# A Paraconsistentist Approach to Chisholm's Paradox

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**Abstract.** The *Logics of Deontic (In)Consistency* (LDI's) can be considered as the deontic counterpart of the paraconsistent logics known as *Logics of Formal (In)Consistency*. This paper introduces and studies new LDI's and other paraconsistent deontic logics with different properties: systems tolerant to contradictory obligations; systems in which contradictory obligations trivialize; and a bimodal paraconsistent deontic logic combining the features of previous systems. These logics are used to analyze the well-known Chisholm's paradox, taking profit of the fact that, besides contradictory obligations do not trivialize in LDI's, several logical dependencies of classical logic are blocked in the context of LDI's, allowing to dissolve the paradox.

**Keywords:** Paraconsistent logic, deontic logic, deontic paradoxes, chisholm's paradox, moral dilemmas, logics of formal inconsistency.

## Introduction

The phenomenon of inconsistency has catastrophic consequences in theories based on classical logic. There is a simple reason to this: in the context of classical logic, a contradiction like $\alpha$ and $\neg\alpha$ trivializes the whole theory, sinc everything is logically derived from a contradiction. Paraconsistent logics, developed independently by da Costa 1963 and Jaśkowski 1948, are logics tolerant to contradictions, and so they admit non-trivial inconsistent (closed) theories.

The *Logics of Formal (In)Consistency* (LFI's), introduced in Carnielli & Marcos 2002 and also studied in Carnielli, Coniglio & Marcos 2007, are a special class of paraconsistent logics in which there is a formal distinction between notions such as contradiction, inconsistency and triviality. This is attained by the use in the formal language of connectives for consistency and inconsistency.

A phenomenon analogous to inconsistency takes place with deontic logics, in the context of moral dilemmas (see, for instance, McConnell 2006). If $\mathsf{O}$ denotes the deontic "obligation" operator, a moral dilemma is a situation of conflicting or contradictory obligations of the form $\mathsf{O}\alpha$ and $\mathsf{O}\neg\alpha$. Note that this is not a particular case of contradictoriness like $\mathsf{O}\alpha$ and $\neg\mathsf{O}\alpha$, but a different (however related) situation. In the presence of conflicting obligations, classical deontic logics trivialize,

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in the same manner as classical logic trivializes when a contradiction occurs. This similarity justify a paraconsistentist approach to deontic paradoxes.

The first paraconsistent deontic system, called $C_1^D$, was introduced in da Costa & Carnielli 1986 with the aim of analyzing moral dilemmas. It is a deontic extension of da Costa's logic $C_1$, being also suitable to the study of deontic paradoxes in general.

The *Logics of Deontic (In)Consistency* (LDI's) were firstly introduced in Coniglio 2007, by considering two deontic systems, **DmbC** and **DLFI1**, based on two different LFI's. It was shown that these systems are appropriate to analyze the well-known deontic paradox of Chisholm introduced in Chisholm 1963.

In Peron & Coniglio 2008 two new LDI's were introduced, **SDmbC** and **BDmbC**, enlarging the possibilities of analysis of Chisholm's paradox, and in Peron 2009 it was presented the distinction between LDI's and deontic LFI's, among other results.

The present paper extends our previous discussions on the subject, in particular the analysis of Chisholm's paradox in this context. Moreover, two new paraconsistent deontic systems are introduced, which can be considered, in some sense, as minimal: **DPI**, a LDI which is not a LFI, and **SDPI**, both a LDI and a LFI such that conflicting obligations trivialize, provided that the involved sentence is deontically consistent. Concerning Chisholm's paradox, it is concluded here that the violation of the Principle of Deontic Consistency is not the unique source of problems in deontic paradoxes based on contrary-to-duty obligations. Indeed, the rigidity of classical logic, in which too much sentences are logically identified, originates unnecessary logical dependencies among the premises of a given information set, and then several plausible formalizations of the given premises must be discarded. The use of a richer language based on a more flexible logic allows more translations from the natural language to the formal one, and so the paradox can vanish. If, in addition, the Principle of Deontic Consistency is weakened, there are still more possibilities to avoid the paradox.

The organization of the paper is as follows: in Section 1 the basic notions concerning paraconsistency in general and LFI's in particular are briefly recalled. In Section 2 it is firstly analyzed the Standard Deontic Logic **SDL** and then the LDI's are introduced, describing succinctly the main systems to be studied in the paper. Section 3 is devoted to the analysis of Chisholm's paradox under the perspective of some systems, namely **SDL**, **DmbC**, **SDmbC**, **DPI**, **SDPI** and **BDmbC**. Finally, Section 4 concludes with a brief evaluation of what was done in this paper.

## 1. Paraconsistency and Formal (In)Consistency

Da Costa's approach to paraconsistency is based on the idea that certain sentences of the object language can be marked as being "classical" or "well-behaved" (cf. da

Costa 1963). More precisely, in a logical context in which contradictions are possible, in the sense that a contradiction does not necessarily trivialize, it is possible to define or to assume as hypothesis inside a formal deduction that an specific sentence, say $\alpha$, has classical properties. The classical (or well) behavior of $\alpha$ is another sentence, $\neg(\alpha \wedge \neg\alpha)$, usually denoted by $\alpha^\circ$. In order to simplify the exposition we limit ourselves to the logic $C_1$, the first logic in the decreasing hierarchy of paraconsistent logics $(C_n)_{n\geq 1}$. Being so, in general

$$\alpha, \neg\alpha \nvdash \beta$$

for some $\beta$, but always

$$\alpha^\circ, \alpha, \neg\alpha \vdash \beta$$

for every $\beta$ (here, '$\vdash$' is the symbol for the consequence relation, and so '$\nvdash$' means "does not follow"). That is, contradictions do not necessarily trivialize, but contradictions of classically (or well) behaved sentences always trivialize (or "explode").

There is another way of interpreting da Costa's approach to paraconsistency: a contradiction is not enough to trivialize, but a contradiction of a contradiction trivializes:

$$\alpha, \neg\alpha \nvdash \beta$$

for some $\beta$, but always

$$(\alpha \wedge \neg\alpha), \neg(\alpha \wedge \neg\alpha) \vdash \beta$$

for every $\beta$. In other words, contradictions are not logically trivial, but a contradiction involving a contradiction is!

It can be observed that the statement that $\alpha$ is well-behaved, namely $\alpha^\circ$, is expressed in terms of the other connectives of the object language. Thus, if $(\alpha \wedge \neg\alpha)$ can be read as "$\alpha$ is contradictory" then $\alpha^\circ$ says that "$\alpha$ is not contradictory". Being so, non-contradictoriness is equivalent to well-behaviorness, or classicality, or *consistency*. But, are the notions of non-contradiction and consistency (or, dually, the notions of contradiction and inconsistency) necessarily the same?

This is the standpoint of *Logics of Formal (In)Consistency* (in short, LFI's), introduced in Carnielli & Marcos 2002 with the aim of generalizing da Costa's C-systems. In this approach the notion of well-behavior or consistency of a sentence $\alpha$ is represented by the sentence $\circ\alpha$, where $\circ$ is a primitive connective of the object language (and, different to the logics $C_n$, in general this connective cannot be expressed in terms of the others). Dually, it is possible to consider a inconsistency connective $\bullet$ as a primitive, such that the interdefinability laws $\circ\alpha \equiv \neg\bullet\alpha$ and $\bullet\alpha \equiv \neg\circ\alpha$ hold in general.

In term of consistency, LFI's has the following property (compare with $C_1$ above):

$$\alpha, \neg\alpha \nvdash \beta$$

for some $\beta$, but always

$$\circ\alpha, \alpha, \neg\alpha \vdash \beta$$

for every $\beta$. But, different to da Costa's systems, the consistency operator $\circ$ is primitive and is not necessarily identical to non-contradiction: in the weaker LFI's, $\circ\alpha$ and $\neg(\alpha \wedge \neg\alpha)$ are not equivalent. Moreover, in general

$$\circ\alpha \vdash \neg(\alpha \wedge \neg\alpha) \quad \text{but} \quad \neg(\alpha \wedge \neg\alpha) \nvdash \circ\alpha;$$

$$\neg\circ\alpha \vdash (\alpha \wedge \neg\alpha) \quad \text{but} \quad (\alpha \wedge \neg\alpha) \nvdash \neg\circ\alpha.$$

Thus, in the context of LFI's the notions of consistency and non-contradiction, as well as the notions of inconsistency and contradiction, can be separated. The steps through the definition of LFI's are briefly outlined below (the interested reader can consult Carnielli & Marcos 2002 and Carnielli, Coniglio & Marcos 2007).

Let **L** be a (propositional) logic having a negation connective $\neg$. A theory $\Gamma$ of **L** (that is, a set $\Gamma$ of sentences of **L**) is *contradictory* if $\Gamma \vdash \alpha$ and $\Gamma \vdash \neg\alpha$ for some sentence $\alpha$. Obviously, $\Gamma$ is not contradictory if and only if, for every $\alpha$, either $\Gamma \nvdash \alpha$ or $\Gamma \nvdash \neg\alpha$. A theory $\Gamma$ is *trivial* if $\Gamma \vdash \alpha$ for every $\alpha$, and is non-trivial otherwise. If $\Gamma, \alpha, \neg\alpha \vdash \beta$ for every $\alpha$ and $\beta$ the theory $\Gamma$ is said to be *explosive*. A logic **L** is *non-trivial* if it has a non-trivial theory, and it is *explosive* if every theory is explosive (equivalently, if $\alpha, \neg\alpha \vdash \beta$ for every $\alpha$ and $\beta$). A logic **L** is *paraconsistent* if it is non-trivial and non-explosive.

