QUINE AND ONTOLOGY

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Abstract

Ontology played a very large role in Quine’s philosophy and was one of his major preoccupations from the early 30’s to the end of his life. His work on ontology provided a basic framework for most of the discussions of ontology in analytic philosophy in the second half of the Twentieth Century. There are three main themes (and several sub-themes) that Quine developed in his work. The first is ontological commitment: What are the existential commitments of a theory? The second is ontological reduction: How can an ontology be reduced to (or substituted by) another? And what is the most economical ontology that can be obtained for certain given purposes? The third is criteria of identity: When are entities of some kind (sets, properties, material objects, propositions, meanings, etc.) the same or different? In this paper I discuss Quine’s development of these three themes and some of the problems that were raised in connection with his work.

I. Introduction

Ontology played a very large role in Quine’s philosophy and was one of his major preoccupations from the early 30’s to the end of his life. As can be seen from the bibliography in the Schilpp volume, he published extensively on ontology—perhaps more than on any other specific philosophical subject. And Quine’s work on ontology provided a basic framework for most of the discussions of ontology in analytic philosophy in the second half of the Twentieth Century. There are three main themes (and several sub-themes) that Quine developed in his work.

The first main theme is ontological commitment: What are the existential commitments of a theory? As is well known, Quine’s answer is that the commitments of a theory (expressed in logical notation) are
manifested by the variables of quantification of the theory. This is often expressed by the slogan “To be is to be the value of a variable.”

The second main theme is ontological reduction: How can an ontology be reduced to (or substituted by) another? And what is the most economical ontology that can be obtained for certain given purposes? The latter is often related to Ockham’s razor and to Quine’s taste for desert landscapes.

The third main theme is criteria of identity: When are entities of some kind (sets, properties, material objects, propositions, meanings, etc.) the same or different? Inspired by Frege, Quine held that the postulation of entities of a given kind requires for its legitimacy that there be a criterion of identity for them. This is often expressed by the slogan “No entity without identity.”

All three themes are introduced in Quine’s early articles “Ontological Remarks on the Propositional Calculus” and “A Logistical Approach to the Ontological Problem”—which was reworked as “Designation and Existence”. Although these papers were not the most influential in the public discussion—which honor should undoubtedly go to “On What There Is”—they set the tone for all future work by Quine.

In “Ontological Remarks on the Propositional Calculus” we find one of Quine’s main sub-themes, namely his concern about propositions as elusive entities that have no clear conditions of individuation. Although Quine’s distrust of propositions may have been inherited from Russell, who argued against them on a number of occasions, in this paper Quine is not concerned to argue against propositions. What he wants to show is that for the purposes of propositional logic sentences suffice, and that it is not necessary in addition to postulate propositions as the denotation of sentences. This is an example of an ontological reduction in the spirit of Ockham’s razor.

But it is “A Logistical Approach to the Ontological Problem” (and its version “Designation and Existence”) that launched Quine’s ontological program and set up its main bases. The question of the ontological commitments of a theory (or language) is raised and clearly answered in terms of variables of quantification (p. 199):

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(1) We may be said to countenance such and such an entity if and only if we regard the range of our variables as including such an entity. To be is to be a value of a variable.

More importantly, however, Quine raises the question of ontology (p. 201):

(2) What entities there are, from the point of view of a given language, depends on what positions are accessible to variables in that language. What are fictions, from the point of view of a given language, depends on what positions are accessible to variables definitionally rather than primitively. Shift of language ordinarily involves a shift of ontology. There is one important sense, however, in which the ontological question transcends linguistic convention: How economical an ontology can we achieve and still have a language adequate to all purposes of science? In this form the question of the ontological presuppositions of science survives.

And he claims that an ontology of concrete individuals plus classes of such individuals, classes of such classes, etc. suffices:

(3) A language adequate to science in general can presumably be formed ... by annexing [to first order set theory] an indefinite number of empirical predicates. For this entire language the only ontology required—the only range of values for the variables of quantification—consists of concrete individuals of some sort or other, plus all classes of such entities, plus all classes formed from the thus supplemented totality of entities, and so on.

This is the ontology that Quine accepted throughout most of his life, but it is a transcendent ontology, not acceptable to a nominalist. The nominalist can, however, try to show that the transcendent part of the ontology can be dispensed with. There are several important sub-themes present here.

One is the claim that an extensional ontology consisting of concrete individuals and classes suffices for all purposes of science. This is not at all obvious and it will be the brunt of much of Quine’s later work to try
to justify it. Another sub-theme is an important reservation that Quine has concerning this ontology, deriving from the paradoxes of set theory. He claims that the paradoxes can be ruled out by diverse stipulations but that “such stipulations are *ad hoc*, unsupported by intuition”, and therefore that “[a] transcendent universe transcends the controls of common sense.” This leads him to the very important conclusion of the paper (p. 202):

(4) Nominalism is in essence, perhaps, a protest against a transcendent universe. The nominalist would like to suppress “universals”—the classes of our universe—and keep only the concrete individuals (whatever these may be). The effective consummation of nominalism in this sense would consist in starting with an immanent (non-transcendent) universe and then extending quantification to classes by some indirect sort of contextual definition. The transcendent side of our universe then reduces to fictions, under the control of the definitions. Such a construction would presumably involve certain semantic primitives as auxiliaries to the logical primitives. If, as is likely, it turns out that fragments of classical mathematics must be sacrificed under all such constructions, still one resort remains to the nominalist: he may undertake to show that those recalcitrant fragments are inessential to science.

Although it is clear that Quine does not declare himself a nominalist, this is his view of the nominalistic project and as such it has been attempted by a number of people since, including Quine himself on at least two occasions.

