

# COMPUTATION OF PROBABILITIES IN CAUSAL MODELS OF HISTORY OF SCIENCE

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## Abstract

*The aim of this paper is to investigate the ascription of probabilities in a causal model of an episode in the history of science. The aim of such a quantitative approach is to allow the implementation of the causal model in a computer, to run simulations. As an example, we look at the beginning of the science of magnetism, “explaining” — in a probabilistic way, in terms of a single causal model — why the field advanced in China but not in Europe (the difference is due to different prior probabilities of certain cultural manifestations). Given the number of years between the occurrences of two causally connected advances  $X$  and  $Y$ , one proposes a criterion for stipulating the value  $p_{Y|X}$  of the conditional probability of an advance  $Y$  occurring, given  $X$ . Next, one must assume a specific form for the cumulative probability function  $p_{Y|X}(t)$ , which we take to be the time integral of an exponential distribution function, as is done in physics of radioactive decay. Rules for calculating the cumulative functions for more than two events are mentioned, involving composition, disjunction and conjunction of causes. We also consider the problems involved in supposing that the appearance of events in time follows an exponential distribution, which are a consequence of the fact that a composition of causes does not follow an exponential distribution, but a “hypoexponential” one. We suggest that a gamma distribution function might more adequately represent the appearance of advances.*

Why did the history of a scientific field follow a certain path and not another? The answer for this kind of question usually involves the identification of a set of historical *causes*. Causal relations, however, may be quite complex, and one way of expressing this complexity is by means of probabilistic causal models. The present paper is part of a project for investigating how the history of science may be expressed in terms of such models. The problem to be addressed is how to compute probabilities for an overall process, given the probabilities for intermediate steps, and how to estimate the probabilities of such intermediate steps.

## 1. Causal Model for an Episode in the History of Science

In a previous work (Pessoa 2005), an overview was presented of the approach to the history of science based on causal models.<sup>1</sup> This project started out as an exploration of a method for postulating counterfactual histories of science (Pessoa 2001), and led to the development of a theory of science based on general units of knowledge, which may be called “advances” (or “achievements”, or “contributions”). Advances are passed on from scientist to scientist, and may be seen as “causing” the appearance of other advances. This results in networks which may be analyzed in terms of probabilistic causal models, which are readily encodable in computer language.<sup>2</sup>

Consider the following representation for the steps leading to the development of a rudimentary form of the magnetic compass (Fig. 1). This is a modification of the model presented in Pessoa (2005), but in which no calculations of overall probabilities were given. In a single diagram, consisting of advances connected by causal relations, one attempts to account for two independent factual histories of the early science of magnetism, those occurring in China and Europe. According to this reconstruction, the difference between the two histories was due mainly to the strong presence of divination techniques in China (Needham 1962). Although such cultural manifestations associated with the lodestone (magnetic ore) were also present (to a lesser extent) in Europe, for instance in Samotracia,<sup>3</sup> we have simplified the situation by considering that the prior probabilities for the divination techniques *B* and *E* in Europe were zero, while in China they were 1.

The path leading to the first magnetic compass, the lodestone spoon compass (*F*), started from the discovery and exploration of the “lodestone effect” (*A*) (the mutual attraction of magnetic ore and the attraction of iron to magnetic ore) which occurred both in China and in Europe. However, in China there was a divination technique done with a greased iron needle floating in water (*B*), which led to a variation involving a floating lodestone needle (*C*). With this practical arrangement, the discovery that the lodestone needle aligns along the North-South direction (*D*) was highly probable, and in fact occurred in China around the beginning of the Christian Era, but not in the West. After this discovery, the development of a rudimentary compass (*F*) was a small step.

Causal connections are represented as probabilistic relations, the values of which are a rough estimate of the probability for the occurrence of an effect in a typical reference time interval, in this case taken to be  $T_{ref} = 400$  years. A