Let $\overline{\bigcirc}(p)$ be a set of sentences depending exactly on the propositional letter $p$ such that

(a) $\overline{\bigcirc}(\alpha), \alpha \nvdash \beta$

(b) $\overline{\bigcirc}(\alpha'), \neg\alpha' \nvdash \beta'$

for some $\alpha$, $\alpha'$, $\beta$ and $\beta'$. A logic **L** is *gently explosive* (w.r.t. $\overline{\bigcirc}(p)$) if

(c) $\overline{\bigcirc}(\alpha), \alpha, \neg\alpha \vdash \beta$

for every $\alpha$ and $\beta$. Given a sentence $\alpha$, the set $\overline{\bigcirc}(\alpha)$ expresses the consistence of $\alpha$ relative to logic **L** (and negation $\neg$). Whenever the set $\overline{\bigcirc}(p)$ is a singleton, the unique element of it will be denoted by $\circ p$ and then the sentence $\circ\alpha$ denotes the consistency of $\alpha$, where $\circ$ is called a *consistency operator*. By the very definition of consistency operator, $\circ\alpha, \alpha, \neg\alpha \vdash \beta$ for every $\alpha$ and $\beta$.

**Definition 1.1.** A *Logic of Formal Inconsistency* (**LFI**) is a paraconsistent logic having a set $\overline{\bigcirc}(p)$ satisfying properties (a), (b) and (c) as above. ■

It is immediate to see that logic $C_1$ is a LFI with $\circ\alpha = \alpha^\circ = \neg(\alpha \wedge \neg\alpha)$. Moreover, every logic $C_n$ (for $n \geq 1$) is a LFI with an appropriate definition of the consistency operator $\circ_n$. In fact, consider the following recursive definition: $\alpha^1 = \alpha^\circ$, and $\alpha^{n+1} = (\alpha^n)^\circ$ (for $n \geq 1$). Finally, let $\alpha^{(1)} = \alpha^\circ$ and $\alpha^{(n+1)} = \alpha^{(n)} \wedge \alpha^{n+1}$ (for $n \geq 1$). Then, consistency in logic $C_n$ is given by the operator $\circ_n p = p^{(n)}$ or, equivalently, by the set $\overline{\bigcirc}_n(p) = \{p^1, p^2, \ldots, p^n\}$ (for $n \geq 1$). It is interesting to observe that the logic $C_{Lim}$ (cf. Carnielli & Marcos 1999), the deductive limit of the hierarchy $(C_n)_{n\geq 1}$, is a LFI in which consistency is expressed by the infinite set $\overline{\bigcirc}(p) = \{p^1, p^2, p^3, \ldots\}$. That is, there are LFI's in which consistency is expressed by an infinite set of sentences instead of a single consistency operator.

The simplest LFI introduced in Carnielli & Marcos 2002 is called **mbC**, defined as follows:

**Definition 1.2.** The logic **mbC** is defined over the language $\circ, \neg, \wedge, \vee, \to$ by means of the following:

**Axiom schemas:**

**(Ax1)** $\alpha \to (\beta \to \alpha)$

**(Ax2)** $(\alpha \to \beta) \to (\alpha \to (\beta \to \gamma)) \to (\alpha \to \gamma))$

**(Ax3)** $\alpha \to (\beta \to (\alpha \wedge \beta))$

**(Ax4)** $(\alpha \wedge \beta) \to \alpha$

**(Ax5)** $(\alpha \wedge \beta) \to \beta$

**(Ax6)** $\alpha \to (\alpha \vee \beta)$

**(Ax7)** $\beta \to (\alpha \vee \beta)$

**(Ax8)** $(\alpha \to \gamma) \to ((\beta \to \gamma) \to ((\alpha \vee \beta) \to \gamma))$

**(Ax9)** $\alpha \vee (\alpha \to \beta)$

**(Ax10)** $\alpha \vee \neg\alpha$

**(bc1)** $\circ\alpha \to (\alpha \to (\neg\alpha \to \beta))$

**Inference rule:**

**(MP)** $$\frac{\alpha \qquad \alpha \to \beta}{\beta}$$

■

From now on, the set of sentences defined over the basic language $\neg, \wedge, \vee, \rightarrow$ will be denoted by $For$, and the set of sentences of **mbC** (which is obtained by adding the unary connective $\circ$) will be denoted by $For^\circ$.

It is worth noting that the system defined over $For$ by means of **(Ax1)–(Ax9)** and **(MP)** is the so-called Positive Classical Logic, denoted by **PCL**. Classical Logic **CL** defined over $For$ is obtained from **PCL** by adding **(Ax10)** and the "explosion law"

**(exp)** $\alpha \rightarrow (\neg\alpha \rightarrow \beta)$

Of course a version of **CL** over $For^\circ$, called **eCL**, can be obtained from **mbC** by adding the axiom schema $\circ\alpha$; in this sense **CL** is a deductive extension of **mbC** in which every sentence is consistent (and so **(bc1)** collapses to **(exp)**). On the other hand, $\bot =_{def} \circ\alpha \wedge (\alpha \wedge \neg\alpha)$ is a bottom sentence in **mbC**, that is, $\bot \vdash_{\mathbf{mbC}} \beta$ for every $\beta$ and so $\sim\alpha =_{def} (\alpha \rightarrow \bot)$ is a classical negation in **mbC**. Therefore **mbC** defined over the set of sentences $For^{\circ\sim}$ generated by $\sim, \circ, \neg, \wedge, \vee, \rightarrow$ (that is, with classical negation $\sim$ as a primitive) can be seen as the conservative extension of **CL** (now defined over the language $\sim, \wedge, \vee, \rightarrow$) obtained by adding axiom schemas **(Ax10)**, **(bc1)** and $\sim\alpha \leftrightarrow (\alpha \rightarrow \bot)$ where, as usual, $(\alpha \leftrightarrow \beta)$ stands for $(\alpha \rightarrow \beta) \wedge (\beta \rightarrow \alpha)$.

The logic **mbC** (as most of the LFI's presented in Carnielli & Marcos 2002 and Carnielli, Coniglio & Marcos 2007) does not satisfy the *replacement property*, namely:

$$\text{if } \alpha \dashv\vdash \beta \text{ then } \varphi(\alpha) \dashv\vdash \varphi(\beta)$$

for every $\alpha$, $\beta$, $\varphi$ (here, $\alpha \dashv\vdash \beta$ means that $\alpha$ and $\beta$ are interderivable, that is, $\alpha \vdash \beta$ and $\beta \vdash \alpha$). For instance, $(p \wedge q)$ is interderivable with $(q \wedge p)$, but $\neg(p \wedge q)$ is not interderivable with $\neg(q \wedge p)$; analogously, $\circ(p \wedge q)$ is not interderivable with $\circ(q \wedge p)$. This means that $\neg$ and $\circ$ are non-truth-functional connectives and so semantics is not a trivial issue for **mbC** (nor for the other LFI's with non-truth-functional connectives). In Carnielli, Coniglio & Marcos 2007 the following two-valued semantics was proposed for **mbC**:

**Definition 1.3.** Let $\mathbf{2} = \{0, 1\}$ be a set of truth-values, where 1 denotes "true" and 0 denotes "false". A ***mbC**-valuation* is a mapping $v : For^\circ \longrightarrow \mathbf{2}$ satisfying the following clauses:

(v1) $v(\alpha \wedge \beta) = 1$ iff $v(\alpha) = v(\beta) = 1$,

(v2) $v(\alpha \vee \beta) = 0$ iff $v(\alpha) = v(\beta) = 0$,

(v3) $v(\alpha \rightarrow \beta) = 0$ iff $v(\alpha) = 1$ and $v(\beta) = 0$,

(v4) $v(\neg\alpha) = 1$ implies $v(\alpha) = 0$,

(v5) $v(\circ\alpha) = 1$ implies $v(\alpha) = 0$ or $v(\neg\alpha) = 0$. ■

As usual, we define $\Gamma \vDash_{\mathbf{mbC}} \alpha$ iff $v(\alpha) = 1$ for every **mbC**-valuation $v$ such that $v(\Gamma) \subseteq \{1\}$. Thus, the logic **mbC** is sound and complete for its valuation semantics, cf. Carnielli, Coniglio & Marcos 2007:

**Theorem 1.4.** Let $\Gamma \cup \{\alpha\}$ be a set of sentences in $For^{\circ}$. Then, $\Gamma \vdash_{\mathbf{mbC}} \alpha$ if and only if $\Gamma \vDash_{\mathbf{mbC}} \alpha$.

## 2. Logics of Deontic (In)Consistency

### 2.1. Standard Deontic Logic and Paraconsistency

Deontic logics study notions such as "obligation", "permission", "prohibition" and similar concepts. Deontic logics were strongly influenced by notions from modal logic. Despite the analogy between modal and deontic concepts can be traced to the XIV century (cf. McNamara 2006), it can be argued that the symbolic and mathematical approach to deontic logics was inaugurated in the celebrated paper (von Wright 1951). In that article, von Wright distinguishes three kinds of modalities: alethic, epistemic and deontic. The first ones deal with concepts such as "necessary" and "possible"; the second ones concern notions as "verifiable" and "falsifiable", whereas the latter are related to notions such as "obligatory" and "permitted".

There exist several systems formalizing deontic logic (see for instance Prakken & Sergot 1994). The basic system is called **SDL** – *Standard Deontic Logic*. The idea behind this logic is to extend the basic modal system, in which the usual box operator $\Box$ for necessitation is substituted by $\mathsf{O}$ (meaning "it is obligatory that") by adding the restriction that conflicting obligations are not allowed. This restriction is formulated by means of axiom **(O-E)** below. The following axiomatization of **SDL** presented in Coniglio 2007 is appropriate to our purposes.