Quine’s ideas were very exciting to many people, including myself, for a number of reasons. To begin with, they seem to rescue some traditional philosophical preoccupations from the criticisms of logical positivism; and this is done in a way that is compatible with the demands for precision and clarity emphasized by analytic philosophers (including the positivists). Moreover, Quine proposes a program for work on ontology for logically minded philosophers and philosophically minded logicians. If one is sympathetic to the idea of ontological parsimony—and who wasn’t at the time?—then the idea of finding the most economical ontology sufficient for the purposes of science is an

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almost irresistible grand project, and one that can be tackled by means of logical and analytic methods. Finally, there seemed to be a clear analogy with Tarski’s work on truth: Quine’s ideas are to ontology as Tarski’s ideas are to truth. Just as Tarski’s work, based on his criterion of truth, led to important mathematical and philosophical developments, one might expect that Quine’s work, based on his criterion of ontological commitment, may also lead to important mathematical and philosophical developments. Although there is no evidence that such a comparison played a role for Quine, it is a suggestive idea for anyone interested in a logical approach to philosophical problems. It is true that a number of questions were raised concerning Quine’s ideas, especially in the fifties and sixties, but it wasn’t clear that the problems and objections that were raised could not be overcome; and many of them could be laid down either to misunderstandings or to a certain dislike and rejection of logical techniques in philosophy.

I turn now to the discussion of Quine’s main themes and to an evaluation of the results of his work.

II. Ontological Commitment

1. Formulations. The question that Quine raises is: What in a given discourse reveals ontological commitments? One of his basic insights in the 1939 papers is that the use of a proper name such as ‘Bucephalus’ or ‘Sherlock Holmes’, or of a common name such as ‘round’ or ‘unicorn’, or of an abstract name such as ‘roundness’ is not a sign of existential commitment. I might use the name ‘Sherlock Holmes’ in a context such as ‘John reasons like Sherlock Holmes’ without supposing (or presupposing) that ‘Sherlock Holmes’ denotes an entity. I might treat the whole context ‘... reasons like Sherlock Holmes’ as a predicate for which I can give conditions of applicability that do not depend on there being a denotation for ‘Sherlock Holmes’. If, however, from the context ‘John reasons like Sherlock Holmes’ I go on to infer ‘There is an x such that John reasons like x’, then it appears that I am treating ‘Sherlock Holmes’ as the name of an entity. I say “appears” because even the use of the existential quantifier may not reveal an existential
commitment, for I may be able to explain away the quantification. The statement ‘There is an $x$ such that John reasons like $x$’ in that particular discourse may be short for, say, ‘John reasons like Sherlock Holmes or John reasons like Poirot’. Hence Quine’s notion of fiction, as in (2) above. Quine’s conclusion is that the existential commitments of a given piece of discourse are revealed by the non-fictional uses of quantification in that discourse.

In “Designation and Existence” Quine puts the matter thus (p. 49):

(5) Perhaps we can reach no absolute decision as to which words have designata and which have none, but at least we can say whether a given pattern of linguistic behavior construes a word $W$ as having a designatum. This is decided by judging whether existential generalization with respect to $W$ is accepted as a valid form of inference.

And a little later (p. 50):

(6) Here then are five ways of saying the same thing: “There is such a thing as appendicitis”; “The word ‘appendicitis’ designates”; “The word ‘appendicitis’ is a name”; “The word ‘appendicitis’ is a substituent for a variable”; “The disease appendicitis is a value of a variable”. The universe of entities is the range of values of variables. To be is to be the value of a variable.

An important kind of example that Quine discusses in this paper (pp. 45–47) is the case of negative existentials such as ‘There are no unicorns’, ‘Pegasus does not exist’, ‘There is no such thing as hyperendemic fever’. The first statement seems rather unproblematic, for it means ‘It is not the case that there is an $x$ such that $x$ is a unicorn’. Quine’s strategy for dealing with the other cases is to appeal to Russell’s theory of descriptions and to say that ‘Pegasus’ means something like ‘the winged horse captured by Bellerophon’, and that by ‘hyperendemic fever’ one might mean something like ‘the disease which killed or maimed four-fifths of the population of Winnipeg in 1903’. In this case the second and third statements are of the same form as the first, because they mean ‘It is not the case that there is an $x$ such that $x$ is a
winged horse captured by Bellerophon' and ‘It is not the case that there is an \( x \) such that \( x \) is a disease which killed or maimed four-fifths of the population of Winnipeg in 1903’, respectively. This strategy for eliminating names is another of Quine’s main sub-themes, and it is discussed more systematically in *Mathematical Logic* (§27) and in many later works.5

2. Objections. Although Carnap6 and Church7 started discussing Quine’s ideas in the forties, the debate about ontological commitment and ontology began in earnest in the early fifties with the symposium “On What There Is”, the symposium “Semantics and Abstract Objects”, and several papers and reviews published throughout the decade.8 Some of these discussions were rather fiery, but while showing the rhetorical abilities of the participants, the issues remained somewhat elusive. The first really clear challenge to Quine’s criterion was Cartwright’s in “Ontology and the Theory of Meaning.” By this time Quine had already distinguished sharply what he called “the theory of reference” (including the notions of truth, reference, satisfaction, extension, etc.) from “the theory of meaning” (including the notions of meaning, analiticity, synonymy, necessity, intension, etc.). The case against the notions of the theory of meaning was made forcefully in “Two Dogmas of Empiricism” and in many later works. It turned out, however, that several of Quine’s formulations of his criterion of ontological commitment involved notions from the theory of meaning.9 Here are some samples (with my italics):

(7) The ontology to which an (interpreted) theory is committed comprises all and only the objects over which the bound variables of the theory have to be construed as ranging in order that the statements affirmed in the theory be true. (“Ontology and Ideology,” p. 11.)

(8) [W]e are convicted of a particular ontological presupposition if, and only if, the alleged presupposition has to be reckoned among the entities over which our variables range in order to render one of our affirmations true. (“On What There Is,” p. 13.)

(9) [A] theory is committed to those and only those entities to which

the bound variables of the theory must be capable of referring in order
that the affirmations made in the theory be true. (“On What There
Is,” pp. 13-14.)

(10) [A]n entity is assumed by a theory if and only if it must be
counted among the values of the variables in order that the statements
affirmed in the theory be true. (“Logic and the Reification of Univer-
sals,” p. 103.)

Yet Quine also claims that his criterion of ontological commitment
belongs to the theory of reference:

(11) Now the question of the ontology of a theory is a question purely
of the theory of reference. (“Ontology and Ideology,” p. 15.)

(12) As applied to discourse in an explicitly quantificational form of
language, the notion of ontological commitment belongs to the theory

And he offers the purely extensional formulation:

(13) [T]o say that a given existential quantification presupposes ob-
jects of a given kind is to say simply that the open sentence which fol-

‘∃x(x is a unicorn)’ presupposes unicorns if, and only if, ∃x(x is a unicorn & ‘x is a unicorn’ is true of x) & ∀x(x is not a unicorn → ‘x is a unicorn’ is not true of x).

