L–M knowledge and practice. Indeed, if NE were true, L–M knowledge and practice would be impossible. But L–M knowledge and practice are not impossible. Therefore, NE is incorrect.

Malfunction, error, and correctness

But perhaps we can do better. Perhaps I've been selling the physicalist NE short. Maybe the perplexity can be resolved by focusing on the conditions under which *P* conducts voltage pulses for sequences of 10 or fewer, such that if these conditions were extended indefinitely (e.g., to sequences of more than 10 voltages pulses) *P* would compute the identity function. The physical makeup of *P*, one might argue, guarantees this. Perhaps something along these lines can be said (Stabler 1987, 11):

We say that a physical system computes a function f if, and only if,

(1) there is an "interpretation" or "realization" function *In* which maps a set of finite sequences of physical 'input' states of the system onto the domain of f, and an interpretation function *Out* which maps a set of finite sequences of physical 'output' states onto the range of f, such that

(2) physical laws guarantee that (in certain circumstances C, and *if the system satisfied conditions of normal operation N for long enough*), if the system went successively through the states of an input sequence i, it would go successively through the states of the corresponding output sequence f where $\mathrm{Out}(f) = f(\mathrm{In}(i))$.

Stabler here gives us a counterfactual constraint on the physical implementation of an L–M rule or function that, he thinks, preserves the objectivity of the L–M computational powers of certain physical mechanisms. If we examine *P*'s physical makeup, we can assess, for example, what the conductor will do on account of its physical properties (absent extraneous interference) and what the conductor would do if some physically relevant conditions were to obtain or continue to obtain. Suppose we physically enhance the conductor so that it can process sequences of more than 10 voltage pulses. Then physical laws guarantee that, if nothing extraneous intervenes, *P* would continue to do what it has been doing all along: transmit sequences of n voltage pulses from point I to point O. On this basis, Stabler concludes that if *P* were to continue operating normally or properly for long enough, it would compute the identity function.

But as Jeff Buechner rightly observed (drawing on Kripke's earlier insights), this response is a *petitio principii*. What counts as *P* operating "normally" or "properly," in the relevant sense, already depends on which rule and design *P* is supposed to be implementing—and this is precisely what is in question. If *P* were a quidentity-computing device, then its conditions of normal operation would include its failing to emit voltage pulses for sequences of more than 10; in that case, emitting such pulses would itself be a sign of inadequacy (error, malfunction, defective architecture, etc.). The antecedent of the counterfactual—"if *P* were to continue operating normally or properly for long enough, it would compute the identity function"—thus covertly presupposes the identity rule. In effect, it says: "if *P* were to continue operating in accord with the identity rule for long enough, it would compute the identity function." And this is true, but trivially so—for it is circular. This is one of the reasons why counterfactual responses to the indeterminacy considerations tend to fail: they beg the very question at issue.[26]

Let us now generalize. Both the input–output behavior of a physical mechanism and the processing steps that occur between input and output are equally compatible with a range of mutually incompossible (standard and non-standard) L–M rules. Any finite sequence of input states will be processed and transmitted to an output state, up to whatever threshold the mechanism's constitution allows. Beyond that threshold, the mechanism will fail to produce further outputs. These are the brute facts. There are several possible reasons for this,

(1) **Finite processing capacity.** A mechanism's processing capabilities may be unable to handle inputs beyond a certain threshold—however specified. Thus, although it can continue to process inputs below that threshold, it will simply not be hable to handle or excute all tha values of the infinite L–M function.[27]

(2) **Decay and breakdown.** The mechanism may physically decay, wear out, or break down after a finite number of operations, so that it can no longer perform the relevant transitions.