**Definition 2.1.** The logic **SDL** is defined over the language $\mathsf{O}, \neg, \wedge, \vee, \rightarrow$ by means of the following:

**Axiom schemas:**

**(Ax1)–(Ax10)** from **mbC** (recall Definition 1.2) plus

**(exp)** $(\alpha \rightarrow (\neg\alpha \rightarrow \beta))$

**(O-K)** $\mathsf{O}(\alpha \rightarrow \beta) \rightarrow (\mathsf{O}\alpha \rightarrow \mathsf{O}\beta)$

**(O-E)** $\mathsf{O}\mathbf{f}_\alpha \rightarrow \mathbf{f}_\alpha$ where $\mathbf{f}_\alpha =_{def} (\alpha \wedge \neg\alpha)$

**Inference rules:**

**(MP)** $\dfrac{\alpha \qquad \alpha \to \beta}{\beta}$

**(O-Nec)** $\dfrac{\vdash \alpha}{\vdash \mathsf{O}\alpha}$ ■

From now on, the set of sentences generated by the language $\mathsf{O}, \neg, \wedge, \vee, \to$ will be denoted by $For^{\mathsf{O}}$.

The usual axiomatizations of **SDL** depart from the set of axioms formed by the instances in the deontic language of the classical tautologies (arguably, this is a lazy and inelegant way to present an axiomatic extension of classical logic) and then consider, instead of **(O-E)**, the axiom schema

**(D)** $\mathsf{O}\alpha \to \neg\mathsf{O}\neg\alpha$

(usually called *the Principle of Deontic Consistency*) and the inference rules **(MP)** and **(O-Nec)**. It is useful and convenient to consider the derived deontic operator $\mathsf{P}\alpha =_{def} \neg\mathsf{O}\neg\alpha$ (meaning "$\alpha$ is permitted" or "$\alpha$ is permissible"). Thus, **(D)** can be interpreted as follows: whatever is obligatory is permitted. Since classically $\neg\beta$ is equivalent to $\beta \to \mathbf{f}_\alpha$ (for any $\alpha$ and $\beta$) then **(D)** can be reformulated as:

**(D*)** $\neg(\mathsf{O}\alpha \wedge \mathsf{O}\neg\alpha)$

stating that conflicting obligations are always false. Moreover, it can be proved that **(O-E)**, **(D)** and **(D*)** are equivalent in the presence of classical logic plus **(O-K)**, **(O-Nec)** and **(MP)**.

An interesting alternative axiomatization of **SDL** is proposed in Chellas 1980 by substituting **(O-K)** and **(O-Nec)** by the following:

**(ROM)** $\dfrac{\alpha \to \beta}{\mathsf{O}\alpha \to \mathsf{O}\beta}$

**(OC)** $(\mathsf{O}\alpha \wedge \mathsf{O}\beta) \to \mathsf{O}(\alpha \wedge \beta)$

**(ON)** $\mathsf{O}\top$

**(OD)** $\neg\mathsf{O}\bot$

where $\top$ can be taken as a classical tautology, for instance $\alpha \vee \neg\alpha$. Indeed, **(OD)** is an alternative formulation of **(D*)**, while **(ON)** and (**O-Nec)** are closely related. By its turn, **(O-K)** can be derived from **(ROM)** and **(OC)**.

Concerning semantics, and being deontic logics particular cases of modal logics, the well-known Kripke semantics of possible-worlds offer an appropriate framework.

**Definition 2.2.** A (generalized) *Kripke struture* is a triple $\langle W, R, \{v_w\}_{w \in W}\rangle$ where:

1. $W$ is a non-empty set (of *possible-worlds*);
2. $R \subseteq W \times W$ is a relation (of *accessibility*) between worlds;
3. $\{v_w\}_{w \in W}$ is a family of mappings $v_w : For^{\mathsf{O}} \to \mathbf{2}$ such that:

(k1) $v_w(\alpha \wedge \beta) = 1$ iff $v_w(\alpha) = v_w(\beta) = 1$,

(k2) $v_w(\alpha \vee \beta) = 0$ iff $v_w(\alpha) = v_w(\beta) = 0$,

(k3) $v_w(\alpha \to \beta) = 0$ iff $v_w(\alpha) = 1$ and $v_w(\beta) = 0$,

(k4) $v_w(\neg\alpha) = 1$ iff $v_w(\alpha) = 0$,

(k5) $v_w(\mathsf{O}\alpha) = 1$ iff $v_{w'}(\alpha) = 1$ for every $w'$ in $W$ such that $wRw'$. ■

Let $M = \langle W, R, \{v_w\}_{w \in W}\rangle$ be a a Kripke structure, $w \in W$ and $\alpha$ a sentence. We say that $\alpha$ is satisfied by $M$ in $w$, denoted by $M, w \Vdash \alpha$, if $v_w(\alpha) = 1$; $\alpha$ is satisfiable in $M$ if $M, w \Vdash \alpha$ for some $w \in W$, and $\alpha$ is true in $M$, denoted by $M \Vdash \alpha$, if $M, w \Vdash \alpha$ for every $w \in W$. Given a class $\mathscr{K}$ of Kripke structures, a sentence $\alpha$ is valid (w.r.t. $\mathscr{K}$) if, for every $M \in \mathscr{K}$, $M \Vdash \alpha$; and $\alpha$ is satisfiable (w.r.t. $\mathscr{K}$) if it is satisfiable for some $M \in \mathscr{K}$. Finally, the consequence relation $\vDash_{\mathscr{K}}$ is defined as follows: $\Gamma \vDash_{\mathscr{K}} \alpha$ iff there exists a finite set $\{\alpha_1, \ldots, \alpha_n\} \subseteq \Gamma$ such that $(\alpha_1 \wedge \ldots \wedge \alpha_n) \to \alpha$ is valid (w.r.t. $\mathscr{K}$).

**Theorem 2.3.** Let $\mathscr{K}_{\mathbf{SDL}}$ be the class of Kripke strutures such that the accessibility relation $R$ is serial, that is: for every $w \in W$ there exists $w' \in W$ such that $wRw'$. Then **SDL** is sound and complete for $\mathscr{K}_{\mathbf{SDL}}$, that is: $\Gamma \vdash_{\mathbf{SDL}} \alpha$ iff $\Gamma \vDash_{\mathscr{K}_{\mathbf{SDL}}} \alpha$.

Now it is convenient to establish a parallel between some metalogical notions from classical and deontic logics. Recall that in classical logic a contradictory theory is trivial, and vice-versa. On the other hand, as observed in da Costa & Carnielli 1986, if a theory $\Gamma$ derives conflicting obligations $\mathsf{O}\alpha$ and $\mathsf{O}\neg\alpha$ in **SDL** then $\Gamma$ is deontically trivial, that is, $\Gamma \vdash \mathsf{O}\beta$ for every $\beta$ (and, *a posteriori*, it is trivial: $\Gamma \vdash \beta$ for every $\beta$). The converse also holds, and so deontic contradictoriness is equivalent to deontic triviality, transposing to the deontic context a situation from classical logic.

In the previous section it was shown that LFI's incorporate metalogical notions such as consistency into the object language. This suggest us that some axioms as **(ON)** and **(OD)** can be treated as metalogical notions which can be internalized in the object language. Adapting to the modal context the definitions presented in Section 1, the following definitions are proposed:

**Definition 2.4.** Let **L** be a logic defined over the set $For^{\mathsf{O}}$, and let $\Gamma$ be a theory of **L**. Then, $\Gamma$ is said to be *deontically contradictory* or *deontically conflicting* (w.r.t. $\mathsf{O}$) if $\Gamma \vdash \mathsf{O}\alpha$ and $\Gamma \vdash \mathsf{O}\neg\alpha$ for some sentence $\alpha$. The theory $\Gamma$ is *deontically trivial* if $\Gamma \vdash \mathsf{O}\alpha$ for every $\alpha$. If $\Gamma, \mathsf{O}\alpha, \mathsf{O}\neg\alpha \vdash \mathsf{O}\beta$ for every $\alpha$ and $\beta$ then $\Gamma$ is said to be *deontically explosive*. The logic **L** is *deontically non-trivial* if it has a deontically non-trivial theory, and it is *deontically explosive* if every theory is deontically explosive (equivalently, if $\mathsf{O}\alpha, \mathsf{O}\neg\alpha \vdash \mathsf{O}\beta$ for every $\alpha$ and $\beta$). The logic **L** is *deontically paraconsistent* if it is deontically non-trivial and deontically non-explosive. ■

It is worth noting that a deontic contradiction is of the form $\mathsf{O}\alpha \wedge \mathsf{O}\neg\alpha$ instead of $\mathsf{O}\alpha \wedge \neg\mathsf{O}\alpha$. Continuing with the analogy with LFI's, let $\overline{\boxminus}(p)$ be a set of sentences in $For^{\mathsf{O}}$ depending exclusively on the propositional letter $p$ such that, for some sentences $\alpha$, $\alpha'$, $\beta$ and $\beta'$,

(Da) $\overline{\boxminus}(\alpha), \mathsf{O}\alpha \nVdash \mathsf{O}\beta$

(Db) $\overline{\boxminus}(\alpha'), \mathsf{O}\neg\alpha' \nVdash \mathsf{O}\beta'$

A logic **L** is *deontically gently explosive* (w.r.t. $\overline{\boxminus}(\alpha)$) if

(Dc) $\overline{\boxminus}(\alpha), \mathsf{O}\alpha, \mathsf{O}\neg\alpha \vdash \mathsf{O}\beta)$

for every $\alpha$ and $\beta$. Given a sentence $\alpha$, the set $\overline{\boxminus}(\alpha)$ expresses the *deontic consistence* of $\alpha$ relative to logic **L** (and negation $\neg$ and obligation $\mathsf{O}$). Whenever the set $\overline{\boxminus}(p)$ is a singleton, their unique element will be denoted by $\boxminus p$ and then the sentence $\boxminus\alpha$ denotes the deontic consistency of $\alpha$, where $\boxminus$ is called a *deontic consistency operator*. The following definition is the deontic version of Definition 1.1

**Definition 2.5.** A *Logic of Deontic Inconsistency* (**LDI**) w.r.t. $\mathsf{O}$ and $\neg$ is a deontically paraconsistent logic (w.r.t. $\mathsf{O}$ and $\neg$) having a set $\overline{\boxminus}(p)$ satisfying properties (Da), (Db) and (Dc) as above. ■

It should be noticed that a deontically paraconsistent logic (w.r.t. a negation $\neg$) is necessarily paraconsistent (w.r.t. the same negation $\neg$). Indeed, by **(exp)**, **(O-Nec)** and **(O-K)**, an explosive deontic logic is, *a posteriori*, deontically explosive. Therefore, a deontically paraconsistent logic is, by force, a paraconsistent deontic logic.