Since there are no unicorns, the right-hand-side is false and, hence, ‘∃x(x is a unicorn)’ does not presuppose unicorns.10

This was a serious challenge and while Quine never acknowledged Cartwright’s objections, or the related objections by Scheffler and Chomsky, in the late sixties he tried other formulations of his criterion, as for example:

(14) My remaining remark aims at clearing up a not unusual misunderstanding of my use of the term ‘ontic commitment’. The trouble comes of viewing it as my key ontological term, and therefore identifying the ontology of a theory with the class of all things to which the theory is ontically committed. This is not my intention. The ontology is the range of the variables. Each of various reinterpretations of the range (while keeping the interpretations of predicates fixed) might be compatible with the theory. But the theory is ontically committed to an object only if that object is common to all those ranges. And the theory is ontically committed to ‘objects of such and such kind,’ say dogs, just in case each of those ranges contains some dog or other. (“ Replies,” p. 237.)

This formulation seems to suggest a model-theoretic criterion of ontological commitment.11 Given a theory T and an interpretation  of that is a model of T, then the ontology of T is the universe of  . How we formulate the other part depends on our understanding of Quine’s qualification “keeping the interpretations of the predicates fixed.” Since Quine certainly does not mean to keep the extensions of the predicates fixed—which would defeat his own example—we must either understand the qualification intensionally or in some alternative extensional way. An intensional interpretation will work, but will not do for Quine’s purposes. A possible extensional alternative is to restrict one’s discussion to substructures of a given model. Thus, given T, a model  of T and a non-empty class C, T is ontologically commit-
ted to entities of \( C \) if and only if \( C \) has non-empty intersection with the universe of every model of \( T \) that is a substructure of \( \mathfrak{A} \). This will work well for Quine’s example of a theory that implies \( \exists x (x \text{ is a dog}) \) but will not work for theories that imply \( \exists x (x \text{ is a unicorn}) \), because they have no models—at least not in any straightforward sense. Hence we cannot talk either of the ontology or of the ontological commitments of such theories and we are back to the problem raised by Cartwright.

Another issue, raised in the fifties by Alston and later taken up by Searle, is the dependence of Quine’s criterion on formalization. Although many people, including Quine, pointed this out, Searle offered the following argument. Suppose that \( 'K' \) is an “abbreviation for (the conjunction of statements) that state all existing scientific knowledge” and consider the predicate \( 'P_x' \) defined as \( x = \text{this pen} \& K' \). Searle claims that by asserting \( \forall x( P_x ) \) we are asserting “the whole of established scientific truth” while being “committed only to the existence of this pen.” At first sight this seems a ridiculous claim, and one is tempted to reply that if \( 'K' \) is an abbreviation for a conjunction of statements, then part of what is being asserted is that conjunction. Now either the statements in the conjunction are written in the notation of the logic of quantification, in which case there will be all kinds of commitments, or they are not, in which case Quine’s criterion cannot be (directly) applied. But this would miss Searle’s point “because the criterion does not determine how a theory should be formalized.” He is quite right; in fact, the criterion is not even supposed to do that. We see then that by means of his extreme example Searle is dramatizing the criterion’s dependence on formalization. He remarks: “I think that \( ['\exists x P_x'] \) is an absurd formulation of scientific knowledge, but there is nothing in the criterion that excludes it as a statement of theory.” With this Quine would agree, although he might suggest that whatever you save in ontology you pay for in ideology. Searle, on the other hand, maintains that “the stipulative definition of \( 'K' \) guarantees precisely that it contains the same commitments” as the statements it abbreviates. I think that the question that is really being raised by this argument is the question of what we may call the implicit commitments of a theory (or remark, or discourse).

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A standard claim by Quine is that such statements as ‘∃x(x is a number)’ or ‘∃x(x is a set)’ are committed to universals, and hence to abstract entities. These claims involve inferences to the effect that all numbers (or sets) are universals, and that all universals are abstract entities. But a theory that implies ‘∃x(x is a number)’ need not be committed to universals. The theory might imply that there are an infinite number of concrete particulars and that numbers are among them. Or it might not imply anything as to whether numbers are universals or not, but then why should Quine conclude that it is committed to universals? Should we distinguish the explicit commitments of a theory from its implicit commitments? In this case we might say that whereas Searle’s assertion ‘∃xPx’ is only explicitly committed to the existence of this pen, it is (via K) implicitly committed to all kinds of things—such as the existence of electrons, for example. This is a very natural tack, but it opens a real Pandora’s box.

Take Quine’s example of a theory that implies ‘∃x(x is a dog)’. Could there be dogs without there being hearts, livers, blood, cells, proteins, electrons? Wouldn’t then the theory be implicitly committed to such things? Where do the implicit commitments of a theory end? From the point of view of a Platonist, the existence of dogs might imply the existence of a property of being a dog, and hence a theory that implies ‘∃x(x is a dog)’ would be just as much committed to universals as a theory that implies ‘∃x(x is a number)’. It would seem that (at least) the implicit commitments of a theory would depend on how they are being judged. I think that this is right, however, and that (as in other cases in logic) adjudications as to whether a theory is committed to the existence of entities of a given kind will depend on a meta-theory within which the adjudications are made. If a meta-theory is based on second order logic, for example, then a theory that asserts ‘Fido is a dog’ and allows the inference to ‘∃x(x is a dog)’, may also allow the inference to ‘∃z(Fido is a z)’.

We should distinguish, therefore, the explicit commitments of a theory T to entities of a certain kind K as those where there is an explicit assertion within the theory that there are such entities, from the implicit commitments of T (relative to a meta-theory T’) as those where the existence of these entities follows from the assertions of T.
by (non-trivial) theoretical considerations in $T'$. This doesn’t really go against the spirit of Quine’s proposal, and it is an idea that fits in quite well with his discussion of relativity to a background theory in “Ontological Relativity” and other later works. Moreover, to the extent that one is concerned with the “whole conceptual scheme” or with a “language sufficient for the whole of science,” the explicit commitments will suffice, for in this case the theory is the background theory.