(3) **Finitude of lifespan.** Even if we idealize the mechanism as having "infinite" processing power, its lifespan is finite. This means that it can only ever carry out finitely many processes or operations, however large that number might be. Rules R and R^*, by contrast, are defined over an infinity of numbers (e.g., the natural numbers). Evidently, no physical mechanism can apply R (or R^*) to all of the cases for which the rule is mathematically or logically defined.[28]

(4) **Descriptive vs. prescriptive.** The (finite) dispositions of a physical mechanism cannot ground the normativity characteristic of L–M rules, nor can they ground the infinite counterfactuals that follow from the application of such a rule. It is one thing for a mechanism to be physically disposed to do x; it is another to ask whether that disposition to do x is logically or mathematically correct—that is, whether the disposition is a disposition to apply R or R^*.[29]

A finite physical mechanism that "does mathematics or logic" will have its causal–structural profile (input–output relations, state-transitions, vehicles, etc.) mapped onto the domain and range of the relevant L–M rule(s), and its state-transitions mapped onto an ideal structure (e.g., an algorithm or automaton) that encapsulates the operation. Yet our concern has been with the purported *objectivity* of the physical instantiation of a logical or mathematical rule. Physically, however, the objective difference between the identity rule and the quidentity rule is not preserved.[30]

Notice that incompossible L–M rules ground different counterfactuals *in* their actual applications. Suppose no mistakes are made and no intervening forces are present. Let φ and γ be two incompossible rules. Then:

- If one is φ-ing, and one were to continue φ-ing beyond the present cases, the *mathematically or logically correct* thing to do would be to generate the φ-value for any new input.
- If one is not φ-ing but rather γ-ing, and one were to continue γ-ing beyond the present cases, the *mathematically or logically correct* thing to do would be to generate the γ-value for any new input (that is, to not φ).

Thus, in the second scenario, it would be mathematically incorrect to generate the φ-value if one is in fact γ-ing. In each case, the mathematically correct output is determined by the particular rule being applied. And it is precisely this (that is, which L–M rule a physical process or mechanism is applying) that I argue is physically indeterminable. The rule or function, even in a single case, determines that what one is now doing is what *would* yield such-and-such if one were to compute a different value under that rule or function.[31] In this way, L–M rules ground counterfactuals. But such rules cannot be determinately instantiated in matter in any objective sense, and thus the physical cannot ground or cause such features.

To sum up. Does a physical threshold constitute a sign of deficiency or a sign of aptness in a physical object as a follower of L–M rules? The correct answer depends on which L–M rule the object is supposed to follow. Similarly, does decay, wear, or breakdown constitute a malfunction, or rather a sign of proper function, in such a computational physical object? Again, the answer depends on which L–M rule the object is supposed to realize, and in what manner. The *normative* or *evaluative* character of the physical conditions for implementing an L–M rule depends, at a fundamental level, on the very L–M rule those conditions are taken to implement. Categories such as "conditions of normal operation," "proper functioning," "deficiency," and "malfunctioning," in relation to the following of L–M rules, are therefore *consequent categories*: they pick out features of the physical mechanism that presuppose, and derive from, the rule it is (supposed to be) implementing.[32] For instance, whether *P*'s failure to conduct 11 voltage pulses is a vice or a virtue depends entirely on what rule *P* is supposed to be following. Hence, to respond by saying, "If we were to fix the conductor, or if the conductor were to continue to operate normally for long enough, *P* would compute *f*" presupposes the very point at issue.

Some further responses

Lastly, let us consider some other common responses to the argument developed thus far.

One response to the indeterminacy considerations is conjunctivism: at any given time t, P is (in the same respect) both an φ-device and a γ-device. But this would require P to be both φ and not-φ at the same time and in the same respect. As we have seen, this cannot be right. P would make contradictory counterfactuals true, which is absurd. Moreover, this response does not explain which physical facts could make the conjunction true. If our argument is sound, the conjunction should have no physical truthmakers. Hence, conjunctivism does not save physicalism and NE.

Another response is disjunctivism: P follows either φ or γ at any given moment, but never both. This, however, amounts to a false dilemma. The thought experiment is meant to show that there is an indefinite number of non-standard L–M rules (incompossible with one another) that any given physical mechanism will, on account of its physical properties alone, satisfy. The disjunction is therefore potentially infinite. And once again, the response does not specify what kind of physical fact could make any of the disjuncts true. According to our argument, the physical facts of P can neither confirm nor disconfirm any of the possible rule-attributions compatible with its profile. Thus, as with conjunctivism, the disjunctive strategy leaves us with no truthmakers and hence no truth-value.[33]