On the other hand, it is possible to define paraconsistent deontic logics, that is, to enrich a given paraconsistent logic with a deontic operator, while preserving the property that conflicting obligations trivialize. Thus, a paraconsistent deontic logic is not necessarily a deontically paraconsistent logic. The analysis of systems of both types is the topic of the next section.

## 2.2. Paraconsistent Deontic Logics and Deontically Paraconsistent Logics

The first paraconsistent deontic system proposed in the literature, called $C_1^D$, was introduced in da Costa & Carnielli 1986 and it was also studied in Puga, da Costa & Carnielli 1988. As suggested by its name, it is a deontic extension of $C_1$, in which some interesting relations between the consistency operator $\cdot^\circ$ and the deontic operator $\mathsf{O}$ arise, as we shall see below.

Related to the approach mentioned above, in Cruz 2005 it was studied a deontic extension of $C_1$ but, instead of taking **(O-E)**, a stronger version was considered: $\mathsf{O}\alpha \to \alpha$ (observe that **(O-E)** is a particular case, with $\alpha$ substituted by $\mathbf{f}_\alpha$). Another difference with the previous works is that the deontic operator $\mathsf{O}$ cannot be iterated, and so sentences such as $\mathsf{O}\mathsf{O}\alpha$ and $\mathsf{O}(\alpha \to \mathsf{O}\beta)$ are not allowed. A paraconsistent dyadic modal logic was also considered, extending $C_1$ with a dyadic deontic operator $\mathsf{O}_\beta\alpha$ meaning "$\alpha$ is obligatory in context (or under condition) $\beta$". But, again, an excessively strong axiom was considered: $\mathsf{O}_\beta\alpha \to (\alpha \wedge \beta)$. Being so, some deontic paradoxes as Chisholm's paradox (see Section 3 below) cannot be satisfactorily treated in this framework.

In the rest of this section some paraconsistent deontic system will be analyzed.

### 2.2.1. The logic DmbC

In Coniglio 2007 a deontic dimension was added to **mbC**, a minimal LFI (recall Definition 1.2), by appropriately adapting the axiomatization of **SDL** presented in Definition 2.1. The resulting system, called **DmbC**, has the consistency operator $\circ$ as primitive (since it is based on **mbC**) but no interaction between $\circ$ and $\mathsf{O}$ is required.

**Definition 2.6.** The logic **DmbC** is defined over the language $\mathsf{O}, \circ, \neg, \wedge, \vee, \to$ by adding to **mbC** (recall Definition 1.2) the following:

**Axiom schemas:**

**(O-K)** $\mathsf{O}(\alpha \to \beta) \to (\mathsf{O}\alpha \to \mathsf{O}\beta)$

**(O-E)**$^\circ$ $\mathsf{O}\bot_\alpha \to \bot_\alpha$ where $\bot_\alpha =_{def} (\alpha \wedge \neg\alpha) \wedge \circ\alpha$

**Inference rule:**

**(O-Nec)** $\dfrac{\vdash \alpha}{\vdash \mathsf{O}\alpha}$ ■

Alternatively, **DmbC** can be defined from the rules and axioms of **SDL** over $For^{\circ\mathsf{O}}$ (the set of sentences generated by the language $\mathsf{O}, \circ, \neg, \wedge, \vee, \to$) by replacing **(exp)** by **(bc1)** and **(O-E)** by **(O-E)**$^{\circ}$, respectively. It can be proved that **DmbC** is both a LFI and a LDI. Thus, it has explicit operators for both consistency and deontic consistency. This fact will be proved in Theorem 2.8 below. The first step is to define a Kripke semantics for **DmbC**. This is obtained by modifying appropriately Definition 2.2 as follows:

**Definition 2.7.** A *Kripke struture for* ***DmbC*** is a triple $\langle W, R, \{v_w\}_{w \in W}\rangle$ where:

1. $W$ is a non-empty set (of *possible-worlds*);
2. $R \subseteq W \times W$ is a relation (of *accessibility*) between worlds;
3. $\{v_w\}_{w \in W}$ is a family of mappings $v_w : For^{\circ\mathsf{O}} \to \mathbf{2}$ such that:

(kc1) $v_w(\alpha \wedge \beta) = 1$ iff $v_w(\alpha) = v_w(\beta) = 1$,
(kc2) $v_w(\alpha \vee \beta) = 0$ iff $v_w(\alpha) = v_w(\beta) = 0$,
(kc3) $v_w(\alpha \to \beta) = 0$ iff $v_w(\alpha) = 1$ and $v_w(\beta) = 0$,
(kc4) $v_w(\neg\alpha) = 0$ implies $v_w(\alpha) = 1$,
(kc5) $v_w(\alpha) = v_w(\neg\alpha) = 1$ implies $v_w(\circ\alpha) = 0$,
(kc6) $v_w(\mathsf{O}\alpha) = 1$ iff $v_{w'}(\alpha) = 1$ for every $w'$ in $W$ such that $wRw'$. ■

Soundness and completeness of **DmbC** w.r.t. the semantics above was proved in Coniglio 2007. As a matter of fact, this result, as well as a similar result for system **SDmbC** to be defined below, are particular cases of a general completeness theorem stated in Bueno-Soler 2008 for logics based on LFI's enjoying the so-called $G^{k,l,m,n}$ axiom (cf. Carnielli & Pizzi 2008).

**Theorem 2.8.** **DmbC** is both a LFI and a LDI.

**Proof.** Consider a Kripke model with $W = \{w\}$, $wRw$ and $v_w$ such that $v_w(p) = v_w(\neg p) = 1$ and $v_w(q) = 0$ for propositional letters $p \neq q$. Then it is easy to see that $p, \neg p \nVdash_{\mathbf{DmbC}} q$ and $\mathsf{O}p, \mathsf{O}\neg p \nVdash_{\mathbf{DmbC}} \mathsf{O}q$, and so **DmbC** is neither explosive nor deontically explosive. In particular, **DmbC** is neither trivial nor deontically trivial. Using axiom **(bc1)** it follows that **DmbC** is a LFI in which the consistency of $\alpha$ is expressed by $\circ\alpha$. Finally, let $\boxminus\alpha =_{def} \mathsf{O}\circ\alpha$. By **(O-Nec)** applied to **(bc1)**, followed by **(O-K)** and **(MP)** it follows that **DmbC** is a LDI in which the deontic consistency of $\alpha$ is expressed by $\boxminus\alpha$. ■

It is easy to see that by adding to **DmbC** the axiom schema $\circ\alpha$ it is obtained a version of **SDL** over $For^{\circ\mathsf{O}}$, denoted by **eSDL**. In fact, as proved above for **mbC**, axiom **(exp)** follows in that axiomatic extension of **DmbC**. On the other hand, by **(O-Nec)**, $\mathsf{O}\circ\alpha$ is a theorem for every $\alpha$ and so **(O-E)** follows.

As observed in Carnielli, Coniglio & Marcos 2007, the *inconsistency operator* $\bullet$ can be defined in **mbC** as $\bullet\alpha =_{def} \sim\circ\alpha$ in terms of the classical negation $\sim$, where $\sim\alpha =_{def} (\alpha \to \bot)$ (the expected definition $\bullet\alpha =_{def} \neg\circ\alpha$ in terms of the paraconsistent negation $\neg$ just works in stronger systems). Inspired by this, it is possible to define in **DmbC** an operator for *deontic inconsistency* as follows:

$$\boxtimes\alpha =_{def} \mathsf{O}\sim\circ\alpha.$$

Since $\circ\alpha, \sim\circ\alpha \vdash \bot$ then $\mathsf{O}\circ\alpha, \mathsf{O}\sim\circ\alpha \vdash \mathsf{O}\bot$ and so $\boxminus\alpha, \boxtimes\alpha \vdash \bot$, by **(O-E)**°. This means that $\boxtimes\alpha \vdash \sim\boxminus\alpha$ and $\boxminus\alpha \vdash \sim\boxtimes\alpha$, but the converses are not true (as it can be easily proved by using Kripke structures). Thus, the analogy between $\bullet$ and $\boxtimes$ is not complete: $\bullet\alpha \equiv \sim\circ\alpha$ and $\circ\alpha \equiv \sim\bullet\alpha$, but neither $\boxtimes\alpha \equiv \sim\boxminus\alpha$ nor $\boxminus\alpha \equiv \sim\boxtimes\alpha$.