There is another concern, however, that Quine raises in “Ontological Relativity”, “Existence and Quantification” and “Grades of Theoreticity”, which has to do with the distinction between objectual quantification and substitutional quantification. It is not always possible to tell whether the quantifiers that are being used in a certain discourse are substitutional or objective, and as he says in “Existence and Quantification” (p. 107) “[w]here substitutional quantification serves, ontology lacks point.” In “Ontological Relativity” (p. 64) he remarks:

(15) Ontology is thus meaningless for a theory whose only quantification is substitutionally construed; meaningless, that is, insofar as the theory is considered in and of itself. The question of its ontology makes sense only relative to some translation of the theory into a background theory in which we use referential quantification. The answer depends on both theories and, again, on the chosen way of translating the one into the other.

From this we can conclude that even the “explicit” ontological commitments of a theory may depend on a meta-theory, and therefore that all ontological commitments may be implicit commitments relative to a meta-theory.

After these papers published in the late sixties Quine essentially dropped the technical discussion of ontological commitment, while continuing to maintain his slogan “To be is to be the value of a variable.” In fact, I was told by colleagues at Princeton in 1973 that at a talk Quine gave there he proposed to treat “ontological commitment” as a term of ordinary language, rather than as a technical term. This seems to me quite reasonable because, largely thanks to Quine’s work,
“ontological commitment” did become a term of ordinary (philosophical) language (at least in the analytic tradition). Also, as I have tried to illustrate above, the technical formulations of the criterion run into similar sorts of difficulties as other technical criteria intended to explicate ordinary notions (e.g., ‘empirical meaning’).

As a term of ordinary language the notion of ontological commitment serves as a guide in discussions of ontology, and the existential assertions of a theory (discourse, remark, person) carry a presumption of commitment to entities of the kind that are asserted to exist. If one wishes to deny that the theory (discourse, remark, person) makes such commitments, then one has the burden of proof to explain them away. This is the sense in which Carnap, Church and others welcomed Quine’s criterion. And in the same spirit one can also talk of the implicit commitments of a theory (discourse, remark, person) relative to various kinds of philosophical or theoretical considerations. As a technical device, on the other hand, the criterion neither has a satisfactory formulation nor seems to lead to interesting conclusions. The latter is particularly clear in connection with mathematical theories. When we talk about the ontological commitments of a mathematical theory we are normally talking about an uninterpreted theory and we are asking about properties of the structures that can be models of the theory. It is not very helpful to say that, e.g., first order Peano’s arithmetic is committed to numbers, when we also say of any theory that implies ‘∃x(x is a number)’ that it is committed to numbers; but it is interesting to say that any model for first order Peano’s arithmetic must include an initial part that is isomorphic to the standard number structure. But I will discuss this issue in connection with Quine’s second major theme.

III. Ontological Reduction

1. Formulations. Whereas Quine proposed a criterion of ontological commitment in his earliest papers, he did not propose a criterion (or even a rough characterization) of ontological reduction until the mid sixties. Nevertheless, the basic idea of ontological reduction that we
find in his early works seems to be that a theory T can be ontologically reduced to a theory T’ if the purposes for which T is used can be shown to be served equally well by T’. It might happen that the ontology of T’ is less comprehensive than the ontology of T, but this is not necessary. Euclidean geometry, for example, can be reduced to the theory of real numbers and sets of real numbers, but this is not a reduction to a less comprehensive ontology. In some cases one does indeed drop some entities, as when reducing the theory of real numbers with infinitesimals to the theory of real numbers by means of limiting processes, but these are rather special cases, and most of the time what one seems to be doing is interpreting one theory in another—as in the case of Euclidean geometry mentioned above. In the sixties Quine became worried by the Löwenheim-Skolem theorem and by the kind of ontological reduction characterized in terms of models that it seems to make possible.

The Löwenheim-Skolem theorem has been a motive of ontological controversies from the moment that Skolem published his second paper on it.¹⁹ In its basic formulation, the theorem says that if a first order theory T formulated in a denumerable language has an infinite model, then it has a denumerable model (i.e., a model of the lowest infinite cardinality). This led Skolem to the conclusion that if a first order system of set theory has a model, then it has a denumerable model. But in any of the standard systems of set theory it is possible to prove that there are non-denumerable infinite sets. Skolem realized, however, that this paradoxical conclusion (Skolem’s paradox) is not an actual contradiction. The reason for this is that the way that cardinalities are attributed to sets is by means of 1-1 functions, and to say that a set A is non-denumerable is to say that there is no 1-1 function f that correlates A with the set N of natural numbers. In a model Σ of set theory, sets are elements of (the universe of) the model and (the interpretation of) the membership relation relates such elements. If A is an element of Σ, then the elements of A in Σ are those elements of Σ that bear the Σ-membership relation to A. If Σ is denumerable, then there can be at most denumerably many elements of Σ that bear the Σ-membership relation to any element of Σ. A 1-1 function f in Σ that correlates two sets A and B in Σ is also an element of Σ, and it can

very well happen that even though there is a 1-1 function “outside” $\mathfrak{A}$ that correlates the elements of $\mathcal{A}$ with the elements of $\mathcal{B}$, there is no such function in $\mathfrak{A}$. Thus, there is no contradiction, and it seems correct to claim that any infinite ontology for a first order theory can be reduced to a denumerable ontology—and hence to an ontology consisting exclusively of natural numbers. Skolem concluded that set-theoretic notions such as denumerability and non-denumerability are not absolute but rather relative notions.

In “Ontological Reduction and the World of Numbers” Quine argues that:

(16) ...a doctrine of blanket reducibility of ontologies to natural numbers surely trivializes most further ontological endeavor. If the universe of discourse of every theory can as a matter of course be standardized as the Pythagorean universe, then apparently the only special ontological reduction to aspire to in any particular theory is reduction to a finite universe. Once the size is both finite and specified, of course, ontological considerations lose all force; for we can then reduce all quantifications to conjunctions and alternations and so retain no recognizably referential apparatus. (P. 216.)

And then proposes the following criterion of ontological reduction designed to block reductions by means of the Löwenheim-Skolem theorem:

(17) The standard of reduction of a theory $\theta$ to a theory $\theta'$ can now be put as follows. We specify a function, not necessarily in the notation of $\theta$ or $\theta'$, which admits as arguments all objects in the universe of $\theta$ and takes values in the universe of $\theta'$. This is the proxy function. Then to each $n$-place primitive predicate of $\theta$, for each $n$, we effectively associate an open sentence of $\theta'$ in $n$ free variables, in such a way that the predicate is fulfilled by an $n$-tuple of arguments of the proxy function always and only when the open sentence is fulfilled by the corresponding $n$-tuple of values. (P. 218.)