Now, a physicalist can respond instead by denying that there is any such thing as an L–M rule. This amounts to an eliminativist response: L–M rules, functions, and operations do not exist. Something real cannot implement what does not exist. Hence, the indeterminacy considerations do not prove that physicalism is false. At most, they show that if there were L–M rules, then physicalism would be false—but, since there are no such rules, the objection does not apply.[34] This is a radical move, but still, one available to the physicalist. She can agree with the indeterminacy considerations while resisting the conclusion that they refute physicalism. The cost, however, is exorbitant. For if physicalism is to be maintained in this way, it must be purchased at the price of denying the existence of L–M rules altogether. Yet if this is so, then one cannot explain how logic, mathematics, and their many applications (in fields such as engineering, architecture, and economics) are even possible. Worse still, one cannot explain how science itself is possible, for science is rife with the application of L–M rules. To insist that there are no such things as L–M rules is to suggest that science, mathematics, and logic are mere illusions. Scientific theorizing is saturated with L–M structure (proof, derivation, equivalence, inference, idealization). Treating all of this as *non-existent* undercuts the very methods physicalism relies on to defend itself and leaves no account of how these practices are even *possible*.

What is worse, this response appears self-undermining. It is not clear how one could, on the basis of indeterminacy considerations, conclude that L–M rules do not exist. For to reach that conclusion, one must do precisely what her denial forbids: think in logically and mathematically determinate ways. Thus, the eliminativist response to L–M indeterminacy not only empties NE of explanatory power, but also undermines the very reasoning needed to defend it. The inference, "Therefore, L–M rules do not exist," aims to be a *logically warranted* conclusion from premises (e.g., from indeterminacy). But if there are no L–M rules, there is no norm of validity to underwrite that inference. The denial presupposes what it denies.

The physicalist can, of course, reject the eliminativist stance and instead claim that L–M rules exist, but not abstractly. Perhaps, she will say, L–M rules are simply rules for concrete behavior. But this seems a red herring. If our indeterminacy argument is sound, then even if L–M rules are not abstract (whatever that means), they are still not physical. Purely physical behavior would remain just as L–M indeterminate as any other purely physical process. If anything, this response would leave the physicalist with non-physical *concrete* rules for behavior—a position that hardly rescues physicalism. Now, if the physicalist presupposes that if L–M rules exist they must exist concretely—because everything that exists is physical and all physical things are concrete—then this amounts to the claim that L–M rules are just physical rules.[35] But it is difficult to see how this sidesteps the indeterminacy considerations, for part of what those considerations show is that the identity conditions of L–M rules and the identity conditions of physical properties are categorically different.

Conclusion

Kripke's great insight was to notice that the rule or norm a PM follows cannot be read off from the physical properties of the mechanism alone, because the rule simply is not there. I have expanded on this insight and developed a general argument against physicalist NE. In order for the L–M profile of a physical computing mechanism to be complete, an external determinant source is necessary: in some cases, this determinant will be the intentions or designs of its human creators; in others, the logico–mathematical practices of scientists, mathematicians, or users. It is not that the physical is irrelevant or unnecessary, but that it is insufficient. In any attribution of L–M properties to the physical, there will be present—whether explicitly or tacitly—the mark of the mental.[36] This need not entail scientific irrealism, if by that is meant the claim that science does not provide knowledge of the world but merely manifests the intellectual habits of its practitioners. This is not the place to defend against that confusion, but it is important to emphasize that the absence of logical or mathematical properties in nature does not preclude our acquiring objective knowledge of the natural world through logical reasoning and through the mathematization of physical reality. What our argument should be understood to imply, rather, is that the material world approximates the mathematizations we impose upon it, but is neither identical with them nor an exact realization of them.[37]

Our main idea has been this: for any logical or mathematical rule *R* that a PM purportedly follows, there will always be some other rule *R**, incompossible with *R*, that the PM is nonetheless physically compossible with in all relevant respects. But then mathematical compossibility and physical compossibility are not equivalent, and the former does not supervene on the latter. Mutually exclusive mathematical facts turn out to be inclusively satisfied by the very same set of physical facts. "If it realizes all, it realizes none" is the slogan. L–M exclusion is not physical exclusion, and physical properties cannot capture, preserve, or respect L–M exclusion. The space of logico–mathematical rules and the space of physical facts are categorically different. Thus we come to what we originally set out to prove: physical facts alone cannot suffice to determine which logico–mathematical rule an agent follows or applies.