### 2.2.2. The logic DPI

Observe that deontic consistency is a derived connective in **DmbC** (as in some other systems to be defined below). A natural question is: it would be possible to define the deontic counterpart of **mbC**, in the sense of being a LDI such that deontic paraconsistency is expressed by a primitive connective? This logic would be non-explosive but without having a consistency connective. In other words, we are looking for a minimal LDI which is not a LFI.

At this point, we must recall that **mbC**, the minimal LFI, has just one axiom for the consistency operator $\circ$, namely **(bc1)**. By removing that axiom (and by removing $\circ$ from the language) it is recovered the interesting paraconsistent logic **PI** introduced in Batens 1980.

**Definition 2.9.** The logic **PI** is defined over the basic language $\neg, \wedge, \vee, \to$ as follows:
**Axiom schemas:** Axioms **(Ax1)-(Ax10)** (recall Definition 1.2)
**Inference rule:** **(MP)** (recall Definition 1.2) ■

The logic **PI** can be transformed in a LDI by adding the modality $\mathsf{O}$, a connective for deontic consistency and appropriate rules and axioms, as follows.

**Definition 2.10.** The logic **DPI** is defined over the language $\mathsf{O}, \boxminus, \neg, \wedge, \vee, \to$ by adding to **PI** the following:
**Axiom schemas:**

**(O-K)** $\mathsf{O}(\alpha \to \beta) \to (\mathsf{O}\alpha \to \mathsf{O}\beta)$

**(Dbc1)** $\boxminus\alpha \to (\mathsf{O}\alpha \to (\mathsf{O}\neg\alpha \to \mathsf{O}\beta))$

**Inference rules:**

**(MP)** $\dfrac{\alpha \qquad \alpha \to \beta}{\beta}$

**(O-Nec)** $\dfrac{\vdash \alpha}{\vdash \mathsf{O}\alpha}$ ■

Concerning semantics, **DPI** can be characterized by Kripke structures as those of **DmbC**, but now $R$ is not necessarily serial, and the valuations must satisfy clauses (kc1)-(kc4) and (kc6) of Definition 2.7, plus the following:

(kc5.1) $v_w(\boxminus\alpha) = v_w(\mathsf{O}\alpha) = v_w(\mathsf{O}\neg\alpha) = 1$ implies $v_w(\mathsf{O}\beta) = 1$ for every $\beta$.

Using the usual techniques, it can be proved that **DPI** is sound and complete w.r.t. its Kripke semantics.

As observed in Carnielli & Marcos 2002, the logic **PI** is not a LFI and the same applies to **DPI**. On the other hand, by using Kripke structures it is easy to see that **DPI** is deontically paraconsistent, being obviously a LDI. So, the following result follows:

**Theorem 2.11.** **DPI** is a LDI but it is not a LFI.

The semantics for **DPI** deserves some comments. Firstly, it should be observed that it is possible to have a trivial valuation $v_{true}$ for **PI** such that $v_{true}(\alpha) = 1$ for every $\alpha \in For$. This is the semantical counterpart of the fact that **PI** is not finitely trivializable, and so there is no bottom formula, that is, a sentence $\perp$ such that $\perp \vdash_{\mathbf{PI}} \alpha$ for every $\alpha$. Being so, it would be expected that its deontic extension **DPI** should maintain this feature and so it would be possible to satisfy $\mathsf{O}\alpha$ for every $\alpha$. This is why the accessibility relation of its Kripke frames is not necessarily serial: it is possible to have an isolated world $w$ in which $v_w(\alpha) = 1$ (and, in particular, $v_w(\mathsf{O}\alpha) = 1$) for every $\alpha \in For^{\mathsf{O}}$.

The lack of a bottom formula (and so the absence of a consistency operator) prevents the definition of a strong (that is, a explosive) negation $\sim$ in both **PI** and **DPI**. Thus, the logic **DPI** is a relatively weak logic system.

### 2.2.3. The logic SDPI

From the last analysis, it seems that the logic **DPI** could be strengthened. A stronger deontic extension of **PI** is possible, closer to **SDL**, by allowing logical explosion from conflicting, deontically consistent obligations; in particular, this would allow the definition of a bottom formula $\perp$. It is enough to slightly modify **DPI** as follows:

**Definition 2.12.** The logic **SDPI** is defined over the language $\mathsf{O}, \boxminus, \neg, \wedge, \vee, \rightarrow$ by substituting in **DPI** axiom **(Dbc1)** by the following:

**(SDbc1)** $\boxminus\alpha \rightarrow (\mathsf{O}\alpha \rightarrow (\mathsf{O}\neg\alpha \rightarrow \beta))$ ■

The Kripke semantics for **SDPI** consists of the Kripke structures of **DPI** satisfying the following clause:

(kc5.2) $v_w(\mathsf{O}\alpha) = v_w(\mathsf{O}\neg\alpha) = 1$ implies $v_w(\boxminus\alpha) = 0$.

Note that (kc5.1) is a consequence of (kc5.2). It is straightforward to prove that **SDPI** is sound and complete w.r.t. its Kripke semantics.

It should also be noted that **SDPI** admits the definition of a bottom formula: indeed, $\bot_\alpha =_{def} (\mathsf{O}\alpha \wedge \mathsf{O}\neg\alpha) \wedge \boxminus\alpha$ is such that $\bot_\alpha \vdash_{\mathbf{SDPI}} \beta$ for every $\alpha$ and $\beta$. However, it is still possible to define a Kripke structure for **SDPI** with an isolated world $w$ such that $v_w(\mathsf{O}\alpha) = 1$ for every $\alpha$ (and, in particular, with $v_w(\mathsf{O}\bot_\alpha) = 1$). This is why the accessibility relation is not required to be serial for **SDPI**, despite having a bottom formula definable in it.

By using any bottom $\bot$, it is possible to define a strong (that is, explosive) negation in **SDPI** as $\sim\alpha =_{def} (\alpha \rightarrow \bot)$. Note that

$$\mathsf{O}\sim\alpha \vdash_{\mathbf{SDPI}} \mathsf{O}\neg\alpha \text{ but } \mathsf{O}\neg\alpha \nvdash_{\mathbf{SDPI}} \mathsf{O}\sim\alpha.$$

Moreover (and this is a general fact of paraconsistent logics having a bottom formula), **SDPI** is a LFI such that the consistency operator is defined *à la* da Costa as follows: $\circ\alpha =_{def} (\alpha \wedge \neg\alpha) \rightarrow \bot$. That is, $\circ\alpha = \sim(\alpha \wedge \neg\alpha)$.

**Theorem 2.13.** **SDPI** is both a LFI and a LDI. Moreover, conflicting obligations involving a deontically consistent sentence trivialize.

This shows that this system has a reasonable expressive power, despite being based on a LFI weaker than **mbC**. Arguably, **SDPI** could be considered as the minimal paraconsistent version of **SDL** or, more precisely, of **eSDL***, the version of **SDL** over $For^{\mathsf{O}\boxminus}$ obtained from **SDPI** by adding the axiom schema $\boxminus\alpha$. As expected, the accessibility relation of any Kripke structure for **eSDL*** must be serial.

To finalize, it can be observed that while **DmbC** and **SDPI** are both LFI's and LDI's, they are different in nature. While the former has the consistency operator $\circ$ as a primitive and the deontic consistency operator is derived, the latter follows the opposite way: the deontic operator $\boxminus$ is primitive, and the consistency operator is derived.

### 2.2.4. The logic $C_1^D$

Now we shall briefly analyze $C_1^D$. As mentioned above, $C_1^D$ is obtained from $C_1$ by adding some deontic axioms. Recall that $C_1$ is defined over the basic language $For$ (where $\alpha^\circ =_{def} \neg(\alpha \wedge \neg\alpha)$) by means of rule **(MP)**, axioms **(Ax1)-(Ax10)** (see Definition 1.2) and:

**(bC1)** $\alpha^\circ \to (\alpha \to (\neg\alpha \to \beta))$

**(ca1)** $(\alpha^\circ \wedge \beta^\circ) \to (\alpha \wedge \beta)^\circ$

**(ca2)** $(\alpha^\circ \wedge \beta^\circ) \to (\alpha \vee \beta)^\circ$

**(ca3)** $(\alpha^\circ \wedge \beta^\circ) \to (\alpha \to \beta)^\circ$

The last three axioms state that consistency is “propagated” through the connectives (it can be proved that $\alpha^\circ \to (\neg\alpha)^\circ$). Different to **mbC**, the classical negation is defined as $\sim\alpha =_{def} \neg\alpha \wedge \alpha^\circ$.

**Definition 2.14.** The logic $C_1^D$ is defined over language $For^{\mathsf{O}}$ by adding to $C_1$ the following:

**Axiom schemas:**

**(O-K)** $\mathsf{O}(\alpha \to \beta) \to (\mathsf{O}\alpha \to \mathsf{O}\beta)$

**(O-D)** $\mathsf{O}\alpha \to \sim\mathsf{O}\sim\alpha$

**(ca4)** $\alpha^\circ \to (\mathsf{O}\alpha)^\circ$

**Inference rule:**

**(O-Nec)** $\dfrac{\vdash \alpha}{\vdash \mathsf{O}\alpha}$ ■

Despite being formally analogous to axiom **(D)** of **SDL**, axiom **(O-D)** is equivalent to **(O-E)**$^\circ$ of **DmbC**, at it can be easily proved. Thus, the conception of $C_1^D$ is close to that of **DmbC**. It could be said that the modal extension of $C_1$ producing $C_1^D$ is in essence the same modal extension of **mbC** producing **DmbC**: the only additional modal axiom is **(ca4)**, concerning the preservation of consistency through the operator $\mathsf{O}$. But this makes sense, because the preservation of consistency through the connectives is a general principle of $C_1$. By defining deontic consistency as $\boxminus\alpha =_{def} \mathsf{O}(\alpha^\circ)$, it can be proved the following:

**Theorem 2.15.** $C_1$ is both a LFI and a LDI.

#### 2.2.5. The logic SDmbC

The next step is the presentation of deontic systems which are LFI's but are not LDI's. This is the case of **SDmbC** and **BDmbC**, introduced in (Peron & Coniglio 2008). As we shall see, the former is a paraconsistent logic deontically explosive; the latter is a bimodal deontic system, being deontically explosive with respect to one modality, and deontically paraconsistent with respect to the other.