Quine gives as motivation for this criterion Carnap’s reduction of so-called impure numbers (e.g., 5 miles, 7.3 °C, 52 light years, etc.) to

pure numbers and the reductions by Frege, Dedekind and von Neumann of various kinds of numbers to sets. In Carnap’s reduction, for example, the proxy function maps the impure number 7.3 °C onto the rational number 7.3 (and similarly for 7.3 miles, 7.3 light years, etc.) and to the predicates ‘the temperature of x is 7.3 °C’, ‘the distance between x and y is 7.3 miles’, and so on, are associated the predicates ‘the temperature in °C of x is 7.3’, ‘the distance in miles between x and y is 7.3’, etc. In the case of the reductions of natural numbers to sets the proxy function maps the numbers 0, 1, 2, ... to ∅, {∅}, {{∅}}, ... (Zermelo) or to ∅, {∅}, {∅, {∅}}, ... (von Neumann), and so on, and to the numerical predicates are associated appropriate set-theoretical predicates.

2. Objections. A number of questions have been raised with respect to Quine’s proposed criterion. To begin with, there is a question of interpretation of the very notion of ontological reduction and of the role of the Löwenheim-Skolem theorem. Whereas Quine interprets ontological reductions as a relation between models of theories, many of the examples that he cites in his works are examples of interpretations of a theory in another. This is the case even for the examples of the reduction of impure numbers to pure numbers and of the reduction of numbers to sets. Secondly, it is not necessary to interpret the Löwenheim-Skolem theorem as establishing a relation between models. The theorem can be formulated by saying that any (deductively) consistent first order theory (in a denumerable language) has a countable model. In this sense, the theorem should be interpreted as telling us something about the possible models of a first order theory (whether interpreted or not). Finally, as I pointed out earlier, there are cases in which a theory T is neither interpreted in another theory T’ nor a model of T reduced to a model of T’, but T is simply replaced by a theory T’ which serves the same purposes as T and has different ontological commitments. Let me comment on these points.

Let’s take the reduction of (the theory of) natural numbers to (the theory of) sets by one of the various methods (Frege, Russell, Zermelo, von Neumann, etc.). Quine claims that these are ontological reductions of numbers to sets, but it seems much more natural to say that they are...
conceptual reductions of number theory to set theory; i.e., that the conceptual apparatus of set theory is sufficient to obtain number theory. In fact, I would say that what is being shown is that set theory is ontologically committed to natural numbers. For, what are the natural numbers? According to Quine what characterizes numbers is that they constitute a progression, and that there are such progressions of sets is precisely what is shown by the reduction of number theory to set theory.\(^{22}\) Unless one attributes a distinctive ontological character to numbers, as distinct from sets, concepts, or any other entities, this kind of reduction is not ontological in any clear sense. And this actually holds even for the case of the impure numbers. Impure numbers are ordered pairs of a number and a tag—i.e., a unit of measurement, which could just as well be a number—and any reasonable theory that can deal with numbers will also have the means to deal with ordered pairs of numbers.\(^ {23}\) Therefore, if one takes a structural approach to mathematical entities, Quine's suggested criterion fails to distinguish between conceptual reduction, ontological reduction and ontological commitment.

Concerning the second point, it is strange for Quine to offer these model-theoretic criteria. In fact, he even remarks (p. 219) that his “formulation suffers from a conspicuous element of make believe” in that “it belongs, by its nature, in an inclusive theory that admits the objects of \(\theta\), as unreduced, and the objects of \(\theta'\) on an equal footing.” And although Quine concludes that (17) “seems, if we overlook this imperfection, to mark the boundary we want,” his criterion would seem to be confronted by problems similar to those that came up in connection with his model-theoretic criterion (14) for ontological commitment. However, in “Ontological Relativity” (p. 58) Quine argues that such reductions really have the character of a *reductio ad absurdum*. But then, why not follow Skolem and interpret the reductions by means of the Löwenheim-Skolem theorem as *reductio ad absurdum*? In fact, it is actually quite surprising that Quine is so determined to reject the reductions provided by the Löwenheim-Skolem theorem, for it seems to me that his overall position with respect to logic and set theory should make him sympathetic to Skolem’s conclusions. Quine held fast over the years to the claim (which I

Oswaldo Chateaubriand quoted in connection with (3) above) that the paradoxes destroyed any intuitive notion of set and that all solutions to the paradoxes propounded by the various set theories are an *ad hoc* patching up of the problem. He has also maintained over the years that the only true logic is (classical) first order logic. In particular he has rejected second order logic as logic, claiming that the content of second order logic should be expressed by means of a first order set theory. It is well known that the Löwenheim-Skolem theorem is one of the characteristic marks of first order logic and that it does not apply to second order logic. So on what is Quine’s belief on the existence of non-denumerably many sets based? Why should he suppose that a (natural) model of set theory consists of non-denumerably many sets? How could he tell? It would seem reasonable, given these positions, to claim with Skolem that such set theoretical notions as denumerability and non-denumerability are relative notions. What does it mean (for Quine) to talk about truth in an absolute set theoretic sense?

Let’s turn now to the third point. Consider the reduction of the theory of real numbers with infinitesimals to the theory of real numbers. As Grandy argues in “On What There Need Not Be,” the infinitesimals do not go proxy to anything but are simply dropped. What happens, in effect, is that a theory $T$ is replaced by another theory $T’$ with different ontological commitments. In this particular case the commitments of $T’$ are among the commitments of $T$, but this is not necessary in general. In a note to the reprint of “Ontological Relativity” Quine acknowledges Grandy’s point and amends his criterion to allow for such “deflations.” But then Grandy argues in a postscript to his paper that by means of a series of reinterpretations of the original theory (consisting of inflations and deflations) one can recover the Löwenheim-Skolem reductions in all their original force, and concludes that one may want “to seek a less model-theoretic approach to ontological questions” (p. 812). As in the case of ontological commitment, this seems to me the right conclusion and for essentially the same reasons. Quine’s idea of ontological reduction is an interesting idea, but his technical model-theoretic formulation is an *ad hoc* solution that does not give any clear insight into the problems that it is designed to solve.