Physicalist NE cannot coherently maintain that L–M rules do not exist, for in doing so it would have to deny that we can understand and apply such rules—something we evidently do—and it would have to deny the existence of rational and intellectual practices whose very nature consists in their application and development. The cost of such a denial is too high: it would deprive us of coherence, intelligibility, and the very intellectual practices that physicalist NE itself relies upon.[38]

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Notes

[1]Originally, it seems naturalism was defined in contraposition to supernaturalism. It also came to denominate a position that was anti-dualist and against the existence of non-physical or non-material substances or properties. At any rate, there is no one single definition of 'naturalism'. For a useful overview, see (Papineau 2023).

[2]I will talk of physical 'entities', 'objects', 'events', 'properties', 'things', 'substances', 'processes', etc., interchangeably. My argument does not depend on any particular ontological category other than the category of *physical* or *material*.

[3]See Chalmers (Chalmers 1997; 2003) for the distinction between naturalism and physicalism and the view that naturalism is not equivalent to physicalism. Chalmers espouses instead a position he dubs "naturalistic dualism".

[4]The argument developed here, though situated within contemporary analytic philosophy and cognitive science, has striking affinities with Edmund Husserl's critiques of psychologism and empiricism in logic and mathematics (see, for example, Husserl 2001). In his arguments against functionalism as a physicalistic theory of the mind, Kripke observed that such a view could not account for the normativity of logical and mathematical functions (a concern also central to Husserl's critique of psychologism), and that, when pursued to its full logical consequence, leads to absurdity (a result parallel to Husserl's charge that psychologism and empiricism about logic are ultimately self-refuting). There are, of course, important differences —just as there are between my own development of Kripke's argument and his own original formulation. But these affinities suggest possible fruitful avenue for future research: namely, exploring how both Husserl's and Kripke's views might be brought together in a critical engagement with contemporary forms of naturalism.

[5]Two points. First, a parallel version of this dilemma can be formulated in terms of the *existence* of L–M rules: either L–M rules exist and naturalism is false, or naturalism is true and L–M rules do not exist. If L–M rules do not exist, then there is no such thing as logico–mathematical knowledge or practice. But there is such knowledge and practice, hence L–M rules exist, and NE is false. In fact, the very possibility of the eliminativist response on the part of the physicalist (see penultimate section) helps us see how the argument developed here grounds that existence–based dilemma. Second, a variant of the dilemma can be developed with respect to the "*abstract*" or *ideal* character of L–M rules and functions (not be interpreted, necessarily, in a Platonist way):

> Either logico-mathematical (L–M) rules exist ideally and physicalist naturalism (pNE) is false, or physicalist naturalism is true and L–M rules do not exist ideally. But if L–M rules do not exist ideally, they must either exist concretely in some material way—for example, a mathematical function is a physical function, or a mathematical function is caused to exist by some physical function—or not exist at all, as eliminativism maintains. If they exist concretely in some material way, it turns out that they cannot exist at all [*per* the indeterminacy arguments]; but the claim that they do not exist at all leads to absurdity. Hence, L–M rules exist, and they must exist ideally rather than materially. Hence, pNE is false.

This line of reasoning, though in my opinion defensible, is markedly different from the one I advance in this paper. Each version stands or falls on its ow merits, and a full treatment of the variant dilemma would require a separate paper.

[6]The argument was first presented in a series of talks that remain unpublished. For articles that discuss this unpublished material, see (Buechner 2011; Burgess 2016; for an early critical reaction to one of these talks, see Stabler 1987).

[7]I use the word *ideal* rather than *abstract* (Kripke's term) in order to avoid a philosophically loaded notion—not that *ideal* is unproblematic, but it carries less philosophical baggage than *abstract* in contemporary analytic philosophy. Much could be said about this Kripkean argument and its implications for the philosophy of computation, particularly regarding the physical application and computation of L–M rules and functions. However, I cannot enter into those issues here. What matters for present purposes is that the following argument is not a triviality argument and therefore does not entail either limited or unlimited pancomputationalism. Nor does it undermine the possibility of physical computation—though it does challenge the view that the physical computation of L–M rules and functions is intrinsic and entirely mind-independent. For two excellent (though by no means uncontroversial) contemporary expositions of the debates surrounding the notion of physical computation, see (Piccinini 2015; Shagrir 2022).

