# LOGIC AND ONTOLOGY

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**Abstract**

*In view of the present state of development of non classical logic, especially of paraconsistent logic, a new stand regarding the relations between logic and ontology is defended In a parody of a dictum of Quine, my stand may be summarized as follows* To be is to be the value of a variable in a specific language with a given underlying logic *Yet my stand differs from Quine's, because, among other reasons, I accept some first order heterodox logics as genuine alternatives to classical logic I also discuss some questions of non classical logic to substantiate my argument, and suggest that my position complements and extends some ideas advanced by L Apostel*

The term 'ontology' usually denotes a part of metaphysics Ontology is the theory of the most general principles of being *qua* being, and may also be called general metaphysics In other words, in ontology one studies the basic traits of all reality, therefore, the common ontological theories are in a certain sense formal, since they do not consider this or that particular being, but being as such, being in general

In ontology, we are concerned especially with what there is and with the various fundamental classes of beings Typical ontological questions include, for example, the following Are there abstract entities (universals)? Are there substances? What is an attribute? Is the (ontological) law of contradiction universally valid?

On the other hand, logic is, at least partly, the doctrine of valid inference Clearly, in order to investigate valid inference, logic must start from certain basic ingredients, such as the notions of object, predicate, relation, and sentence In the analysis of these ingredients,

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an ontology seems to be implicit The main problem is, does logic commit us to ontological presuppositions?

Of course, there are philosophers who deny that logic has any significant connection with ontology at all This is precisely the case of Nagel (see Nagel 1949), who defends the thesis that logic has nothing to do with ontology, because he believes that logic neither tries to describe the way in which men in fact think about the world nor is related to the real world which we think about In Nagel's view, logic has only a normative role, prescribing rules of sound thinking and having no ontological import

G Berry (1975, p 243) employed the neologism 'anontologism' to designate the above position, according to which logic does not have any ontological significance Other important philosophers, leaving aside Nagel, accept the anontological view of logic, for instance, Ayer, Carnap and Kemeny, as well as the neopositivists in general

Here I shall argue in favor of a new manner of regarding the interconnections between logic and ontology, which is in opposition to all strict anontological conceptions

The paper is divided into four parts In the first, I give some reasons to substantiate a certain kind of ontological view of logic In the second, a particular system of paraconsistent logic is described and discussed In the third, by recourse especially to notions introduced in part two, I try to show how my view is in agreement with the most recent discoveries in paraconsistent logic and in some other fields of non-classical logic (though only paraconsistency receives full treatment) Finally, in the fourth, I observe that my conception constitutes a natural development of some views of L Apostel

## I

In previous works (see da Costa 1980, 1981), I have maintained the thesis that logic does not have any *direct* philosophical import Nonetheless, it has really an important *indirect* relevance to philosophy, and especially to ontology To begin with, let me make clear my position

Two main arguments may be presented in favor of the philosophical neutrality of logic 1) As a matter of fact, logic, conceived as the result of the logician's activity, shows itself to be independent of any philosophical thesis, and, as a consequence, of any direct ontological commitment whatsoever This circumstance seems so obvious to me, that I shall not pursue this line of thought further, 2) When the remark is made that a given development of logic implies (or depends on) some philosophical hypothesis, one can always answer that the development in question constitutes simply a *façon de parler*, that it does not in fact get involved with any philosophical thesis at all, but that it only feigns to this end For example, when our attention is called to the Platonist bias of extant set theory, the most obvious reply is to say that we are working as if Platonism were true, feigning to give countenance to Platonism, but that really we stay above such criticisms (sometimes, ontological issues are even condemned as meaningless, see, for instance, Carnap 1950) When we stay at the level of pure logic, our stand remains unassailable Undoubtedly, such disputes can only be settled, if ever, by means of some extra philosophical assumptions, i e , if we leave the domain of pure logic and get into the field of philosophy

Anyhow, the indirect philosophical significance of logic seems quite obvious For instance, Godel's incompleteness theorems and the non-classical logics led to a wealth of philosophical problems and disputes, much progress has arisen from the philosophical analysis of those topics The indirect import of logic to philosophy may be sum marizes in a few words it means that logic shows itself to be important to the domain of philosophy when supplemented by philosophical principles, i e , considered from the point of view of philosophy

From now on, expressions such as 'philosophical import of logic', 'ontological relevance of logic', etc , will respectively mean 'indirect philosophical import of logic', 'indirect ontological relevance of logic', etc That is, the qualification 'indirect' will normally be im plicitly understood