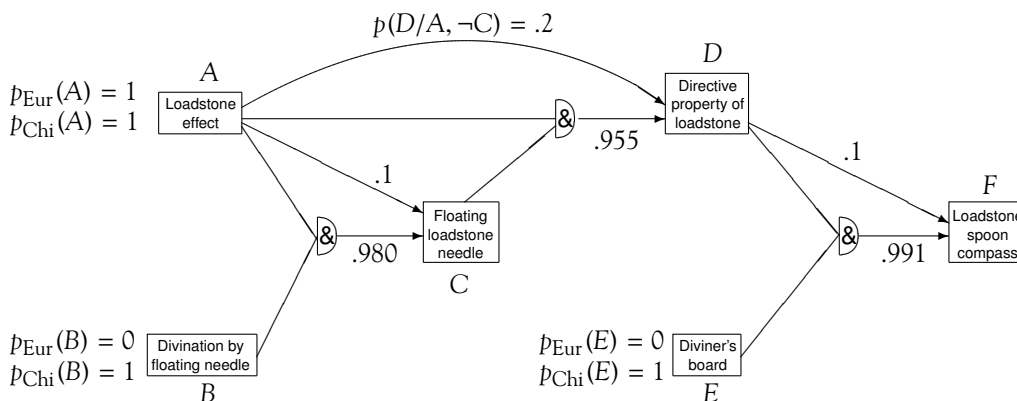


Figure 1: Simplified causal model for the beginning of the science of magnetism. The difference between the developments in China and in Europe, up to around the 5th century, are accounted for by different prior probabilities for divination techniques. Probabilities with three significant digits were calculated from the empirical time span between the advances in China, according to eq. (4) and also eq. (6), as explained in section 8. The probabilities with only one significant digit were guesses.

certain advance such as  $D$  may be represented with two arrows leading to it: this expresses a *disjunction* of causal paths, i.e., the effect may arise from either one of two different paths. A *conjunction* of causes is represented by lines flowing to the symbol “&”. The succession of two causal relations, for example  $A \rightarrow D$  and  $D \rightarrow F$ , will be called *composition* of causes, resulting in the overall relation  $A \rightarrow F$ .

The probability for an event  $F$ , given a set of causes such as  $A$ ,  $B$ , and  $E$ , has been expressed for the reference time interval  $T_{ref}$ . What would the expression for  $p_T(F/A, B, E)$  be if the time interval  $T$  chosen were different from the reference time?

## 2. Probability as a Function of Time

To treat this problem, we will change the notation, and write  $p_{Y/X}(t)$  for the conditional probability of occurrence of a generic advance  $Y$  up to the time  $t$  after the occurrence of an advance  $X$ . How should  $p_{Y/X}(t)$  be expressed as a function of time  $t$ ?

First of all, this probability  $p_{Y/X}(t)$  should increase monotonically with  $t$  (i.e.,

it should never decrease with  $t$ ). Drawing an analogy with a fisherman in a pond, after he throws his bait, the probability of catching a fish in the next hour cannot be smaller than the probability of catching a fish after only five minutes. Another obvious restriction on  $p_{Y/X}(t)$  is that it can never be greater than 1 (Fig. 2b).

One simple way of expressing the amount of time until some specific event occurs is by means of an *exponential* distribution:  $f(t) = x \cdot e^{-xt}$ . This is used in physics, for example, for the law for radioactive decay. If there are initially  $N_0$  “parent” nuclei which decay independently, with the same probability, then after a time  $t$  there will be  $N_0 \cdot e^{-xt}$  nuclei, where  $x$  is the decay constant or *rate* of the distribution. On the other hand, assuming that the end product of the decay is a stable nucleus, the number of such “daughter” nuclei will increase as  $N_0 \cdot (1 - e^{-xt})$  (see Evans 1955, ch. 15).

The probability for the appearance of a single daughter nucleus is obtained by taking this curve of growth with  $N_0 = 1$ :

$$p_{Y/X}(t) = 1 - e^{-xt}. \quad (1)$$

Figure 2 shows a graph of this *cumulative probability function*, which is the time integral of the *distribution function*  $f(t) = x \cdot e^{-xt}$ , shown in Figure 2b.

In most of this paper, we will assume that, in the history of science, the probability of an effect following a cause will obey such a law. With this assumption, we can calculate what the probability of an effect will be for any interval of time  $T$ , by simply computing  $p_{Y/X}(T)$ . Such a supposition of exponential decay may be, of course, criticized, and we will consider some of its drawbacks in section 8.