[8]See (Stabler 1987). Stabler was present in one of the talks Kripke given against the *objectivity* of the notion of computation.

[9]For Stabler's precise formal characterization of this mapping relation between the physical mechanism and the identity rule, see (Stabler 1987, 11).

[10]A radical naturalistic project might, for example, employ the notion of physical computation in cognitive science to explain the cognitive functions associated with doing mathematics (like, for example, our ability to carry out arithmetical operations) as, in some sense, explicable in terms of the physical implementation of mathematical rules. But as we hope to show, albeit intuitively here, there is no sense in which the physical implements an L–M rule in an intrinsic, objective and mind-independent way. So physical explanations of such cognitive functions are ontologically misplaced.

[11]Though I will not address the matter here, it should be noted that the formal indeterminacy at issue has little to do with complexity considerations on the part of the physical system. Moving from FSAs to CSAs in the manner proposed by Chalmers (Chalmers 1994; 1996; 2012) will not resolve the problem. I will develop this point more fully in a separate paper on the nature of logic, mathematics, and physical computation.

[12]The point here has nothing to do with the number "10" as such, but with the finitude that characterizes all physical mechanisms. "10" serves only as a case in point—a placeholder for the limiting threshold beyond which any finite mechanism ceases to operate, whether because it exhausts its processing capacities, wears out, breaks down, or otherwise malfunctions. In this sense, the number "10" stands simply for the finitary material limit inherent in any physical computational mechanism.

[13]This is a variation of Kripke's well-known *quaddition* function. See (Kripke 1982).

[14]I am assuming, for the sake of argument, that the judgments in question have metaphysical import. That is, I am assuming that whether a physical mechanism computes a logico-mathematical function is a matter of *de re* determination. If physicalism is true, then the truth-makers of such judgments have to be physical.

[15]I set aside "extended" versions of physicalism, according to which *P* together with *P*'s physical environment should factor into the determinants or truth-makers of judgments about

P. The argument can be developed equally well against these extended versions, provided that what is "extended" is ultimately cashed out in physicalistic terms. For an overview of physicalism, see (Stoljar 2024). For a different but interesting critique of physicalism on the basis of the mathematical nature of physics, see (Schneider 2017a; 2017b).

[16]The literature on supervenience is both extensive and complex. But the core idea is straightforward: there can be no change in the supervening properties without a corresponding change in the supervenient base. So, if A supervenes on B, A cannot change without there being a change in B. "A set of properties A supervenes upon another set B just in case no two things can differ with respect to A-properties without also differing with respect to their B-properties. In slogan form, 'there cannot be an A-difference without a B-difference'." (McLaughlin and Bennett 2023) Our claim is that L–M rules and operations violate supervenience. See n. 18.

[17]In one sense, this is trivial: any physical mechanism is "very limited" when measured against the domain and range of infinitary functions, since no such mechanism possesses infinite processing capacity or an infinite lifespan.

[18]Alternatively, suppose P^* is a physical duplicate (Döppelgänger) of P that exists on TE. On TE, P^* is regarded as a quidentity-computing device, whereas its physical counterpart on Earth (namely P) is taken by earthians to be an identity-computing device. Kripke offers a similar example involving a device that suddenly appears before two designers. Suppose that device is P. Now imagine an earthian and a twin-earthian encountering P in a shared "middle world" (a middle world where earthians and twin-earthians can meet). Both examine the device and reach different conclusions: the earthian judges it to be an identity-computing device, while the twin-earthian judges it to be a quidentity-computing device. As Kripke observes, there is nothing in the physical facts of P that makes either judgment correct (or incorrect). See the discussion in (Buechner 2011) for a similar claim. For the original formulation of the Twin Earth thought experiment, though used for a different purpose and in a different context, see (Putnam 1973; 1975b).

[19]Of course, a defender of supervenience might object that it is impossible for the logico–mathematical properties of P to change without a corresponding change in its physical properties. But this simply begs the question. Naturally, such a change would be impossible—*if supervenience physicalism were true*. Yet it is precisely this thesis that is at issue.