**Definition 2.16.** The logic **SDmbC** is defined over $For^{\circ \mathsf{O}}$ by replacing in **DmbC** (recall Definition 2.6) axiom **(O-E)**$^\circ$ by

**(O-E)**$^*$ $\mathsf{O}\mathbf{f}_\alpha \rightarrow \bot_\alpha$ where $\mathbf{f}_\alpha = (\alpha \wedge \neg\alpha)$ and $\bot_\alpha = (\alpha \wedge \neg\alpha) \wedge \circ\alpha$ ■

It is worth noting that **SDmbC** is a paraconsistent deontic logic, but it is not a deontically paraconsistent logic (cf. Theorem 2.17 below). As in the case of **DmbC**, if the axiom schema $\circ\alpha$ is added to **SDmbC** then it is obtained **eSDL**, the version of **SDL** over $For^{\circ \mathsf{O}}$. This situation is analogous to the relation between **mbC** and **eCL**, the version of **CL** over $For^\circ$, described in Section 1.

With respects to the semantics for **SDmbC**, it is enough to extend the Kripke semantics for **DmbC** (cf. Definition 2.7) by adding to the valuation mappings the following clause:

(kc7) $v_w(\mathsf{O}\neg\alpha) = 1$ implies $v_{w'}(\alpha) = 0$ for every $w'$ in $W$ such that $wRw'$.

Clause (kc7) expresses that it is impossible to have conflicting obligations in a world $w$. As shown in (Peron 2009), the following results hold:

**Theorem 2.17.**
(i) **SDmbC** is a LFI.
(ii) **SDmbC** is deontically explosive and so it is not a LDI.

**Theorem 2.18.** **SDmbC** is sound and complete with respect to its Kripke semantics.

Another interesting feature of **SDmbC** is that the strong negation $\sim$ and the paraconsistent negation $\neg$ collapse within the scope of the deontic operator.

**Theorem 2.19.** $\mathsf{O}\neg\alpha \dashv\vdash_{\mathbf{SDmbc}} \mathsf{O}{\sim}\alpha$.

### 2.2.6. The logic BDmbC

The systems **DmbC** and **SDmbC** can be interesting in the analysis of deontic paradoxes, as we shall see in the next section. However, a logic combining both systems can enrich this analysis. Such a logic, called **BDmbC**, is obtained by combining **DmbC** and **SDmbC**, in a way that one deontic operator behaves as in **DmbC**, whereas the other behaves as in **SDmbC**.

The idea of considering paraconsistent bimodal deontic logics is not new. In Puga, da Costa & Carnielli 1988 a bimodal system defined over $C_1$ called $\mathbb{C}_1$ having a deontic operator $\mathsf{O}$ and an alethic operator $\Box$ was presented. Some interactions between both operators where forced, namely $\mathsf{O}\alpha \to \Diamond\alpha$ (where, as usual, $\Diamond\alpha =_{def} \neg\Box\neg\alpha$ is the "possibility" operator) and $\Box\alpha \to \mathsf{O}\alpha$. This principles are based on ideas by Kant and Hintikka, respectively. Since $\Box\alpha \to \alpha$ is an axiom of $\mathbb{C}_1$ then this logic is both a LFI and a LDI with respect to both modalities, where consistency and alethic consistency of $\alpha$ are expressed by the same sentence, namely $\alpha^\circ$; on the other hand, the deontic consistency of $\alpha$ is expressed by $\mathsf{O}(\alpha^\circ)$.

Our proposal, however, is different, by several reasons. On the one hand, both modalities of **BDmbC** are deontic, in contrast to $\mathbb{C}_1$ in which one modality is alethic. On the other hand, **BDmbC** is deontically explosive with respect to one modality and deontically paraconsistent with respect to the other, in contrast to $\mathbb{C}_1$ in which both modalities are not explosive, as mentioned above.

**Definition 2.20.** The logic **BDmbC** – *Bimodal Deontic mbC* – is defined in the set $For^{\circ\mathsf{O}\overline{\mathsf{O}}}$ of sentences generated by $\overline{\mathsf{O}}, \mathsf{O}, \circ, \neg, \wedge, \vee, \to$ by adding to **mbC** the following:

**Axiom schemas:**

**(O-K)** $\mathsf{O}(\alpha \to \beta) \to (\mathsf{O}\alpha \to \mathsf{O}\beta)$

**($\overline{\mathsf{O}}$-K)** $\overline{\mathsf{O}}(\alpha \to \beta) \to (\overline{\mathsf{O}}\alpha \to \overline{\mathsf{O}}\beta)$

**(O-E)$^\circ$** $\mathsf{O}\bot_\alpha \to \bot_\alpha$ where $\bot_\alpha =_{def} (\alpha \wedge \neg\alpha) \wedge \circ\alpha$

**($\overline{\mathsf{O}}$-E)** $\overline{\mathsf{O}}\mathbf{f}_\alpha \to \bot_\alpha$ where $\mathbf{f}_\alpha =_{def} \alpha \wedge \neg\alpha$

**(BA)** $\overline{\mathsf{O}}\alpha \to \mathsf{O}\alpha$

**Inference rules:**

**(O-NEC)** $\dfrac{\vdash \alpha}{\vdash \mathsf{O}\alpha}$

**($\overline{\mathsf{O}}$-NEC)** $\dfrac{\vdash \alpha}{\vdash \overline{\mathsf{O}}\alpha}$

■

The operator $\overline{\mathsf{O}}$ can be interpreted as "classically obligatory"; this notion is expressed in **($\overline{\mathsf{O}}$-E)**. On the other hand, $\mathsf{O}$ is interpreted as "weakly obligatory", as expressed in **(O-E)**$^{\circ}$. However, an additional interaction between both operators is intended: namely, it is convenient that $\overline{\mathsf{O}}$ be "stronger" or "stricter" than $\mathsf{O}$, which justifies the inclusion of **(BA)**. This axiom was inspired by the analogous one presented in the bimodal system **KT**$^{\boxdot}$, cf. Carnielli & Pizzi 2008.

The logic **BDmbC** suggest an additional analogy between LFI's and LDI's. Just as **mbC** is $\neg$-paraconsistent but it is not $\sim$-paraconsistent, the theorem below shows that **BDmbC** is $\mathsf{O}$-paraconsistent but it is not $\overline{\mathsf{O}}$-paraconsistent. The details of the proof are left to the reader.

**Theorem 2.21.**
(i) **BDmbC** is a LFI.
(ii) **BDmbC** is a LDI with respect to $\mathsf{O}$.
(iii) **BDmbC** is deontically explosive with respect to $\overline{\mathsf{O}}$ and so it is not a LDI with respect to $\overline{\mathsf{O}}$.

As expected, the semantics for **BDmbC** is given by Kripke structures having two accessibility relations, one for each modality. In details:

**Definition 2.22.** A *Kripke structure for* ***BDmbC*** is a tuple $\langle W, R, \overline{R}, \{v_w\}_{w\in W}\rangle$ where:

1. $W$ is a non-empty set (of *possible-worlds*);
2. $R \subseteq W \times W$ and $\overline{R} \subseteq W \times W$ are relations (of *accessibility*) between worlds which are serial;
3. $R \subseteq \overline{R}$;
4. $\{v_w\}_{w\in W}$ is a family of mappings $v_w : For^{\circ\mathsf{O}\overline{\mathsf{O}}} \to \mathbf{2}$ satisfying clauses (kc1-kc6) of Definition 2.7 plus the following clauses:

(kc6.1) $v_w(\overline{\mathsf{O}}\alpha) = 1$ iff $v_{w'}(\alpha) = 1$ for every $w'$ in $W$ such that $w\overline{R}w'$,

(kc7.1) $v_w(\overline{\mathsf{O}}\neg\alpha) = 1$ implies $v_{w'}(\alpha) = 0$ for every $w'$ in $W$ such that $w\overline{R}w'$. ■

The proof of the completeness of **BDmbC** with respect to its Kripke semantics can be found in Peron 2009. Since in **BDmbC** the operator $\mathsf{O}\alpha$ behaves as in **DmbC**, as long as $\overline{\mathsf{O}}$ has the same behavior as the deontic operator of **SDmbC**, it is easy to prove (by semantical means) the following result:

**Theorem 2.23.** In **BDmbC**:
$\overline{\mathsf{O}}\neg\alpha \dashv\vdash_{\mathbf{BDmbC}} \overline{\mathsf{O}}{\sim}\alpha$ holds,
but $\mathsf{O}\neg\alpha \dashv\vdash_{\mathbf{BDmbC}} \mathsf{O}{\sim}\alpha$ does not hold.