Conversely, if one stipulates a certain  $p_{Y/X}$  for a reference time interval  $T$ , then the decay constant is given by:

$$x = \frac{-1}{T} \ln(1 - p_{Y/X}). \quad (2)$$

### 3. Features of the Exponential Distribution

In the case of radioactive decay, there is a large number of nuclei undergoing the same process at the same time. How can this be applied to the case of histories of science, which usually occur only once (although independent discoveries happen quite frequently)? One strategy is to consider a set or “ensemble” of *possible* histories of science, *all starting at a specific date* (usually involving the real situation of science at a certain date in the past). To simplify matters, we might

consider 100 such possible histories, one of them our actual history, the other 99 counterfactual. We will not go into the details here of how to conceive such an ensemble (see Pessoa 2006), but the idea is that the counterfactual worlds *could* have arisen out of fortuitous events, if a certain scientist (such as Carnot) had not died at a young age, or if a seminal paper had not been ignored (such as Waterston's), or if a bright individual had become a musician (as could have happened with Einstein).

As an illustration, consider two advances  $X$  and  $Y$ , such that the appearance of the first brought sufficient conditions for the discovery of  $Y$  (this is of course a simplification, since there are always many other relevant causes and conditions). This could be the case of the development of a scientific instrument, such as the compound microscope ( $X$ ), developed by J. J. Lister in 1826, which led to the discovery of cellular structures in all plant tissues ( $Y$ ), by M. Schleiden, in 1838. The "empirical" time span elapsed between  $X$  and  $Y$  is 12 years.

Our ensemble of possible histories of science is constructed from the actual situation, right after  $X$  was obtained, in 1826. In the 100 possible histories, would  $Y$  appear after 12 years in exactly every one of them? Probably not: there would be a distribution of possible histories, according to the time elapsed between  $X$  and  $Y$ , and we might take 12 years as a mean value  $\bar{T}$  (in the lack of further information).

Since we are assuming that  $X$  is sufficient for the production of  $Y$ , the latter could have appeared immediately after  $X$ . In fact, the assumption of *exponential distribution*, i.e., of eq. (1) as our cumulative probability function, implies that the year in which  $Y$  would appear the most, in the ensemble of possible worlds, would be the first year, then a few less in the second year, and so forth. This is represented in the exponential decay curve of Fig. 2b, where a histogram represents the distribution of the appearance of  $Y$  in the hundred worlds.

There is something counterintuitive in the *exponential distribution*, since, in our world,  $Y$  appeared after *twelve* years, and one would guess that the year in which most "decays" happen would be around twelve and not one year. A better distribution function might be one with a hump around its mean value. An example of this is the so-called *gamma function*, defined by two parameters  $x$  and  $n$ , according to:

$$f_n(t) = \frac{x^n t^{n-1} e^{-xt}}{(n-1)!}. \quad (3)$$

For  $n = 1$ , the distribution reduces to the exponential case  $f_1(t) = x \cdot e^{-xt}$  seen

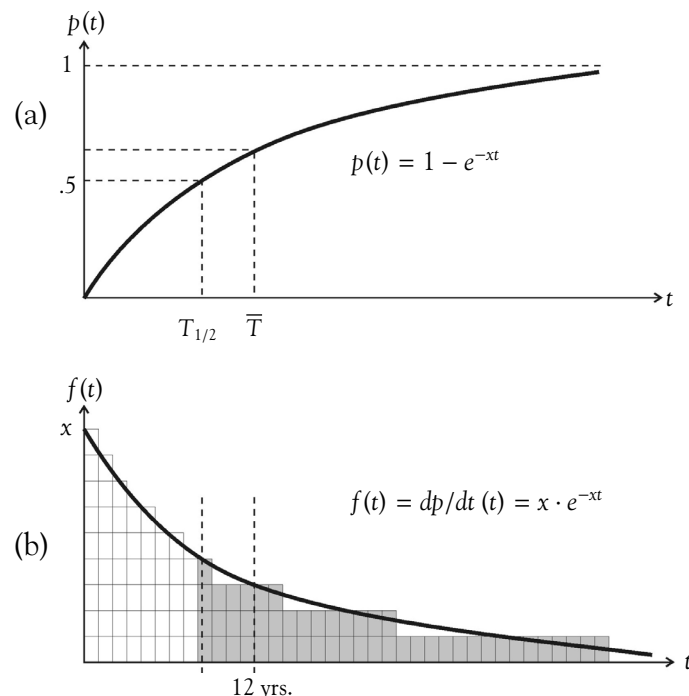


Figure 2: (a) Cumulative probability function  $p_{X/Y}(t)$ , for the exponential case. (b) Exponential distribution function  $f(t)$  of 100 worlds (represented by rectangles) according to the year of occurrence of the relevant event in each one. The mean  $\bar{T}$  is taken to be 12 yrs., the empirical time span of the example mentioned in the text. In half of the worlds the event occurs before the half-life  $T_{1/2}$ .

above. The cases in which  $n = 1, 2, 3, 4$  are presented in Fig. 3a, and a sketch of a gamma-like distribution is shown in Fig. 3b (for other details, see Pessoa 2007). The disadvantage of such a distribution is that two parameters are needed to define it. Only in situations involving two or more independent discoveries would one have enough data for roughly estimating the standard deviation of the underlying distribution function.