What are the principal theoretical reasons which explain that so many philosophers, particularly those belonging to the schools of em piricism and of analytic philosophy, deny the (even indirect) import of logic to ontology? Berry touches the heart of the question when he writes the following

> What are the motives underlying logical anontologism? I think there are two basic ones The first is the empiricist's and the analytic philosopher's felt need to explain the certainty of logic Any empirical principle can be wrong, for later observation may disprove it This uncertainty characterizes *all* empirical hypotheses A similar fallibility, in fact, marks every assertion about the world, for the world can always rise up somewhere along the line to veto it How come, then, that logic *is* certain? The anontologist replies that logic can be certain simply because *it is not about the world*, so the world can never prove it wrong The second motive underlying anontologism is a felt need to explain the *a priori* character of logic, its independence, that is, of observations The pure logician or the pure mathematician works in his study or at his blackboard He is untroubled by laboratories or experiments, and in his investigations he ignores their deliverances And well he may, says anontologism, for since his investigations are not about the world at all, no observation of it can guide ou correct them (Berry 1979, p 244)

Berry also adds that

> A third circumstance, though hardly classifiable as a motive under lying logical anontologism, at least renders it more palatable What Carnap (cf, for instance, 1935, pp 58ff) described as translation of sentences from the material mode into the formal mode of speech, or what Quine (1960, pp 270ff) calls 'semantic ascent' enables one to convert discussion of objects into discussion of that discussion of objects Instead of saying, e g, '4 is a number', you say 'the expres sion "4" is a numeral' Or, to take the trivial sort of case, instead of asserting sentence A, e g, '2+2=4', one can always assert the equiv alent sentence 'A is true', e g, '"2+2=4" is true' Sentences about numbers thus give way to sentences about numerals or even to sentences about sentences, so that logic itself can be replaced by metalogic If we can now clear the way by ingesting Nagel's distinction between the real world and language, particularly ideal language, we can swallow the anontologism of metalogic smoothly (Berry 1975, pp 244–5)

Nonetheless, logic is neither completely certain nor entirely *a priori* The known paradoxes, the difficulties related to the so called grand logics (set theories and higher-order predicate calculi), and the non-classical systems of logic confirm the historical lesson that several uncertainties and doubts are always harassing logicians (for details, see da Costa 1980 and Quine 1953). And logic interpreted as the most general part of science, not as a pure formal game, also can not be envisaged as entirely *a priori*, as Quine and other philosophers have argued (cf., for instance, Quine 1950, 1953 and 1960).

Without getting into details here, I may justifiably contend that certainty and apriority constitute characteristics which do not belong to logic in its entirety.

The mere possibility of semantic ascent can not explain the semantical dimension of a system of grand logic, and in particular the relations between logic and reality. Recourse to the idea of semantical ascent likewise does not solve the philosophical problems originating in a well-elaborated strong syntax (for example, the collection of symbolic expressions is potentially infinite. So, the natural question: What is a symbol?) That is to say, indirect philosophical problems can not properly be replaced by purely linguistic analysis. Logic does give rise to ontological inquiries, which are justifiable (and indirect).

I think that the present situation of logic clearly corroborates the foregoing conclusions on the general relations which link logic to ontology.

But let me leave the domain of generalities and enter into the territory of more definite assertions.

Once Quine wrote that "to be is to be the value of a variable" (1950, p. 15, and 1966, p. 66). Indeed, the ontological commitment of our theories is measured by the domains of their (bound) variables, as he asserts in a more explicit passage:

> To be assumed as an entity is, purely and simply, to be reckoned as the value of a variable. In terms of the categories of traditional grammar, this amounts roughly to saying that to be is to be in the range of reference of a pronoun. Pronouns are the basic media of reference; nouns might better have been named propronouns. The variables of quantification, 'something', 'nothing', 'everything', range over our whole ontology, whatever it may be; and we are convicted of a particular ontological presupposition if, and only if, the alleged presup-

position has to be reckoned among the entities over which our vari ables range in order to render one of our affirmations true (Quine 1953, p 13)

(This remains valid even in languages in which names and variable binding term operators, as the description symbol and the abstraction operator, can not be eliminated)

My stand constitutes basically a modifications os Quine's In a few words, I think that to be is to be the value of a variable in a given language with a determinate logic As a corollary, logic and ontology are so related, that a number of changes in logic entails the possibility of richer and more complex ontologies At first sight, this is surprising, since it implies that there exists in principle a vast class of ontologies whose underlying logics are incompatible Indeed, as there are infinitely many pure geometries, so there exists an infinity of *pure* ontologies And as one of the tasks of the physicist consists exactly in trying to choose the best geometry to be applied in his researches, so the ontologist has to attempt to discover the most appropriate ontologies to cope with reality