IV. Ontology

1. Formulations. Suppose that for a certain kind of entities there is a condition \( C(x,y) \) such that

\[
(i) \quad \forall x \forall y (C(x,y) \rightarrow x = y).
\]

We can reasonably say in this case that the condition \( C(x,y) \) determines the identity of the entities in question, and also that \((i)\) is a criterion of identity for those entities. The condition \( C(x,y) \) may be fairly complex and involve quantifications. If all quantifiers in \( C(x,y) \) range over the entities in question, and all terms and predicates are defined for those entities, then we can think of \((i)\) as an absolute (or intrinsic) criterion of identity. If, on the other hand, we appeal to other kinds of entities, or distinguish some entities of the same kind for which the determinateness of identity is presupposed, then \((i)\) is relative to those entities. Thus, consider the following:

\[
(ii) \quad \forall x \forall y (\forall z (Zx \leftrightarrow Zy) \rightarrow x = y),
\]

\[
(iii) \quad \forall x \forall y (\forall z (z \in x \leftrightarrow z \in y) \rightarrow x = y),
\]

\[
(iv) \quad \forall x \forall y (\forall z (x \in z \leftrightarrow y \in z) \rightarrow x = y),
\]

\[
(v) \quad \forall x \forall y (\forall z (z < x \leftrightarrow z < y) \rightarrow x = y),
\]

\[
(vi) \quad \forall x \forall y (\exists w \exists z(Dir(x,w) \& Dir(y,z) \& w \parallel z) \rightarrow x = y).
\]

\((ii)\) is Leibniz’ principle of identity of indiscernibles formulated in second order logic. As a criterion of identity for objects, say, it is a relative criterion that appeals to all properties of objects. \((iii)\) is the usual principle of extensionality for sets and \((iv)\) is an alternative principle of extensionality for sets. If the quantifier ‘\( \forall z \)’ ranges only over sets, then they are intrinsic criteria. And so is \((v)\), which is Goodman’s principle of identity for individuals in terms of the part-whole relation,
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if the quantifier ‘\( \forall z \)' ranges only over individuals. Frege’s principle of identity for line directions (vi) is a relative criterion that appeals to lines, to a relation between line directions and lines, as well as to a relation between lines.

Quine’s dictum “No entity without identity” is intended as a principle that allows as legitimate entities only those for which either there is an intrinsic criterion of identity, or for which there is a relative criterion that appeals only to entities that have already been legitimately introduced. His conception of the ontology of science, already suggested in (3), is of a hierarchy of sets based on concrete individuals. Assuming that there is a criterion of identity for these individuals, then (iii) is a relative criterion of identity for sets of individuals (where the quantifier ‘\( \forall z \)' ranges over individuals), and is again a relative criterion of identity for sets of individuals and/or of such sets (where the quantifier ‘\( \forall z \)' ranges over individuals and over sets of individuals), and so on. This is the ontology that Quine accepted throughout most of his life with the concrete individuals restricted to physical objects.\(^{26}\)

2. Objections. There are two natural questions that can be raised with respect to Quine’s position. The first is to what extent he has been able to show that such an ontology of physical objects and sets is really sufficient for science. He has argued that one can dispense with propositions, properties, meanings and mental entities, among other things, but one can question these claims. The second question is whether the ontology that he accepts can be justified on his own terms. This is the question that I will discuss in my concluding remarks.

An obvious problem for Quine’s position is the question of identity for physical objects. This has been a major preoccupation of philosophers and it is somewhat surprising that for many years Quine seemed to ignore it. In the mid-seventies he finally came to terms with it, however, and in “On the Individuation of Attributes” he argues that the criterion of identity for physical objects is coextensiveness.\(^{27}\) Now what is a physical object supposed to be, according to Quine, and what is the appropriate sense of “coextensiveness”? If a physical object is not a set, but is made up of molecules, then we must interpret the object as

being some kind of structure. Perhaps the most general sense would be to think of material objects as space-time sums (in Goodman’s sense) of whatever they are made up. Should we say that two material objects are the same when every elementary particle that is part of one is part of the other? I.e., should we use principle (v) above as a relative principle with the variables ‘x’ and ‘y’ ranging over physical objects and the quantifier ‘\(\forall z\)’ ranging over elementary particles? This would reduce the problem of identity for physical objects to the problem of identity for elementary particles, and raises the question of the criterion of identity for elementary particles. This is actually quite problematic, and in “Whither Physical Objects?” and “Things and Their Place in Theories” Quine examines the matter and argues for an alternative solution. He proposes to identify physical objects with the space-time region they “occupy” and to identify these space-time regions with sets of quadruples of real numbers. Moreover, since real numbers can themselves be defined set-theoretically, Quine finally reaches the rather remarkable conclusion that the entire ontology of science can be restricted to pure sets:

\[
(18) \text{We are left with just the ontology of pure set theory, since the numbers and the quadruples can be modeled within it. There are no longer any physical objects to serve as individuals at the base of the hierarchy of classes, but there is no harm in that. It is common practice in set theory nowadays to start merely with the null class, form its unit class, and so on, thus generating an infinite lot of classes, for which all the usual luxuriance of further infinities can be generated.}\]

Aside from its inherent implausibility, there are two main problems with this conclusion. One is that the proxy function that Quine needs to effect the reduction of physical objects to sets of quadruples is simply not well defined. (Which set of quadruples of real numbers is to be attributed to a given physical object?) The other is that, in terms of a criterion of identity, the ontology of pure sets is not really better off than an ontology of properties. Although Quine visualizes his ontology as starting from the empty set, there is nothing that intrinsically distinguishes the empty set from any other set. As with any set, which set is

the empty set depends on all sets. The problem is that the axiom of extensionality for sets (iii), which for Quine is the paradigm criterion of identity, is highly impredicative, because the quantifier ‘∀z’ ranges over all sets. And so is the empty set, which is defined by the condition that no set belongs to it. Hence, the identity of the empty set depends on all the sets in the hierarchy of pure sets, no matter how high up, and so does the identity of every other set. In the case of the ontology of sets built upon a basis of individuals, one presupposes that identity is well-defined for the individuals and then one uses (iii) as a schema for a sequence of relative criteria for sets of individuals, sets of those sets, etc. with a restricted range for the quantifier ‘∀z’. In the case of pure sets, on the other hand, (iii) has an absolute interpretation. In fact, by an argument of Frege’s, if one were to map the whole universe of pure sets one-to-one onto itself—with the empty set mapped onto whatever set you may wish—(iii) would continue to hold.