In this paper, for simplicity, we will consider calculations with the exponential decay curve of Fig.2b. It represents a situation in which the probability of an event occurring in a certain interval of time  $T$  (assuming it has not yet occurred) is constant, irrespective of the instant  $t$  chosen. This is related to the curious feature of the system being “memoryless”: if after, say, five years we verify that  $Y$  has not appeared, the probability function for its appearance will be the *same*

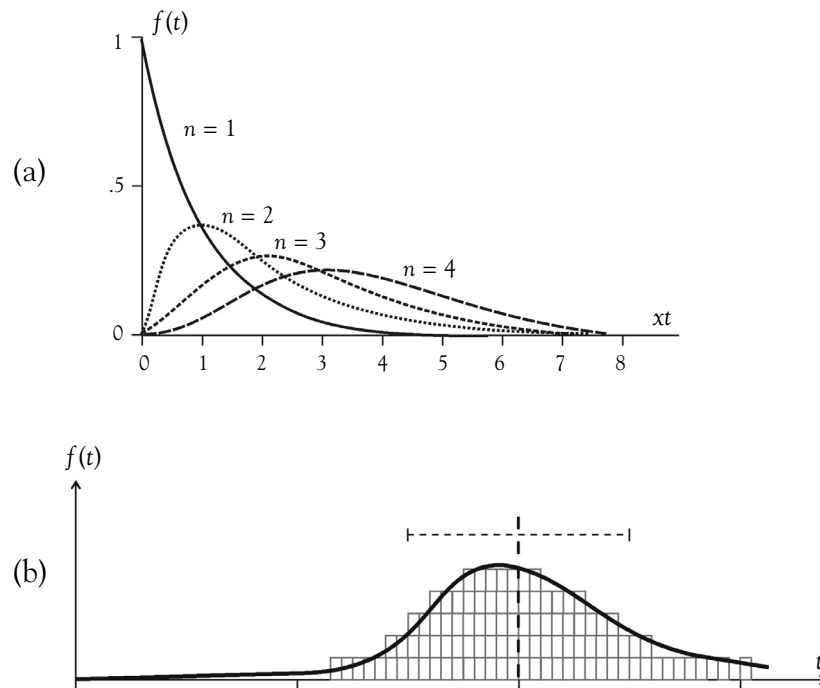


Figure 3: (a) Probability densities  $f_n(t)$  as a function of  $xt$ , according to the gamma distribution (eq. 3), for  $n = 1, 2, 3$ , and 4. (b) Example of a gamma-like distribution, with the indication of the mean and the standard deviation.

exponential curve, initiating at this new date. In other words, the fact that an effect has not appeared after five years does not increase the probability of its appearance in the following year; there is no memory of the time that has elapsed (Ross 1997, p. 237).

This may be illustrated by the example of 100 fishermen who set out to catch fish in a pond. Every time one of them catches a fish, he has a picture taken, throws the fish back into the water and leaves. In this way, the number of fish is constant, and so *the probability of catching them is also constant* (the fish don't learn to avoid baits). Let us suppose that the probability of catching a fish in ten minutes is 10%. After the first ten minutes, roughly 10 of the fishermen make a catch and leave. There are now only 90 fishermen, and in the next ten minutes around 9 of them catch a fish. They leave, and we are left with 81 fishermen. After 10% of them make a catch in the next ten minutes, roughly 8 leave. Draw-

ing a graph similar to Fig. 2b, one obtains the exponential distribution, until the last fisherman is left. This unlucky chap still has the same probability of 10% of making a catch, as he did in the beginning! Exponential distributions represent “memoryless” processes, i.e., ones for which the probability of an event occurring is constant.

In causal models of the history of science, an exponential distribution applies well to discoveries which arise from the *exploration of objects in a certain domain*, such as planets in the sky, or antibiotics in the soil. An astronomer who sets out to find a missing planet might have the same probability of finding it as someone who has already been looking for it for a few years. But, of course, in science this is an exception: usually a long chain of small advances is a prerequisite for a major breakthrough. A long composition of exponential decays is not itself an exponential decay, but something closer to a gamma distribution (more precisely, to the “hypoexponential” distribution, to be mentioned below, in section 5).