The simple recognition of the existence of *logically* distinct ontologies may be regarded as an advance in the philosophical domain, as is plain

The preceding conception is illustrated and defended in the next two sections

## II

Suppose one wants to formulate a theory of *Cantor's Absolute*, i e , a set theory capturing most of the properties of sets, as they appear in our intuitive and naive handling of them The works of Cantor and of the first mathematicians who undertook to develop the theory of sets may be considered as typical in this respect Some of the most important characteristics of such a theory seem to be the following

1) There are sets and atoms (*Urelemente*) Among the sets there are the void set and the universal set Any set can always belong to other sets In general, most of Zermelo-Fraenkel axioms must be

true of sets (Anyhow, if the atoms are not taken into account in the theory, this does not fundamentally change its nature )

2) The theory of Cantor's Absolute has to be contradictory, as the common paradoxes (for instance, Russell's, Burali-Forti's, and Cantor's) make evident Nonetheless, the theory should not be trivial — that is, some set-theoretical propositions should not be theorems of our theory, because they are not intuitively true regarding Cantor's Absolute

3) Most of the usual arguments of naive set theory should be admitted in the theory I am seeking Consequently, its underlying logic has to be classical logic in the sense that deduction from consistent premises should be classical But, since the theory should not be trivial, the function of classical logic in it has to differ from he standard uses of that logic Because my theory will belong to the category of paraconsistent theories,[1] I may say that I will employ classical logic within a paraconsistent logic

4) Essentially, in order to adapt classical logic to its new paraconsistent function, I shall modify the standard concept of deduction, as follows The sentence $\alpha$ is said to be a consequence of the sentences belonging to $A$ if, and only if, the following conditions are fulfilled (i) There exist sentences $\alpha_1, \alpha_2, \quad, \alpha_n$ belonging to $A$, or already shown to be consequences of $A$, such that $\alpha$ is a classical consequence (logical consequence) of $\alpha_1, \alpha_2, \quad, \alpha_n$, (ii) $\{\alpha_1, \alpha_2, \quad, \alpha_n\}$ is consistent (of course, the members of $A$ and the logical truths are included among the consequences of $A$ too) Owing to the fact that the notion of consistency in first order predicate calculus is not decidable and to other technical difficulties, I also introduce modal terms in my theory, however, the modal terms could be dispensed with altogether, and we could keep only extensional concepts These and other details will be made patent by the exposition below

In order to formulate the desired system, which reflects Cantor's Absolute, I introduce, to start with, an auxiliary logic, which serves several other purposes $S5^*$, that is, $S5$ extended with quantification and (contingent) equality

The language of $S5^*$ is a standard first-order language (without function symbols) to which I add the symbol of necessity $\Box$ ($\Diamond$ is definable in terms of $\Box$) Therefore, in this language we have the common connectives ($\rightarrow$, $\wedge$, $\vee$, $\neg$, $\leftrightarrow$), the modal operators ($\Box$, $\Diamond$), the quantifiers ($\forall$, $\exists$) the symbol of equality ($=$), predicate symbols, individual terms (constants and variables), and auxiliary symbols (parentheses) The formulas of $S5^*$ will be denoted by small Greek letters, and classes of formulas by capital Greek letters In this system the variables and constants are to be thought of as referring to functions which select from each possible world and element of that world (the only terms allowed are variables and constants An atomic formula $Rxy$ is true in world $\iota$ (for the assignment of $f, g$ to $x, y$) if $R_\iota f_\iota g_\iota$

Axioms of $S5^*$

*S1* $\alpha$, where $\alpha$ is an instance of a tautology (in $\rightarrow, \wedge, \vee, \neg, \leftrightarrow$)

*S2* $\forall x(\alpha \rightarrow \beta) \rightarrow (\alpha \rightarrow \forall x\beta)$, where the variable $x$ does not occur free in $\alpha$

*S3* $\forall x\alpha(x) \rightarrow \alpha(t)$, where the term $t$ is free for $x$ in $\alpha(x)$

*S4* $x = x$, where $x$ is any variable

*S5* $x = y \rightarrow (\alpha(x) \leftrightarrow \alpha(y))$, whit the usual restrictions for contingent equality[2]

*S6* $\Box(\alpha \rightarrow \beta) \rightarrow (\Box\alpha \rightarrow \Box\beta)$