In “On the Individuation of Attributes” (pp. 101–102) Quine argues that to use (iv) as a criterion of identity for attributes (with the variables ‘x’ and ‘y’ ranging over attributes and the quantifier ‘∀z’ ranging over sets of attributes) will not do, because we would still have the problem of identity for sets of attributes. But what if we use (iv) again as a criterion of identity for sets of attributes (with the variables ‘x’ and ‘y’ ranging over sets of attributes and the quantifier ‘∀z’ ranging over sets of sets of attributes), and so on? Quine would presumably consider this solution to be illusory, involving an infinite regress. My point is that his idea that he can conform to the dictum “No entity without identity” by appealing to an ontology of pure sets is equally illusory.

V. Conclusion

Evidently I have not been able in the space of this paper to examine all the details of Quine’s work on ontology and of the fairly large literature that it generated. My conclusions in relation to Quine’s development of his main themes have been negative, in the sense that I do not think that his (technical) solutions to the questions of ontological
commitment, ontological reduction and ontology can be sustained. Nevertheless, the questions that he raised, and his work on them, have had an enormous impact on our appreciation of the issues relating to ontology. Quine’s work has been a source of inspiration for several generations of philosophers and logicians in the analytic tradition, and undoubtedly it will continue to be a source of inspiration for future generations as well.

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Keywords
W. V. Quine, ontological commitment, ontological reduction, ontology, criteria of identity.

A ontologia desempenha um papel muito importante na filosofia de Quine, e foi uma de suas maiores preocupações desde o início dos anos 30s até o final de sua vida. Sua obra sobre a ontologia ofereceu uma armação básica para a maior parte das discussões de ontologia na filosofia analítica na segunda metade do século XX. Há três temas principais (e diversos sub-temas) que Quine desenvolveu em sua obra. O primeiro é o compromisso ontológico: quais são os compromissos de uma teoria quanto ao que existe? O segundo é a redução ontológica: como pode uma ontologia ser reduzida a (ou substituída por) uma outra? E qual é a ontologia mais econômica que pode ser obtida para determinados propósitos? O terceiro são os critérios de identidade: quando as entidades de certo tipo (conjuntos, propriedades, objetos materiais, proposições, significados, etc.) são iguais ou diferentes? Neste artigo, discutimos o desenvolvimento que Quine deu a estes três temas e alguns dos problemas que foram levantados em relação a sua obra.

Palavras-chave
W. V. Quine, compromisso ontológico, redução ontológica, ontologia, critérios de identidade.

Notes

1 But in his autobiography in the Schilpp volume he says (p. 14) that in the years 1932/33 he “felt a nominalist’s discontent with classes.”
2 See, for example, Field Science Without Numbers, Hellman Mathematics without Numbers, Burgess and Rosen A Subject With No Object.
3 One was in his 1947 joint paper with Goodman “Steps Toward a Constructive Nominalism” and another in the preliminary version of The Roots of Reference in his lectures at Irvine in 1971—in the Schilpp volume (p. 39) he says:

"I went to Irvine hoping to get by with substitutional quantification over abstract objects, and I came away disabused."

4 See Quine’s method for eliminating propositions in “A Logistical Approach to the Ontological Problem”, p. 200.

5 E.g., O Sentido da Nova Lógica (§41), Methods of Logic (§42), “On What There Is” (pp. 5-8), Philosophy of Logic (pp. 25–26).

6 Carnap discussed Quine's views in Meaning and Necessity (§10) and in “Empiricism, Semantics and Ontology”. He did not object to Quine's criterion of ontological commitment as such but he objected to the use of the word 'ontology', because “it might be understood as implying that the decision to use certain kinds of variables must be based on ontological, metaphysical convictions,” whereas he considers that “the decision ... is a practical decision like the choice of an instrument.” (Meaning and Necessity, p. 43.) In “Empiricism, Semantics and Ontology” Carnap elaborates the point making his now classical distinction between questions that are external or internal to a linguistic framework (or theory). In his reply “On Carnap’s Views on Ontology”, Quine (rhetorically?) misunderstands Carnap’s distinction as a distinction between category questions and subclass questions, and considers it trivial (pp. 207-210). As Myhill rightly points out in p. 62 of his review, however, the distinction is of a different nature: “a question(-token) is internal relative to T if the asker accepts T at the time of his asking, and is prepared to use T in order to obtain an answer; external otherwise, in particular if the question is part of a chain of reflections and discussions aimed at choosing between T and some rival theory.” Obviously, external questions tend to be category questions in Quine’s sense, but they need not be. The subclass question in set theory whether there are uncountable sets of smaller cardinality than the continuum was asked both as an internal question (before the independence results) and as an external question (before and after the independence results). (But as late as 1968, in “Ontological Relativity” pp. 52–53 and “Existence and Quantification” pp. 91–93, Quine persists in interpreting Carnap as making a distinction between category and subclass questions.)

7 In his reviews of Quine 1939a and Quine 1943 Church is quite sympathetic to Quine’s criterion—calling it “this simple criterion of logical coherence” in his 1958 paper (p. 1009).

8 For the initial discussion see Geach 1951, Ayer 1951, Quine 1951d, Quine 1951b, Black 1951 and Bar-Hillel 1952.

9 Church points out in “Ontological Commitment” that the notion of ontological commitment is an intensional notion in the straightforward sense in
which the notions of belief, or knowledge, are intensional notions. For, as he says (pp. 1013–1014) “ontological commitment to unicorns is evidently not the same as ontological commitment to purple cows, even if by chance the two classes are both empty and therefore identical.”

10 Cartwright considers other extensional formulations, but they either let in too little or let in too much. Scheffler and Chomsky in “What Is Said To Be” raise similar objections. There is a detailed discussion of these papers in §§6–7 of my 1971 dissertation Ontic Commitment, Ontological Reduction, and Ontology.