#### 4. Estimation of the Probability Function

Looking at the history of science, how should one proceed to associate probabilities to the appearance of an advance  $Y$ ? First, one should evaluate what causes contributed to its occurrence, and construct a causal model for advance  $Y$ . In the simplest case, consider a situation in which a single advance  $X$  is sufficient for producing  $Y$ , as in the example given in the previous section.

The time span between  $X$  and  $Y$  was assumed to be 12 years. We will call this the *empirical time span*  $\tau$  of the causal process  $X \rightarrow Y$ . Whatever the probability distribution of the underlying process, the best guess is that  $\tau$  corresponds to the time average or *mean*  $\bar{T}$  of the occurrences of the event in the possible worlds. This is different from the *half-life*  $T_{1/2}$ , which is the time it takes for the event to take place in half of the worlds (see Fig. 2). For the exponential cumulative probability function, the mean is simply the inverse of the decay constant:  $\bar{T} = 1/x$  (Ross 1997, p. 236). So equating the mean  $\bar{T}$  and the actual time span  $\tau$ , one may estimate  $p_{Y|X}(t)$ , according to eq. (1), by taking:

$$x = 1/\tau. \quad (4)$$

Such a result is independent of any hypotheses concerning the typical reference time interval  $T_{ref}$  of an historical setting, which is a nice feature. In our previous example, for  $\tau = 12$  yrs., one finds  $x = .083$  yrs.<sup>-1</sup>. For estimating



probabilities,<sup>4</sup> however, one must fix a typical time interval as a reference. For advances in 19th century European physics, such an interval may be taken to be 10 years. Then, using eq. (1), one obtains  $p_{Y/X}(10 \text{ yrs.}) = .565$ .

### 5. Composition of Causes

Given that  $X$  causes  $Y$  with a certain probability  $p_{Y/X}(T)$  — measured, as we have just remarked, for a typical time interval  $T$  — and that  $Y$  causes  $Z$  with probability  $p_{Z/Y}(T)$ , what is the probability  $p_{Z/X}(T)$  associated with the *composition* of causes?

One assumes eq. (1) and the analogous equation for the second process:  $p_{Z/Y}(t) = 1 - e^{-yt}$ . The situation may be represented by:

$$X \xrightarrow{1 - e^{-xt}} Y \xrightarrow{1 - e^{-yt}} Z.$$

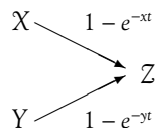
We present the mathematical developments elsewhere (Pessoa 2007), in order not to burden the reader. This problem is analogous to radioactive decay to an unstable state  $Y$  (Evans 1955, ch. 15). The result  $p_{Z/X}(t)$  for the composition (or convolution) of these two causes is:

$$p_{Z/X}(t) = [x \cdot p_{Z/Y}(t) - y \cdot p_{Y/X}(t)] / (x - y). \tag{5}$$

This may be generalized to a composition of any number of causes, and the result follows the so-called “hypoexponential” distribution (Ross 1997, p. 246–8), also called the “Bateman equations” in nuclear physics (Evans 1955, ch. 15), of which the gamma distribution is the special case for equal decay constants.

### 6. Disjunction of Causes

A certain advance may be obtainable by more than one causal path. For example, we might have the following *independent* causal relations:  $X \rightarrow Z$ , with probability  $p_{Z/X}(t)$ , and  $Y \rightarrow Z$ , with  $p_{Z/Y}(t)$ . What is then the probability of the disjunction of possibilities  $p_{Z/X,Y}(t)$ , that is, of the occurrence of either  $X \rightarrow Z$  or  $Y \rightarrow Z$ , given that both  $X$  and  $Y$  are present at a certain initial time?



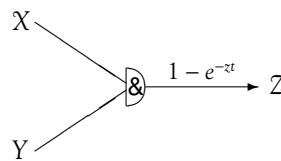
This problem is analogous to the probability of throwing two die and obtaining at least one “six”. There is  $1/6$  of a chance of obtaining a “six” with the first dice,  $1/6$  for the second, but after adding the probabilities one must subtract  $1/36$  because the throw with two “sixes” was counted twice: the result is  $11/36$ .