*S7* $\Box\alpha \rightarrow \alpha$

*S8* $\Diamond\alpha \rightarrow \Box\Diamond\alpha$

Rules

*R1* From $\alpha$ and $\alpha \rightarrow \beta$ to infer $\beta$

*R2* From $\alpha$ to infer $\forall x\alpha$

*R3* From $\alpha$ to infer $\Box\alpha$

$\vdash \alpha$ means that $\alpha$ is provable ins $S5^*$, and we write $\Gamma \vdash \alpha$ if $(\alpha_1 \wedge \alpha_2 \wedge \quad \wedge \alpha_n) \rightarrow \alpha$ is provable in $S5^*$ for $\alpha_1, \alpha_2, \quad, \alpha_n$ in $\Gamma$ (if $\Gamma = \varnothing$, then $\Gamma \vdash \alpha$ means, by extension, simply that $\vdash \alpha$)

I remark, *en passant*, that $S5^*$ can be the basis of a modal set theory similar to the modal higher-order logic of Gallin 1975 Such a set theory proves useful in all contexts in which Gallin's systems IL and $ML_p$ have found applications

$S5^*$ may be regarded as a paraconsistent logic. To show how this can be done, let us suppose that $\Sigma$ is the set of non-logical axioms of a given theory $\mathcal{T}$. The set of theorems of $\mathcal{T}$, denoted by $\Delta$, is defined as follows: $\alpha \in \Delta$ if, and only if, we have

1) $\alpha \in \Sigma$, or
2) $\alpha$ is a theorem of $S5^*$, or
3) $\alpha$ is $\Diamond\beta$, with $\beta \in \Delta$, or
4) There exist $\alpha_1, \alpha_2, \ldots, \alpha_n \in \Delta$, such that
4.1) $\Diamond(\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n) \in \Delta$, and
4.2) $\Box((\alpha_1 \wedge \alpha_2 \wedge \ldots \wedge \alpha_n) \rightarrow \alpha) \in \Delta$. To express that $\alpha$ is a theorem of $\mathcal{T}$, we write $\Sigma \Vdash \alpha$ or $\Vdash_{\mathcal{T}} \alpha$.

It is clear that $\mathcal{T}$ may be inconsistent (for some $\alpha$, $\alpha$ and $\neg\alpha$ are theorems of $\mathcal{T}$), but non-trivial (there exists at least one formula $\beta$ such that it does not belong to the class of theorems of $\mathcal{T}$, i.e., we don't have $\Vdash_{\mathcal{T}} \beta$). In short, with the new definition of consequence ($\Vdash$), our logic is paraconsistent.

Nonetheless, observe that $\Vdash_{\mathcal{T}} \alpha$ and $\Vdash_{\mathcal{T}} \beta$ do not imply that $\Vdash_{\mathcal{T}} \alpha \wedge \beta$. This inconvenience could be eliminated by the introduction of a modal conjunction: $\alpha \& \beta =_{\text{def}} \Diamond\alpha \wedge \Diamond\beta$. Note also that if $\Sigma = \{\alpha \wedge \neg\alpha\}$, then $\mathcal{T}$ is trivial, though $\Sigma = \{\alpha, \neg\alpha\}$ does not entail that $\mathcal{T}$ is trivial.

Now let me pass to the description of an inconsistent but apparently non-trivial set theory $ZF^*$, which reflects Cantor's Absolute better than any of the extant set theories.

The language of $ZF^*$ results from the language of $S5^*$ when the collection of its predicate symbols contains only the symbol $\in$ of membership, the individual constants are deleted, and the classifier, $\{\ \}$, is adjoined. The addition of this last symbol offers no difficulties, and the adaptations we have to make in order to define the concepts of term and of formula are clear enough (for details, see da Costa 1980).

**Definition 1.** *If $t_1$ and $t_2$ are terms, then*
$t_1 \notin t_2 =_{def} \neg(t_1 \in t_2)$
$t_1 \neq t_2 =_{def} \neg(t_1 = t_2)$