[20]To clarify: we are not concerned with *parallel* processing/computation. This is not the issue here. We are after the determinate application of a unique rule, objectively exemplified. That is, the physical properties and conditions that enable a physical mechanism to realize or implement a definite L–M rule such that it is an objective and mind-independent fact that a physical mechanism implement *this* L–M rule as opposed to some other incompossible one. Parallel computation presupposes this, for it takes it for granted that the physical mechanism can compute definite L–M operations.

[21]At this point, it may be objected that it is premature to conclude that, because physical facts fail to instantiate definite L–M rules, physicalism is therefore false. The physicalist may instead claim that L–M rules simply do not exist and that it is thus no surprise the physical cannot implement them. I return to this eliminativist response in the penultimate section.

[22]Notice that the sufficiency principle is formulated so as to cover all physical properties whatsoever, whether internal or external, brain-bound or environmentally extended. If extended cognition is ultimately cashed out in physicalistic terms, then the argument applies

to it just as much as to standard internalist models, as already noted.

[23]This conclusion leaves open whether something physical might realize L–M rules in virtue of properties or capacities *other than purely physical ones*. My point here is only that physical properties *as such* cannot secure determinate L–M realization/instantiation; I do not rule out the possibility that additional, non-physical capacities (e.g., rational capacities) could be necessary for such realization.

[24]There is an epistemological version of this argument, framed in terms of underdetermination rather than indetermination, that is still devastating for physicalism. I will not explore that version here. For further discussion of the epistemological variant, see (Ramos Díaz 2021, 227–29).

[25]I cannot here address the counter-response that the intentions of a designer (or the human mind in general) are themselves just as physical as any other physical thing. If so, then the mind would be just as indeterminate with respect to L–M rules as any other physical system, and in need of further extrinsic determination (through interpretation or design) in order to achieve L–M determinacy. But this move risks an infinite regress: if every extrinsic determinant is itself purely physical, then either we end up with no determination at all, or we must posit a determiner that possesses intrinsic, non-derived "formality" (L–M knowledge). I argue that the buck must stop with the human mind, and this requires rejecting the thesis that everything is physical. The counter-response is, in essence, Daniel Dennett's well-known retort to the problem of intentionality (see Dennett 1987), namely, that nothing possesses intrinsic or non-derived intentionality. I take this view to be outright incoherent, since, among other things, it implies that one could not formulate or grasp the very argument required to deny that human beings can grasp and follow definite L–M rules. If, however, the human mind is where the regress of L–M determination must end, then it follows that the mind cannot be a purely physical thing. For reasons of space, I cannot develop this claim in detail here. For a fuller exposition, see (Ramos Díaz 2019; 2021).

[26]The problem is that "normal" is ambiguous. On the one hand, it can mean "behavior and states in accord with physical propensities, as specified by physical laws." On the other hand, it can mean "behavior in accord with an L–M rule, such that the application is correct." But these two senses do not conceptually coincide: physical conditions of normal operation are not the same as correct applications of L–M rules. Moreover, "normal" can be said in relation to the mathematical or logical behavior of the system (the L–M rules and laws it is supposed to follow or implement and whether it is applying them correctly), or it can be said in relation to the physical behavior one would expect given certain physical laws, structures, and conditions (the physico-chemical or neurobiological structure, states, changes, etc. and whether these changes are happening as they should). These two senses are not coextensive—or at least that is what we have attempted to show. At any rate, the real problem is whether counterfactual constraints can be specified non-circularly, without already presupposing which rule the physical mechanism is supposed to be following (e.g., identity vs. quidentity).

[27]Strictly speaking, the argument does not require that the L–M functions or rules have infinite domains and ranges. It is enough that they involve an upper or lower bound whose attainment would exceed the lifespan of the universe for any physical system. That suffices to show the gap. For simplicity's sake, however, we frame the argument in terms of infinite functions.

[28]It is not necessary that one apply a rule to all of its infinitely many possible cases in order

to follow or apply an infinite L–M rule. Humans do this all the time without exhaustively computing every case. What is required is that the capacities of the realizer of L–M rules or functions be able to discriminate intrinsically between incompossible L–M rules. This is precisely what humans can do but no purely physical system can: humans grasp and follow L–M rules objectively and determinately, such that their thinking and reasoning can become *informed* by those rules. In their case, there is no systemic indeterminacy. Hence, their cognitive powers cannot be wholly physical. For further discussion, see (Ramos Díaz 2021).