The probability in the case of a *disjunction* of causal paths, in which each cause is sufficient for the production of the effect, with probabilities  $p_{Z/X}(t)$  and  $p_{Z/Y}(t)$ , is therefore:

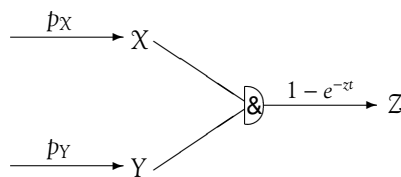
$$p_{Z/X,Y}(t) = p_{Z/X}(t) + p_{Z/Y}(t) - p_{Z/X}(t) \cdot p_{Z/Y}(t). \quad (6)$$

## 7. Conjunction of Causes

The case in which two causes are sufficient only in *conjunction*, for the production of an effect, cannot be related to the probabilities of the single conjuncts, since each of these is by itself insufficient (probability 0). In this case, of course, the joint probability  $p_{Z/X,Y}(t) = 1 - e^{-zt}$  has to be given.

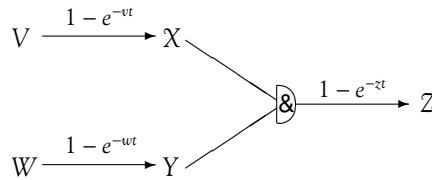


Consider now the situation in which the conjuncts  $X$  and  $Y$  are not assumed to be given, but have probabilities  $p_X$  and  $p_Y$  of occurring, with nothing else known:



In this case, the original joint probability  $p_{Z/X,Y}(t)$  must be multiplied by the probability that *both*  $X$  and  $Y$  occur, which is  $p_X \cdot p_Y$ .

The situation in which  $X$  and  $Y$  arise from previous causal processes with exponential cumulative probability functions is more complicated, but may be readily solved by integration (see Pessoa 2007).



### 8. Example of Computation of Probabilities

The aforementioned results may be used to compute the overall probability  $p_T(F)$  associated to Fig. 1, for the case of China and Europe. The reference time interval is taken to be  $T_{ref} = 400$  years. We have reverted to the notation used in section 1.

In Fig. 1, the probabilities with three significant digits were calculated from the empirical time span  $\tau$  between actual discoveries (section 4). Advance A corresponds to the report of the lodestone effect made by Pu Wei in 220 BCE, while advance C occurred with the report of the floating lodestone needle by Liu An in 120 BCE (Needham 1962, pp. 232, 281). With this data, one may use eq. (2) and (4), with  $\tau_{C/A} = 100$  years, to calculate a probability .982. However, care must be taken to interpret this. Notice that there are two possible paths arriving at C, one involving the set of advances  $\{A, B\}$ , while the other path involves only A, or more precisely, A and not B:  $\{A, \neg B\}$ . What is calculated from the empirical time span is the disjunction of these two paths:  $p(C/A) = .982$ , which we assume obeys an exponential distribution. In Fig. 1, what is represented are the isolated disjuncts,  $p(C/A, \neg B) = .1$  and  $p(C/A, B) = .980$ . The value of the latter is calculated with eq. (6).

The same analysis applies to the occurrence of advance D and of F. In the first case, we assume that the directive property of lodestone was discovered in the year 0 of the Christian Era, so that there is an empirical span (not so empirical!) of 120 years between the occurrence of C and D in China. The probability of the disjunction is  $p(D/A) = .964$ , and since we guessed that  $p(D/A, \neg C) = .2$ , therefore  $p(D/A, C) = .955$ , as shown in Fig. 1. The final case involves an empirical time span of 83 years between the estimated occurrence of D and Wang Chung’s description of the rudimentary compass, the lodestone spoon used with the diviner’s board, in 83 CE (Needham 1962, pp. 233, 237, 261–2). The probability of the disjunction gives  $p(F/D) = .992$ , while  $p(F/D, \neg E) = .1$  and  $p(F/D, E) = .991$ .

In order to calculate the overall probability for the invention of the rudi-

mentary compass in China, we should use the probabilities for the disjuncts, mentioned in the preceding paragraphs and calculated from the empirical time spans in China. Furthermore, one assumes the prior occurrence of  $A$  and  $B$ :  $p_{\text{Chi}}(A) = p_{\text{Chi}}(B) = 1$ . Thus, the probability of  $C$  is simply  $p_{\text{Chi}}(C/A) = .982$ . The composition with  $p_{\text{Chi}}(D/A, C) = .964$  may be computed with eq. (5), where the decay constants are the inverse of the empirical time spans (eq. 2). The result for the composite probability is:  $p_{\text{Chi}}(D/A) = .88$  (which is less than the simple product of the probabilities). This result for  $p_{\text{Chi}}(D/A)$ , however, will not be used for calculating  $p_{\text{Chi}}(F)$ , since one must compute an overall integral (the composite distribution is not exponential).