**Definition 2** *If $t_1$ and $t_2$ are terms and the variable x does not appear free in them, we put*

$$t_1 \subset t_2 =_{def} \forall x(x \in t_1 \rightarrow x \in t_2)$$

**Definition 3** *$t_1$ and $t_2$ are terms and the variable x does not have free occurrences in them*

$$\{t_1, t_2\} =_{def} \{x \quad x = t_1 \vee x = t_2\}$$
$$\{t_1\} =_{def} \{t_1, t_1\}$$
$$\{t_1 \cup t_2\} =_{def} \{x \quad x \in t_1 \vee x \in t_2\}$$
$$\{t_1 \cap t_2\} =_{def} \{x \quad x \in t_1 \wedge x \in t_2\}$$
$$\sim t_1 =_{def} \{x \quad x \notin t_1\}$$
$$\varnothing =_{def} \{x \quad x \neq x\}$$
$$\bigvee =_{def} \{x \quad x = x\}$$
$$\mathbb{P}(t_1) =_{def} \{x \quad x \subset t_1\}$$
$$\bigcup t_1 =_{def} \{x \quad \exists y(y \in t_1 \wedge x \in y)\}$$
$$\bigcap t_1 =_{def} \{x \quad \forall y(y \in t_1 \rightarrow x \in y)\}$$

*Remark* In the last two cases, y must be distinct from x and not occur free in $t_1$ From now on, such obvious restrictions will not be made explicit

**Definition 4** *Under clear conditions, we put*

$\hat{x}\alpha(x)$ *for* $\{x \quad \alpha(x)\}$

$P_1$ *for* $x \in \{t_1, t_2\} \leftrightarrow x = t_1 \vee x = t_2$

$P_2$ *for* $x \in t_1 \cup t_2 \leftrightarrow x \in t_1 \vee x \in t_2$

$P_3$ *for* $x \in t_1 \cap t_2 \leftrightarrow x \in t_1 \wedge x \in t_2$

$P_4$ *for* $x \in \sim t_1 \leftrightarrow x \notin t_1$

$P_5$ *for* $x \in \varnothing \leftrightarrow x \neq x$

$P_6$ *for* $x \in \mathbb{P}(t_1) \leftrightarrow x \subset t_1$

$P_7$ *or* $x \in \bigcup t_1 \leftrightarrow \exists y(y \in t_1 \wedge x \in y)\}$

$P_8$ *for* $x \in \bigcap t_1 \leftrightarrow \forall y(y \in t_1 \rightarrow x \in y)\}$

$Q_1$ *for* $\exists y(\varnothing \in y \wedge \forall x(x \in y \rightarrow \cup\{x\} \in y))$

$Q_2$ *for* $\forall z \exists y \forall x(x \in y \leftrightarrow (\alpha(x) \wedge x \in z))$

$Q_3$ *for any standard formulation of the axiom of replacement in ZF*

$Q_4$ *for any standard formulation of the axiom of choice in ZF*

$Q_5$ *for* $\forall x(x \neq \varnothing \rightarrow \exists y(y \in x \wedge y \cap x = \varnothing))$

Now, I proceed to the statement of the non-logical axioms of ZF* (the restrictions to which the axioms are subjected are the common ones, and every axiom will always stand for its universal closure)

Non-logical axioms of ZF*

$A_1$ $\Box(\{x \;\; \alpha(x)\} = \{y \;\; \alpha(y)\})$

$A_2$ $\Box[\forall x(\alpha(x) \leftrightarrow \beta(x)) \rightarrow \{x \;\; \alpha(x)\} = \{x \;\; \beta(x)\}]$

$A_3$ $\Box(\{x \;\; x \in t\} = t)$

$A_4$ $\Box[\forall z(z \in x \leftrightarrow z \in y) \rightarrow x = y]$

$A_{5\,1}$ $x \in \{x \;\; \alpha(x)\} \rightarrow \alpha(x)$

$A_{5\,2}$ $\alpha(x) \rightarrow x \in \{x \;\; \alpha(x)\}$

$A_5$ constitutes the general separation axiom  It is not presented in the common intuitive version

$$x \in \{x \;\; \alpha(x)\} \leftrightarrow \alpha(x)$$

owing to the fact that it would be self-contradictory (this can be demonstrated, for instance, by the derivation of a form of the Russell paradox (essentially the classical formulation)), and this would turn the system into a trivial one

$A_6$ $\Box(P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5 \wedge P_6 \wedge P_7 \wedge P_8)$

The operations postulated by $A_6$, through $P_1 - P_8$, correspond to basic traits of the concept of a set and have to be unrestrictedly valid

$A_7$ $Q_1 \wedge Q_2 \wedge Q_3 \wedge Q_4 \wedge Q_5$

The statements $Q_1 - Q_5$, which compose $A_7$, though making part of usual ZF, are not supposed to be valid without restrictions, but imposed only on certain sets (i e , on the sets of the actual world, if I make appeal to the modal-semantical jargon)  For other kinds of sets, I could postulate properties similar to those of Quine's NF, nonetheless, this procedure will not be explored in the present paper