[29]For Kripke's critique of dispositionalism, see (Kripke 1982; cf. Buechner 2011; 2018). For a parallel critique in Husserl, see (Husserl 2001).

[30]This is, in part, why there can be no semantic causal theory of meaning for L–M knowledge if the facts that are supposed to cause that knowledge are wholly physical. I hope to develop this argument in a future paper.

[31]Thus, if what I am now doing is identity, then what I am now doing is precisely *the same operation* that would have given me 185 had I applied the rule to "185" (barring error or interference), and so on for all possible counterfactual cases of the rule whatsoever. The single case virtually, or logically, contains them all. This is the kind of determinacy that characterizes the real and objective application of L–M rules or operations.

[32]I am simplifying. Let f and g be two incompossible L–M rules. A designer might say that P is perfectly adequate for f-ing, since all he wants is a device that can carry out f for any $n > 0$ but $n < 10$. The aptness of P for this design is partly constituted by its physical makeup and conditions, but not entirely, for—as we have argued—the physical properties of P leave it indeterminable. Another designer, by contrast, might regard the very same P as a deficient f-ing device, or alternatively as a highly adequate and apt g-ing device.

[33]Note also that both responses (conjunctivism and disjunctivism) require and presuppose that the physical system realizes or exemplifies a determinate logical *form* (whether a conjunction or a disjunction) objectively and intrinsically, as opposed to some other incompossible logical form (e.g., qunjunction, quisyunction, etc.). Yet this is precisely what we have argued cannot happen. Thus, both responses commit a *petitio*.

[34]One might argue that this response could also be described as *fictionalist*, in the sense that it allows us to continue to speak *as if* there are L–M rules while denying their reality. It eliminates the existence of L–M rules altogether and, therefore, any real sense in which logic and mathematical reasoning can be understood as rule-governed activities. For clarity, I will refer to this response as eliminativist, since this is the effect it has in relation to the philosophy of mind.

[35]There are, to be sure, attempts to reduce L–M structures to physical structures in ways that are, ultimately, eliminativist. I cannot go into detail here, but I would argue that these attempts fail, for they too fall prey to the formal indeterminacy considerations, among other difficulties. For a recent physicalist view about logic and mathematics, see (Szabó 2003; 2025).

[36]To demonstrate the strength of the argument, I have assumed—and nothing is lost by doing so—that the problems of pancomputationalism (see Putnam 1975a; cf. 1988) and triviality (cf. Godfrey-Smith 2009) have been successfully resolved. Suppose, for example, that we can establish that a physical mechanism does not compute just any and every L–M function, and that it cannot be said that everything (or anything) computes some L–M function. Even if we grant this, it still follows that any physical mechanism will satisfy incompossible

L–M rules. Not *every* L–M rule, but at least incompossible ones. And since every L–M rule has non-standard analogues with which it is incompossible, PMs will always remain L–M indeterminate—even at the finest granular level of physical analysis.

[37]For natural physical mechanisms, processes, and structures, we still need an extrinsic determinant source of their mathematizations, since no such mathematical properties are found intrinsically in physical reality qua physical. In such cases, the extrinsic determinant source is the quantitative and formal idealization or abstraction carried out by human theoreticians of the physical phenomena (e.g. theoretical physicists), while the immanent determinant source of the mathematization lies in the physical, natural structures, powers, and properties inherent in the phenomena themselves—properties which either approximate the mathematization, or are approximated by it.

[38]Except in rare cases such as Wittgenstein, the philosophy of mathematics has been preoccupied almost obsessively with mathematical *objects* (propositions, numbers, sets, etc.), and far less with mathematical *rules* and the conditions necessary for understanding and applying them. Shifting the emphasis to rules allows us to inquire into the nature of mathematics as a rational activity carried out by rational agents. There is such a thing as *doing* mathematics (mathematicians do it all the time) and L–M rules are constitutive of this multifaceted activity we call mathematics. My contention is that this rational activity or practice cannot be naturalized. It requires that agents have the capacity for logico–mathematical knowledge, which is partly the capacity to understand and apply determinately a host of logico–mathematical rules—something the merely physical cannot do. If such knowledge cannot be naturalized, then neither can the practice that is constituted by it. I hope to develop this further in a forthcoming paper.