To proceed, we assume  $p_{\text{Chi}}(E) = 1$ , which simplifies the calculation of  $p_{\text{Chi}}(F/D, E)$ . The overall probability may thus be computed by calculating the composition of three causes  $A \rightarrow C \rightarrow D \rightarrow F$ , using an expression that we have not presented here. For the probabilities  $p_{\text{Chi}}(C/A) = .982$ ,  $p_{\text{Chi}}(D/C) = .964$ , and  $p_{\text{Chi}}(F/D) = .992$ , one obtains  $p_{\text{Chi}}(F) = .76$  for the probability of the compass being developed in China in the span of 400 years after the discovery of the lodestone effect.

In Europe, we assumed  $p_{\text{Eur}}(A) = 1$ , and  $p_{\text{Eur}}(B) = 0$ . To compute  $p_{\text{Eur}}(D/A)$ , one must consider a disjunction of paths,  $p_{\text{Eur}}(D/A, \neg C)$  or  $p_{\text{Eur}}(D/A, C)$ . The first is simply .2, while the second is calculated by composing probabilities .1 and .955, according to eq. (5), which gives  $p_{\text{Eur}}(D/A, C) = .07$ . The disjunction (eq. 6) furnishes  $p_{\text{Eur}}(D/A) = .26$ . We will simplify the estimate and assume that  $p_{\text{Eur}}(D/A)$  corresponds to an exponential distribution. Furthermore, since  $p_{\text{Eur}}(E) = 0$ , eq. (5) may be used to calculate the composition  $A \rightarrow D \rightarrow F$ , with  $p_{\text{Eur}}(F/D) = .1$ . The final probability is roughly  $p_{\text{Eur}}(F) = .02$ , a quite low estimated probability for the compass being developed in Europe after 400 years of the discovery of the properties of magnetic ore.

## 9. Discussion

In this way, a single causal model “explains” two independent paths of science. Of course, the example is completely ad hoc, but it encapsulates Needham’s interpretation of why the science of magnetism developed in such different ways in these two possible (but factual) worlds. Causal models offer an alternative way of encoding the information painstakingly obtained by historians of science. The use of numbers does not reflect a philosophical assumption that such numbers

really exist out there. The aim of such a quantitative method is to represent the history of science in a computer, so that simulations of the evolution of science may be undertaken. Certain arguable philosophical assumptions *have been made*, such as the existence of units of knowledge and of causal connections between them, but for a discussion of these the reader is referred to Pessoa (2005).

Notice that the partial result obtained for  $p_{\text{Chi}}(D) = .88$  could not by itself be used for finding  $p_{\text{Chi}}(F)$ . Probabilities were computed in a “holistic” way, integrating over all exponential cumulative functions. To proceed in a stepwise manner, one would have to consider not only the partial probabilities, but also the convoluted cumulative functions, which are *not* exponential.

This problem also raises an objection against modeling the appearance of advances by means of an exponential probabilistic function, since in reality the occurrence of any advance is the result of complicated chains of causal events. For this reason, a function resembling the gamma distribution mentioned in section 3 would be more adequate. It is plausible to assume that certain types of advances follow approximately an exponential distribution, such as discoveries arising from the exploration of new territory. Other advances, such as the solution of puzzles involving many ingredients, which require intermediary steps, are surely not adequately modeled by such a memoryless distribution.

One could hold the view that there exist *elementary causal connections*, which do not arise from the composition of smaller causal links. They could be recognized by obeying an exponential distribution, as is the case of radioactive decay (which nonetheless may in fact deviate from the pure exponential). The realist metaphysical position that all causal connections arise from the composition of elementary or “atomic” exponential causal links may be called “causal atomism”.

In our case study, we have modeled the causal relation between the discovery of the lodestone effect and the construction of the first rudimentary compass. If the intermediary steps were ignored, and the relation were considered exponential, the empirical time span of 303 years in China would lead to a probability (relative to a reference interval of 400 years) of .733 (using eqs. (1) and (4)), which is different from the value .758 obtained by taking into consideration intermediary steps.