Taking into account the non standard definition of theorem of a theory founded one $S5^*$, we have

**Theorem 1** *All theorems of (customary) ZF are theorems of ZF* too*

**Corollary 1** *If ZF* is not trivial, then ZF is consistent*

**Theorem 2** *ZF* is inconsistent*

*Proof* Let us put $R = \{x \quad x \notin x\}$ Then, $A_5$ implies that $\Vdash_{ZF^*} \forall x(x \in R \rightarrow x \notin x)$ But, by the definition of theorem of ZF*, one has $\Vdash_{ZF^*} \Diamond\forall x(x \in R \rightarrow x \notin x)$, and also that also $\Vdash_{ZF^*} \Box(\forall x(x \in R \rightarrow x \notin x) \rightarrow (R \in R \rightarrow R \notin R))$ >From the last formulas, one gets that $\Vdash_{ZF^*} R \in R \rightarrow R \notin R$ Therefore, $\Vdash_{ZF^*} \Diamond(R \in R \rightarrow R \notin R)$ By S5*, $\Vdash_{ZF^*} \Box((R \in R \rightarrow R \notin R) \rightarrow R \notin R)$ Consequently, $\Vdash_{ZF^*} R \notin R$ Analogously, $\Vdash_{ZF^*} R \in R$ Therefore, ZF* is inconsistent (We could likewise derive in ZF* a version of Cantor's paradox )

If $\langle t_1, t_2, \quad , t_n\rangle$ denotes the ordered $n$-tuple of $t_1, t_2, \quad , t_n$ (when $n = 1$, $\langle t_1\rangle = t_1$ by convention), then instead of $A_4$ one could similarly have introduced a more general statement of the axiom of separation

$A'_{4\,1}$ $\hat{x}_1 \hat{x}_2 \quad \hat{x}_n\ \alpha(x_1, x_2, \quad , x_n)\ (\langle t_1, t_2, \quad , t_n\rangle) \rightarrow \alpha(t_1, t_2, \quad , t_n)$

$A'_{4\,2}$ $\alpha(t_1, t_2, \quad , t_n) \rightarrow \hat{x}_1\hat{x}_2 \quad \hat{x}_n\alpha(x_1, x_2, \quad , x_n)(\langle t_1, t_2, \quad , t_n\rangle)$

where $\hat{x}_1\hat{x}_2 \quad \hat{x}_n\alpha(x_1, x_2, \quad , x_n)$ can be defined likewise $\hat{x}\alpha(x)$ Under this hypothesis, Russell's paradoxes for relations would be derivable in ZF* too, though in a form which at first sight does not trivialize the system

**Theorem 3** *In ZF** ( $\Vdash$ *abbreviates* $\Vdash_{ZF^*}$)

$\Vdash \quad \Diamond(Q_1 \wedge Q_2 \wedge Q_3 \wedge Q_4 \wedge Q_5)$

$\Vdash \quad P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5 \wedge P_6 \wedge P_7 \wedge P_8$

$\Vdash \quad Q_i,\ i = 1, 2, \quad , 5$

$\Vdash \quad P_j,\ j = 1, 2, \quad , 8$

$\Vdash \quad \Diamond(x \in \{x \quad \alpha(x)\} \rightarrow \alpha(x))$

$\Vdash \quad \Diamond(\alpha(x) \rightarrow x \in \{x \quad \alpha(x)\})$

$\Vdash \quad P_1 \wedge P_2 \wedge P_3 \wedge P_4 \wedge P_5 \wedge P_6 \wedge P_7 \wedge P_8 \wedge Q_1 \wedge Q_2 \wedge Q_3 \wedge Q_4 \wedge Q_5$

$\Vdash \quad \forall x(x \in V \leftrightarrow x = x)$

$\Vdash \quad \{x \quad \alpha(x)\} = \hat{x}\alpha(x)$

$$\Vdash \hat{x}(x \in t) = t$$

$$\Vdash x \in \hat{x}(x \in t) \leftrightarrow x \in t$$

$$\Vdash \forall x(\varphi(x) \leftrightarrow \psi(x)) \rightarrow \hat{x}\varphi(x) = \hat{x}\psi(x)$$

$$\Vdash x \cup \sim x = V$$

$$\Vdash \mathbb{P}(V) = V$$

Some observations on the above technical developments are in order

1) *S5**, with the non-standard definition of consequence, could perhaps be employed in other contexts For instance, in the formalization of portions of dialectics, such as Hegel's Such an application would not mean that dialectics is in principle axiomatizable, but only that certain formalizations would help us to achieve better understanding of dialectical principles Maybe *S5** could likewise find applications in connection with some other theories or hypotheses which fall near paraconsistency, such as, for example, the hypothesis of complementarity in physics