Of course it is possible to consider a plethora of paraconsistent deontic logics and LDI's by taking different LFI's and/or with combinations of **(O-E)** and **(O-E)**$^{\circ}$ (as it was done with **BDmbC**). In particular, in (Coniglio 2007) the system **DLFI1** based on the logic **LFI1** was considered. The latter is a LFI enjoying nice features, for instance some axioms propagating the inconsistency operator $\bullet$. Another interesting point about **LFI1** is that it has a simple 3-valued matrix semantics. The deontic system **DLFI1** is both a LFI and a LDI such that the deontic inconsistency operator $\boxtimes$ satisfies the following: $\boxtimes\alpha \longleftrightarrow (\mathsf{O}\alpha \wedge \mathsf{O}\neg\alpha)$. Moreover, $\boxtimes$ propagates under certain circumstances, which is an useful tool in the analysis of deontic paradoxes. The property of propagation of inconsistency resembles the propagation of consistency enjoyed by $C_1^D$. However, the underlying paraconsistent logics $C_1$ and **LFI1** are quite different: while the latter is a 3-valued logic, the former cannot be characterized by finite matrices. Moreover, the propagation properties are not the same.

A previous and related approach to the analysis of modal paradoxes by using modal LFI's can be found in Costa-Leite 2003, where a paraconsistent alethic system called $\mathbf{Ci}^T$ was proposed in order to analyze Fitch's paradox of knowability. The system $\mathbf{Ci}^T$ is is a modal extension of **Ci**, a LFI introduced in Carnielli & Marcos 2002.

Instead of defining more and more paraconsistent deontic systems and LDI's, the next section will analyze a classical deontic paradox, the so-called Chisholm's paradox, in the context of some of the system considered in this section.

## 3. Chisholm's Paradox

In general, a deontic paradox consists of a set of sentences in natural language, intuitively consistent and without logical dependencies such that, when formalized in a deontic language, it is logically trivial or it has logical dependencies. This situation occurs more frequently when the given sentences refer to norms, laws and moral principles: in this case contradictions can arise. Thus, most of such paradoxes are originated when the given set of premises is formalized in **SDL** as a set $\Gamma$ of logically independent sentences, but conflicting obligations such as $\mathsf{O}\alpha$ and $\mathsf{O}\neg\alpha$ are derived from $\Gamma$ within **SDL**. As pointed out in (McConnell 2006), the problem is related to the difficulties of **SDL** for dealing with *contrary-to-duty obligations*.

In this section, the well-known Chisholm's deontic paradox will be analyzed under the light of some LDI's discussed in the previous section. The aim is to show to what extent the paraconsistentist approach to deontic paradoxes can be fruitful.

One of the first deontic paradoxes was proposed in Chisholm 1963. The following formulation of it was given in Åqvist 2002.

Consider the set $\mathscr{C}$ formed by the following sentences:

(1) It ought to be that John does not impregnate Suzy Mae.

(2) Not-impregnating Suzy Mae commits John to not marrying her.

(3) Impregnating Suzy Mae commits John to marry her.

(4) John impregnates Suzy Mae.

Let $\mathtt{A}$ and $\mathtt{B}$ be propositional letters representing the sentences "John impregnates Suzy Mae" and "John marries Suzy Mae", respectively. It is possible to formulate the set (1)–(4) above in a deontic language in several ways. In order to simplify the presentation, the following notions will be useful.

**Definition 3.1.** Consider in **Dmbc** the following derived deontic operators:
(i) $\mathsf{F}_1\alpha =_{def} \mathsf{O}\neg\alpha$ (*prima-facie* prohibition)
(ii) $\mathsf{F}_2 =_{def} \mathsf{O}{\sim}\alpha$ (strong prohibition) ■

The name of such operators is inspired by Ross 1930.

Returning to the set $\mathscr{C}$, it is clear that (1) admits two formulations in **Dmbc**: $\mathsf{F}_1\mathtt{A}$ and $\mathsf{F}_2\mathtt{A}$. By its turn, (2) has three interpretations: $\neg\mathtt{A} \to \mathsf{F}_1\mathtt{B}$, $\neg\mathtt{A} \to \mathsf{F}_2\mathtt{B}$ and $\mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B})$. Analogously, sentence (3) can be formalized as $\mathtt{A} \to \mathsf{O}\mathtt{B}$ or $\mathsf{O}(\mathtt{A} \to \mathtt{B})$. Being so, there exist 12 possibilities to formalize the set $\mathscr{C}$, which are conveniently arranged below:

$$\Gamma_{1.1} = \{\mathsf{F}_1\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}_1\mathtt{B}, \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{1.2} = \{\mathsf{F}_1\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}_2\mathtt{B}, \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{1.3} = \{\mathsf{F}_2\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}_1\mathtt{B}, \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{1.4} = \{\mathsf{F}_2\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}_2\mathtt{B}, \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{2.1} = \{\mathsf{F}_1\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{2.2} = \{\mathsf{F}_2\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{3.1} = \{\mathsf{F}_1\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathsf{O}(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$$\Gamma_{3.2} = \{\mathsf{F}_2\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathsf{O}(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$$\Gamma_{4.1} = \{\mathsf{F}_1\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}_1\mathtt{B}, \mathsf{O}(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$$\Gamma_{4.2} = \{F_1\mathtt{A}, \neg\mathtt{A} \to F_2\mathtt{B}, O(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$$\Gamma_{4.3} = \{F_2\mathtt{A}, \neg\mathtt{A} \to F_1\mathtt{B}, O(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$$\Gamma_{4.4} = \{F_2\mathtt{A}, \neg\mathtt{A} \to F_2\mathtt{B}, O(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

Clearly some of the sets above are not appropriate because some logical dependencies are present. This is exactly the paradoxical aspect of the set $\mathscr{C}$, because it is consistent and the sentences are logically independent. Now we will analyze Chisholm's paradox in some of the deontic systems defined above.

### 3.1. Analysis in SDL

Given that in **SDL** just one 'prohibition' operator is definable in **SDL** (because just classical negation is present) then the unique candidates to formalize $\mathscr{C}$ in **SDL** are $\Gamma_{1.4}$, $\Gamma_{2.2}$, $\Gamma_{3.2}$ and $\Gamma_{4.4}$. But the following laws hold in **SDL**:

$$\vdash_{\mathbf{SDL}} \alpha \to (\neg\alpha \to O\beta) \;\text{ and }\; \vdash_{\mathbf{SDL}} O\neg\alpha \to O(\alpha \to \beta).$$

Then, (2) and (4) are logically dependent in $\Gamma_{1.4}$; (1) and (3) are logically dependent in $\Gamma_{3.2}$; by its turn, (1) and (3), on the one hand, and (2) and (4), on the other, are logically dependent in $\Gamma_{4.4}$.

So, the unique alternative in **SDL** is $\Gamma_{2.2}$. But then $O\mathtt{B}$ follows by **(MP)** between (3) and (4). On the other hand, $\vdash_{\mathbf{SDL}} O(\neg\mathtt{A} \to \neg\mathtt{B}) \to (O\neg\mathtt{A} \to O\neg\mathtt{B})$ and so, by **(MP)** two times it follows $O\neg\mathtt{B}$. That is, the set $\Gamma_{2.2}$ is logically trivial in **SDL**, arriving so to a paradox.

### 3.2. Analysis in DmbC

Moving now to **DmbC**, there are much more alternatives for choosing a set $\Gamma_{i.j}$, because there are less logical interdependencies.

By taking appropriate Kripke structures, it is easy to prove that

$$\nvDash_{\mathbf{DmbC}} \alpha \to (\neg\alpha \to O\neg\beta) \quad\text{and}\quad \nvDash_{\mathbf{DmbC}} \alpha \to (\neg\alpha \to O{\sim}\beta).$$

Moreover, by using Kripke structures it is easy to see that the sentences in $\Gamma_{1.1}$, $\Gamma_{1.2}$, $\Gamma_{1.3}$ and $\Gamma_{1.4}$ are logically independent in **DmbC**. The independence of the premises in $\Gamma_{2.1}$, $\Gamma_{2.2}$, $\Gamma_{3.1}$, $\Gamma_{4.1}$ and $\Gamma_{4.2}$ is obtained by analogous arguments. By **(O-K)** and **(MP)**, $O\mathtt{B}$ is derived from $\Gamma_{1.j}$ (for $1 \leq j \leq 4$) and also from $\Gamma_{2.1}$ and $\Gamma_{2.2}$. By a similar argument, $O\neg\mathtt{B}$ is derived from $\Gamma_{i.j}$ (for $i.j = 2.1, 2.2, 3.1$). This means that $\Gamma_{2.1}$ and $\Gamma_{2.2}$ are deontically contradictory.

Concerning the set $\Gamma_{3.2}$, note that $\vdash_{\mathbf{DmbC}} \sim\mathtt{A} \to (\mathtt{A} \to \mathtt{B})$ and then, by **(O-Nec)** and **(O-K)** there is a dependence between (1) and (3). Then, this set cannot be used for the analysis. By the same reason the sets $\Gamma_{4.3}$ and $\Gamma_{4.4}$ must be discarded.

Altogether, there are nine sets logically independent in **DmbC** formalizing $\mathscr{C}$ and so being potentially useful for the analysis of Chisholm's paradox, in contrast to **SDL** which have just one. Remarkably, none of them are logically trivial and so the nine sets are relevant for the analysis. It is easy to prove by using Kripke structures that $\Gamma_{i.j}$ does not deduce $\mathsf{O}\neg\mathtt{B}$ for $i.j = 1.1, 1.2, 1.3, 1.4, 4.1, 4.2$. By its turn, $\Gamma_{i.j}$ does not deduce $\mathsf{O}\mathtt{B}$ for $i.j = 3.1, 4.1, 4.2$. Thus, all these sets do not infer conflicting obligations. Finally, $\Gamma_{i.j}$ deduce conflicting obligations for $i.j = 2.1, 2.2$, but there is no deontic explosion (and so there is no logical explosion).