The justification for the use of exponential functions is their simplicity (especially adequate for integration) and our ignorance of the underlying chain of causal processes. Still, the resulting formalism is quite complicated to be applied to large causal networks, which would require one big overall integral. Maybe the use of good computers, numerical integration, and approximation methods

may overcome this problem, not only for the exponential distribution but also for the gamma distribution. But the big advantage of the exponential distribution is that we need only one parameter (the empirical time span) to determine (as our best guess) the exact form of the distribution, while for the gamma case (or any Gaussian looking distribution) we need two parameters (mean value and the standard deviation).

Further work has indicated that hypotheses concerning the exact form of the distribution function may be unnecessary, at least concerning the *composition* of causal processes, since the mean value of the composition of any two distributions is the sum of the mean values of each distribution, and the square of the standard deviation of the composition is the sum of the square of the standard deviations of each distribution. This may greatly simplify the computations, at the price of not obtaining precise values for the calculation of probabilities.

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## Keywords

Philosophy of science, history of ancient science, causal model, probability.

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### Resumo

O objetivo deste trabalho é investigar a atribuição de probabilidades em um modelo causal de um episódio da história da ciência. O objetivo desta abordagem quantitativa é permitir a implementação do modelo causal em um computador, para que se possam rodar simulações. Como exemplo, olhamos para o nascimento da ciência do magnetismo, “explicando” — de uma maneira probabilista, em termos de um único modelo causal — porque o campo avançou na China mas não na Europa (a diferença é devida a diferentes probabilidades a priori atribuídas a certas manifestações culturais). Dado o número de anos entre as ocorrências de dois avanços  $X$  and  $Y$  conectados causalmente, propõe-se um critério para estipular o valor  $p_{Y|X}(t)$  da probabilidade condicional de um avanço  $Y$  ocorrer, dado  $X$ . Além disso, deve-se supor uma forma específica para a função de probabilidade cumulativa  $p_{Y|X}(t)$ , que tomamos como sendo a integral temporal de uma função de distribuição exponencial, como é feito na física de decaimentos radioativos. Mencionam-se regras para o cálculo de funções cumulativas para mais do que dois eventos, envolvendo composição, disjunção e conjunção de causas. Consideramos também os problemas decorrentes da suposição de que o surgimento de eventos no tempo seguiriam uma distribuição exponencial; tais problemas são consequência do fato de que a composição de causas não segue uma distribuição exponencial, mas sim uma “hipo-exponencial”. Sugerimos que uma distribuição gama pode representar mais adequadamente o aparecimento de avanços.

### Palavras-chave

Filosofia da ciência, história da ciência antiga, modelo causal, probabilidade.

### Notes

<sup>1</sup> A causal model may be represented by a structural diagram (a directed acyclic graph), such as that of Fig. 1, where each node stands for a variable and each arrow represents

the causal dependence between variables. More rigorously (Pearl 2000, p. 203), a causal model is a mathematical description of a set of variables  $v_i$ , by means of a set of functions  $f_i$ , the arguments of which are other endogenous variables  $a_i$  and also exogenous variables  $u_i$  (represented in a stochastic way):  $v_i = f_i(a_i, u_i)$ . Alternatively, one may use a probabilistic representation that makes use of Bayes' theorem, in order to compute conditional probabilities in light of new evidence. The interest in causal models, in the last twenty years, arose from the problem of inferring causal relations from a collection of data, which *prima facie* furnishes only correlations, and from appropriate experiments of intervention. In the present study, one cannot use most of the results developed in this field, since history usually happens only once (except in cases of independent discoveries) and it is not possible to intervene in it. We therefore employ only the notation that is used in causal models and the analysis of certain structures which form in a network of causal relations.

<sup>2</sup> We have stored historical information and run some preliminary simulations using the "SCHEME" programming language.

<sup>3</sup> It is worth noting that there is strong evidence that the first to discover the directive property of lodestone were the Olmecs, in Central America, before 1000 BCE (Carlson, 1975).

<sup>4</sup> At this point, we might try to clarify the interpretation of *probability* being adopted in this methodology. The strategy of imagining an ensemble of similar worlds, and considering the frequency with which each time span between two advances would occur, could suggest a frequentist interpretation of probability. However, such worlds are not observed but only imagined, so in fact the probabilities being postulated seem to follow a subjective interpretation. This problem is related to that of the interpretation of *causality* being adopted. As a matter of fact, the definition of causality that has been assumed is counterfactual: "A is cause of B, if A and B occurred, and if the absence of A would alter the probability of occurrence of B".