2) ZF* has a peculiar Kripke semantics, so, notwithstanding its at first sight rather heterodox nature, it has an interpretation within the field of well-established modal ideas It can also be provided with a semantic of valuations (see Arruda & da Costa 1977), which constitutes a basically non-classical kind of interpretation

3) It is not known whether the non-traditional strong use of classical logic to study Cantor's Absolute will contribute to decide problems of set theory, i e , whether there are any set theoretic sentences $\sigma$ undecidable in ZF, but provable in ZF*

4) ZF* may be reinforced in various directions For instance, by new postulates guaranteeing the existence of several other 'contradictory' sets

5) Generally speaking, ZF* and its possible extensions parallel the real process of growth of axiomatic set theory, normally, the growth results from attempts to modify the axiomatic basis of set theory, the most significant to us being those originated by the adjunction of new principles (as, for example, the principles of reflection) [3]

6) The leading ideas of this paper could be carried out using different formulations for the modal logic other than S5* Also other formulations of the existence axioms of attributes could be used, for example the (strong) theory of properties of Reinhardt 1980 and (forthcoming)

## III

Since ZF* is stronger than ZF, it may theoretically replace ZF in all its applications, and in particular in science If one replaces ZF by ZF* as the underlying logic of science, then *possible* contradictions are not in principle excluded from the scientific field Anyway, one should insist that this circumstance does not entail that any contradiction whatever must be accepted or that one is hampered in the utilization of reasonings similar to the *reductio ad absurdum* Such conclusions are false, partly because, as I have already noted, the subjacent (first-order) logic of ZF* is essentially classical logic, though with a different definition of logical consequence

Because ZF* contains, so to say, contradictory sets, such as Russell's class, the ontology of science founded on ZF* becomes more populous than the ontology of the scientific system based, as is customary, on a set theory such as ZF In effect, one has in ZF*, as is easily seen $\Vdash \exists x(x = R)$, $\Vdash R \in R$, and $\Vdash R \notin R$ Therefore, Quine's criterion of ontological commitment, duly broadened, shows that the ontology of ZF* encompasses 'contradictory' objects Thereby, my thesis that to be is to be the value of a variable in a specified language with a given logic, receives confirmation

In principle, of course, I admit that the criteria to infer ontological assumptions either from logical or from scientific theories in general are the same In a few words, the process through which it seems reasonable to infer the existence of electrons and of neutrinos in physics does not differ intrinsically from the one by means of which one asserts the ontological commitments of logical systems (it seems more reasonable to accept certain ontological implications than to adopt positions such as fictionism and instrumentalism)

The subjacent logic of scientific theories and, in general, of rational contexts, results from a series of factors, most pragmatic in essence, as I have shown in da Costa 1980 Among them, I mention the following simplicity, intuitiveness, naturalness, psychological meaningfulness, and power of systematization In all these respects, ZF* does not get behind ZF too much Therefore, only the future development of science can decide, if ever, the issue of choosing ZF or ZF* to function as the logical system of science Ontology and logic are both in some measure historical they do not have Aristotelian essences, but are constructed in the course of history, subject to all historical uncertainties

The handling of modal and intensional logics leads us to ontological assumptions too This constitutes precisely the case of some semantics designed to cope with modal and intensional calculi, it seems, as Quine already has pointed out, that essentialism is linked to modal ideas Here I do not enter into discussions about modal and intensional systems, but only recall that modal and intensional concepts give rise to ontological questions If the topic were studied in some detail, one would perceive that my maxim on ontological commitment would receive further corroboration

The same kind of corroboration would be furnished by a critical examination of intuitionistic logic, with all its basic categories of construction, proof, mathematical entity, etc

My exposition might be supplemented by a discussion of the role of Platonism as the best ontological stance regarding systems like ZF and ZF* The main argument would parallel that of Berry (1975, pp 264–74), but the treatment of Platonism from my perspective will be left for a future work

I repeat that the ontological commitment of a standard theory may be discovered by the investigation of the entities which are referred to by its variables This method involves most Quinean distinctions and corollaries, but not all of them (particularly because I accept logics other than classical first order calculus, moreover, in some non-classical set theories, such as ZF*, the abstraction symbol, or classifier, can not be eliminated) On the other hand, the criterion of ontological commitment proposed should not be envisaged as an eternal logical or philosophical truth it constitutes for me only a heuristic maxim, intended to illuminate, if a little dimly, ontological researches [4]