11 The point that a theory may be committed to entities of a given kind without being committed to any specific entity of that kind was raised explicitly by Scheffler and Chomsky (p. 74). In the revised edition of From a Logical Point of View, Quine reformulates (10) as “entities of a given sort are assumed by a theory if and only if some of them must be counted among the values of the variables in order that the statements affirmed in the theory be true.”

12 Speech Acts, pp. 109–110. Alston directs his arguments mostly against the presentation of Quine’s ideas in White Toward Reunion in Philosophy. Searle’s discussion refers back to Alston’s arguments and is presented as a development of them. A detailed discussion of their arguments can be found in §5 of Chateaubriand 1971.

13 Searle also points out that one can use less extreme examples.

14 In “Logic and the Reification of Universals” (p. 102) Quine says: “... it should ... be possible to point to certain forms of discourse as explicitly presupposing entities of one or another given kind, say universals, and purporting to treat of them; and it should be possible to point to other forms of discourse as not explicitly presupposing these entities.” Although Quine does not elaborate a distinction between explicit and implicit commitments, Chihara develops it in “Our Ontological Commitment to Universals” (pp. 39 ff.). This paper, and his later book Ontology and the Vicious Circle Principle and paper “On Criteria of Ontological Commitment” contain a very good discussion and survey of the main issues.

15 I first made this suggestion in a graduate paper I wrote for Chihara in the mid-sixties and it was presented and discussed by him in his 1968 paper and in his later works of 1973 and 1974. I developed the suggestion in more detail in Chateaubriand 1971 (chapter 3). At that time I discovered that Robbins in “Ontology and the Hierarchy of Languages” had made a similar point. In Church 1958 we can find an appeal to a meta-theory but in a more restricted form—see pp. 1013–1014. What Church (note 4), Cartwright (p. 324) and
others do is to appeal to analiticity, or to semantical rules, but only for una-

16 See Chateaubriand 1971 pp. 113 ff. The qualification “non trivial” is in-

tended to block vacuous (and other) inferences which do not actually depend

17 Later discussions of ontological commitment (where Quine’s slogan is re-

18 This was observed by Wang long ago: “... Quine’s general criterion of using

19 Skolem “Some Remarks on Axiomatized Set Theory.” Skolem returned to


21 Whereas Skolem proved the theorem by reducing an (assumed) infinite

See, for example, *Word and Object*, p. 258. The arguments for this notion of ontological commitment are developed in more detail in Chateaubriand 1971 (chapter 4), Chihara 1973 (chapter 3) and Chateaubriand 1990.

In connection with the reduction of complex numbers to pairs of real numbers (and other similar reductions) Quine comments in "Foundations of Mathematics" (p. 25): “One tends to say not that the complex numbers have been eliminated in favor of ordered pairs, but that they have been explained as ordered pairs. One may say either; the difference is only verbal.” This suggests that Quine might claim that there is only a verbal difference in saying that the reduction of number theory to set theory shows that set theory is ontologically committed to natural numbers and saying that it shows that natural numbers can be ontologically reduced to sets.

See e.g. *Philosophy of Logic* (chapters 6 and 7). For a discussion of Quine’s arguments concerning second order logic see Boolos.

This comes up in connection with certain versions of the Löwenheim-Skolem theorem due to Bernays and Wang, which establish that any deductively consistent axiomatic theory $T$ can be effectively interpreted in first order arithmetic together with the (arithmetical) assumption that $T$ is consistent. (In Survey of Mathematical Logic Wang calls this result Bernays’ Lemma and formulates it as follows (p. 349): “If $S$ is consistent, then we can define in the system $Z_e$ obtained from number theory by adding Cons($S$) as an axiom, certain predicates such that the axioms $A_1$, $A_2$, ... of $S$ all become provable in $Z_e$ if in them we replace all the predicates one by one by predicates defined in $Z_e$ and let all variable range over natural numbers.”) In “Ontological Reduction and the World of Numbers” (p. 215) Quine comments on Wang’s result and argues that the problem is not to interpret axiomatic theories but rather “to accommodate all the truths of $\theta$—all the sentences, regardless of axiomatizability, that were true under the original interpretation of the predicates of $\theta$.” But, surely, unless there is an “original interpretation” and an absolute notion of set-theoretic truth for it, this doesn’t make sense. (For further discussion of Quine’s position vis-à-vis Wang’s result see Jubien 1969 and Grandy 1979.)

See, for example, “Logic and the Reification of Universals”, “The Scope and Language of Science”, *Word and Object* (chapter 7), and “On the Individuation of Attributes.”

"On the Individuation of Attributes" (pp. 100–101). Quine offers an argument using his criterion of ontological reduction to identify physical objects with aggregates of molecules. He then distinguishes between having a criterion of “individuation” for physical objects—which is what I called having a
criterion of identity for when two physical objects are the same—from having a criterion of "specification" for which aggregate of molecules corresponds to a given (ordinary) physical object—a desk, say. He claims that the latter is a problem of "vagueness of boundaries" and that there are many objects (aggregates of molecules) that can serve as the desk, but that "this vagueness of boundaries detracts none from the sharpness of our individuation of desks and other physical objects." For, whereas "there are many almost identical physical objects, almost coextensive with one another, ... [that] could serve as the desk ... they all have their impeccable principle of individuation; physical objects are identical if and only if coextensive." I discuss this argument in more detail in Logical Forms, pp. 166–169.

28 “Things and Their Place in Theories”, pp. 17–18. In “Whither Physical Objects?” Quine puts the matter thus (pp. 501–502): “There remains the question of ground elements. Take the members of my sets; then take the members of those members, if such there be, and so on down, until you get to rock bottom: to non-sets, to individuals in some sense. These are the ground-elements; and what are they to be? Not physical objects; they gave way to space-time regions. But space-time regions gave way in turn to sets of quadruples of numbers; so nothing offers. However, this is all right. Since Fraenkel and von Neumann, a set theory without ground elements has even been pretty much in vogue. There is the empty set, there is the unit set of the empty set, there is the set of these two sets, and all the finite and infinite sets having these as members. Continuing thus we suffer no shortages. This is known as pure set theory, and I seem to have ended up with this as my ontology: pure sets.”

29 These questions are discussed in detail in Logical Forms, especially chapter 10, from which I have extracted the last two paragraphs. The problem with the proxy function is the problem to which I referred in note 27.

30 See The Basic Laws of Arithmetic §10.