Summarizing, in **DmbC** we have the following scenarios for Chisholm's paradox:

$$\Gamma_{i.j} \vdash_{\mathbf{DmbC}} \mathsf{O}\mathtt{B} \;\text{ but }\; \Gamma_{i.j} \nvdash_{\mathbf{DmbC}} \mathsf{O}\neg\mathtt{B} \;\text{ for } i.j = 1.1, 1.2, 1.3, 1.4$$

$$\Gamma_{3.1} \vdash_{\mathbf{DmbC}} \mathsf{O}\neg\mathtt{B} \;\text{ but }\; \Gamma_{3.1} \nvdash_{\mathbf{DmbC}} \mathsf{O}\mathtt{B}$$

$$\Gamma_{i.j} \vdash_{\mathbf{DmbC}} \mathsf{O}\mathtt{B}, \mathsf{O}\neg\mathtt{B} \;\text{ without trivializing deontically, for } i.j = 2.1, 2.2$$

$$\Gamma_{i.j} \nvdash_{\mathbf{DmbC}} \mathsf{O}\mathtt{B} \;\text{ and }\; \Gamma_{i.j} \nvdash_{\mathbf{DmbC}} \mathsf{O}\neg\mathtt{B} \;\text{ for } i.j = 4.1, 4.2$$

That is, **DmbC** has enough expressive power to encompass all the possible situations, namely:

- just $\mathsf{O}\mathtt{B}$ is derived;
- just $\mathsf{O}\neg\mathtt{B}$ is derived;
- both $\mathsf{O}\mathtt{B}$ and $\mathsf{O}\neg\mathtt{B}$ are derived, without having deontic explosion (and so, without trivializing);
- neither $\mathsf{O}\mathtt{B}$ nor $\mathsf{O}\neg\mathtt{B}$ are derived.

### 3.3. Analysis in SDmbC

As happens in **SDL**, in the logic **SDmbC** the deontic operators $\mathsf{F}_1$ and $\mathsf{F}_2$ collapse; let $\mathsf{F}$ be the unique prohibition operator in **SDmbC**. Thus, there are just four possibilities in **SDmbC** formalizing $\mathscr{C}$, namely:

$$\Gamma_{0.1} = \{\mathsf{F}\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}\mathtt{B}, \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{0.2} = \{\mathsf{F}\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathtt{A} \to \mathsf{O}\mathtt{B}, \mathtt{A}\}$$

$$\Gamma_{0.3} = \{\mathsf{F}\mathtt{A}, \mathsf{O}(\neg\mathtt{A} \to \neg\mathtt{B}), \mathsf{O}(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$$

$\Gamma_{0.4} = \{\mathsf{F}\mathtt{A}, \neg\mathtt{A} \to \mathsf{F}\mathtt{B}, \mathsf{O}(\mathtt{A} \to \mathtt{B}), \mathtt{A}\}$

It is easy to see that in **SDmbC** it holds:

$$\nvDash_{\mathbf{SDmbC}} \alpha \to (\neg\alpha \to \mathsf{O}\beta) \quad \text{but} \quad \vDash_{\mathbf{SDmbC}} \mathsf{O}\neg\alpha \to \mathsf{O}(\alpha \to \beta)$$

and so $\Gamma_{0.3}$ and $\Gamma_{0.4}$ are discarded because there are logical dependencies among the premises. On the other hand, $\Gamma_{0.2}$ trivializes in **SDmbC**, because this system does not accept conflicting obligations. Thus, the only admissible set is $\Gamma_{0.1}$ which satisfies the following:

$\Gamma_{0.1} \vdash_{\mathbf{SDmbC}} \mathsf{O}\mathtt{B}$ but $\Gamma_{0.1} \nvdash_{\mathbf{SDmbC}} \mathsf{O}\neg\mathtt{B}$.

That is, Chisholm's paradox can again be avoided, producing the expected result: John has to marry Suzy Mae.

### 3.4. Analysis in DPI and in SDPI

Since **DPI** admits just one negation, namely the paraconsistent one, just the sets $\Gamma_{0.j}$ (for $j = 1, \ldots, 4$) of **SDmbC** are obtained, with $\mathsf{F}\alpha = \mathsf{O}\neg\alpha$. As expected, $\Gamma_{0.1}$ just derives $\mathsf{O}\mathtt{B}$; $\Gamma_{0.3}$ just derives $\mathsf{O}\neg\mathtt{B}$; $\Gamma_{0.2}$ derives both $\mathsf{O}\mathtt{B}$ and $\mathsf{O}\neg\mathtt{B}$ without deontic explosion (and so without trivializing); and $\Gamma_{0.4}$ neither derives $\mathsf{O}\mathtt{B}$ nor derives $\mathsf{O}\neg\mathtt{B}$.

Concerning **SDPI**, the same results obtained for **DmbC** hold in this logic, *mutatis mutandis*.

### 3.5. Analysis in BDmbC

The logic **BDmbC** validates all the inferences in **DmbC** about $\mathsf{O}$, as well as the inferences of **SDmbC** about $\overline{\mathsf{O}}$. Moreover, three prohibition operators can be defined: $\mathsf{O}\neg\alpha$, $\mathsf{O}{\sim}\alpha$ and $\overline{\mathsf{O}}\neg\alpha$. Since two obligation operators are available, namely $\mathsf{O}\alpha$ and $\overline{\mathsf{O}}\alpha$, there are much more alternatives to formalize the set $\mathscr{C}$. Being an extension of **DmbC**, all the possible situations for the paradox described in Subsection 3.2 can also be obtained in **BDmbC**. However, since there are two kinds of obligations in **BDmbC**, there are strictly more possible situations than in **DmbC**. Namely, the following obligations concerning $\mathtt{B}$ are obtained from the different sets of premises:

- just $\mathsf{O}\mathtt{B}$ is derived;
- just $\overline{\mathsf{O}}\mathtt{B}$ (and so $\mathsf{O}\mathtt{B}$) is derived;
- just $\mathsf{O}\neg\mathtt{B}$ is derived;
- just $\overline{\mathsf{O}}\neg\mathtt{B}$ (and so $\mathsf{O}\neg\mathtt{B}$) is derived;

- $\mathsf{O}$B and $\mathsf{O}\neg$B are derived, without having deontic explosion;
- $\overline{\mathsf{O}}$B (and so $\mathsf{O}$B) and $\mathsf{O}\neg$B are derived, without having deontic explosion;
- neither $\mathsf{O}$B (nor $\overline{\mathsf{O}}$B) nor $\mathsf{O}\neg$B (nor $\overline{\mathsf{O}}\neg$B) are derived;
- everything is derived (logical explosion).

### 3.6. Summarizing

To summarize, all the plausible answers to the question "Should John marry Suzy Mae?" are obtained in **DmbC**, **DPI** and **SDPI**. In the case of **BDmbC**, there are still more variants for the answers (even the logical explosion can be attained), as shown above. On the other hand, **SDmbC** produces just one answer, the intuitively expected one. In contrast, **SDL** just produces logical explosion (or logical dependencies).

The main point of our analysis lies in the fact that Chisholm's paradox is not exclusively based on the violation of the Principle of Deontic Consistency as presented frequently in the literature (this position is defended, for instance, in (Prakken & Sergot 1994)). The analysis above suggests that the logical dependence between the premises in $\mathscr{C}$ (dependency arising because of the use of classical logic) plays an important role. Thus, in **SDmbC** the principle of deontic explosion is maintained while rejecting the principle of explosion, and then it is possible to avoid the paradox. That is, by eliminating some logical dependencies typical of classical logic, the paradox vanishes. In the case of **DmbC**, **DPI** and **SDPI**, both principles of explosion are weakened and so there are more possibilities to avoid the paradox. Finally, **BDmbC** combines **DmbC** and **SDmbC**, obtaining still more solutions to the paradox by combination of its deontic operators.

## 4. Final remarks

Our analysis departs from the transposition of the approach to paraconsistency of LFI's to the deontic context. This allows to consider notions such as deontic explosion and deontic paraconsistency.

The Logics of Deontic (In)Consistency seem to encompass several paraconsistent deontic systems already proposed in the literature, suggesting the definition of a taxonomy of such systems, in a similar way as the proposal for LFI's started in Carnielli & Marcos 2002.

The approach to Chisholm's paradox presented here is based on the generality of LDI's, and so it is possible to obtain solutions to the paradox based on the lack of logical dependencies and/or the elimination of the principle of deontic explosion. The distinction between the two sources of paradox can be useful to analyze other deontic paradoxes, as those presented in Prakken & Sergot 1994, Prakken & Sergot 1997 and Carmo & Jones 1997.

Clearly, the research on paraconsistent deontic logic and deontic paradoxes has several possibilities of further development. We hope that our discussion can contribute to some extent to the analysis of the interesting question of deontic paradoxes and related areas.[1]

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**Resumo.** As *Lógicas da (In)Consistência Deôntica* (LDI's) podem ser consideradas como sendo a contraparte deôntica das lógicas paraconsistentes chamadas de *Lógicas da (In)Consistência Formal*. Neste artigo são introduzidas e estudadas novas LDI's e outras lógicas deônticas paraconsistentes satisfazendo diferentes propriedades: sistemas tolerantes a obrigações contraditórias; sistemas em que as obrigações contraditórias produzem trivialização; e uma lógica deôntica paraconsistente bimodal que combina as características de sistemas previamente introduzidos. Estas lógicas são utilizadas para analisar o conhecido paradoxo de

Chisholm aproveitando-se do fato de que, além que as obrigações contraditórias não trivializam nas LDI's, varias das dependências lógicas da lógica clássica são bloqueadas no contexto das LDI's, permitindo assim dissolver o paradoxo.

**Palavras-chave:** Lógica paraconsistente, lógica deôntica, paradoxos deônticos, paradoxo de Chisholm, dilemas morais, lógicas da inconsistência formal.

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