## IV

In his (1960), Apostel defends the thesis that logic represents a tool of great importance for the ontological inquiry He also believes that ontology is intimately connected with science the former depends at least in part on the state of development of the latter He writes

> If I ask 'what is there?', I try to transcend the limitations of what I know about what there is, of what I add, due to perspective and human nature, to what there is, in order to capture what there is, what was there before I began to think, what there will be after my thinking shall have finished and what constantly guides my thinking But only in my thinking and through it, can I find the traces of this, and the common properties, one or multiple, it, the invariant, will have Ontology is an empirical science, that has to compare all various stages of the sciences, in simplified formalized versions, in order to discover the common property of all irreducible elements in this approximate model of what science is about This will be existence The function of logic in ontology will be quite more complex than present day logic claims it to be, but it will be extremely important (Apostel 1960, p 225)

Apostel has been one of the first philosophers to apply paraconsistent logic in his researches For example, he has used paraconsistent logic to clarify dialectics (see Apostel 1979)

He systematically insisted on the fact that logic and mathematics are both linked to experience

> The axiom of infinity states that there exists at least one infinite set We can discuss it supposing either that there is no infinite system in real nature or that such an infinite system really exists Let us take the first alternative In the case that *no infinite system* exists, the acceptability of an axiom of infinity (now, strictly speaking, false) depends upon certain properties of the universe and of human thinking This is no isolated case Indeed there has never been completely empty space, yet it is useful at first to develop mechanics without friction There has never been a pure free market economy, yet it is useful to develop economy from this standpoint Many sciences are typological sciences, studying complex reality through

> studying simplied models of it (schematizations, idealizations, fictions) Saying this however only states the problem the philosopher has to explain *why it necessary to study what is not in order to understand what is and how it is possible to understand what is through the study of what is not*
>
> One answer would be the following the exact number of objects in the universe in unknown to us, and many properties of our universe are independent of the number of objects in it For the human mind on the other hand it may be impossible to understand the consequences of given statements in which parameters appear, although the statements may be independent of these parameters, if we do not set these parameters equal to an extreme value (zero or infinite) This statement has a physical part and a psychological part, and both these parts are empirically confirmable statements
>
> If the universe is infinite (and some empirical arguments seem to point in the directions of this second alternative), then the justifications of the axiom of infinity has to be undertaken along entirely different lines (Apostel 1972, p 203)

Apostel's considerations imply, in particular, that the justification of a system of logic depends on a number of elements, almost all of them pragmatical Therefore, no logical system should be judged only on *a priori* grounds The important point is to know whether it works or not Possibly, this philosophical perspective explains the fact that Apostel has been so liberal relative to the non classical logics

Of course, Apostel does not acquiesce to Quine's criterion of ontological commitment as a definitive philosophical truth (see, for example, Apostel 1960, pp 205–6) But my interpretation of Quine's criterion, as a heuristic rule, capable of helping us to disentangle the ontology subjacent to a theoretical language, seems to be in agreement with his main ideas

Therefore, I think that Apostel would approve of the maxim according to which to be is to be the value of a variable in a language with a fixed underlying logic, as I conceive it, more than this, he would also accept that the inter-relations between logic and ontology are precisely those delineated in previous sections, just as he would be favorable to the arguments presented to corroborate my thesis

Perhaps the new function of logic in ontology, which for Apostel "will be quite more complex than present day logic claims it to be", will constitute the outcome of investigations similar to those here undertaken Moreover, I believe that my position regarding logic and ontology may be considered as a corroboration of some of his views (and other of his works reconfirm my thesis, see for example, Apostel 1963 and 1971) [5]

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## Notes

[1] The reader may consult Arruda 1980 to get a good idea of paraconsistent logic

[2] See Hughes & Cresswell 1974 and Gallin 1975

[3] The subjacent logic of ZF* has plainly a *dual* logical system, which can be utilized in the systematization of incomplete theories (for such theories, see Rescher & Brandom 1979) Those logics for incomplete theories, in some sense dual of paraconsistent systems, could be called *paracomplete*

[4] Logic and ontology are intimately connected Therefore, the Quinean well known thesis of ontological relativity leads, from our point of view, to a correspondent thesis of logical relativity

[5] I would like to thank Professor Rolando Chuaqui, who read a preliminary version of this article and discovered a mathematical error in it, and Professor W N Reinhardt, with whom I discussed its main theses