EPISTEMOLOGIC CONTROVERSY ON QUANTUM OPERATORS OF SPACE AND TIME

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Abstract. Since the very begining of quantum theory there started a debate on the proper role of space and time in it. Some authors assumed that space and time have their own algebraic operators. On that basis they supposed that quantum particles had "coordinates of position", even though those coordinates were not possible to determine with infinite precision. Furthermore, time in quantum physics was taken to be on an equal foot, by means of a so-called "Heisenberg's fourth relation of indeterminacy" concerning time and energy. In this paper, the proper role of space and time in the core of non-relativistic quantum phsysics is analyzed. We will find that, rigorously, that relation for time and energy shows a different root. For the role of space, it will be discussed that there is no "coordinate of position" in the conceptual structure of quantum physics because quantum particles are not point-like objects.

Keywords: Space, time, quantum operators, geometric coordinates, dynamical variables.

1. Introduction

As a departure point, we will assume that an amorphous spacetime must possess intrinsically all of those properties which it clearly must possess but which are not due to the influence of matter. In effect, we are suggesting that the way to discover the intrinsic properties of spacetime is by taking out all those properties which happen to be effects of matter-energy. Now the latter are precisely those features which are represented by the affine and metric properties of the geometric description of the universe. When these are removed, one is not left with a void or strict nonentity, as relationalists would contend, but with the topological structure of a *differentiable manifold* M, taking M to correspond to the amorphous space-time continuum which seems to be presupposed in Grünbaum's philosophy of spacetime.

A differentiable manifold may be viewed as a homogeneous and continuous collection of points on which it is possible to define various differentiable functions. It has a dimensionality and may be coordinatized. Thus, every point may be labelled by an n-tuple of numbers x_i. Lines or curves may be defined on it, although they would lack an intrinsic length. A scalar field could be constructed by associating each point with an invariant quantity, $\Phi(X_i)$. One may also construct a covariant vector field, the gradient of Φ, by differentiating the scalar field: $\partial\Phi/\partial X_i = \Phi_i = A_i$.

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It would also be possible to construct contravariant vectors (tangent vectors) although this requires higher mathematical techniques which are beyond the scope of this paper. Clearly, such a vector could not be treated as a directed line between two points, since such a line lacks a definite length. Furthermore, the raising and lowering of indices would be out of the question in the absence of a fundamental or metric tensor.

The differentiation of a vector field requires an affinity, which is not now at our disposal. Although, therefore, the derivatives of vectors cannot, in general, be defined on a bare differentiable manifold, there is one differential operation which does make sense in the present context. This operation is somewhat analogous to the divergence $\nabla \cdot u$. It is called the curl of a vector field and is classically symbolized by $\nabla \times u$. In component notation, it is defined by $curl\ u = \varepsilon_{ijk} \nabla_j u_k$.

The reason that the curl of a vector is available to us is that $\varepsilon_{ijk} \nabla_j u_k$ is equivalent to $u_{j;k} - u_{k;j}$ where ";" expresses as usual the covariant derivative. It is easy to verify that when this is written in its expanded elementary form, the terms which depend on the affinity cancel out. If u_k is the gradient of Φ, then the curl of u_k will be $\varepsilon_{ijk} \nabla_j \nabla_k \Phi$. Since $\varepsilon_{ijk} = -\varepsilon_{ikj}$ and since j and k are dummy indices, which are therefore interchangeable, it follows that $\varepsilon_{ijk} \nabla_j \nabla_k \Phi = -\varepsilon_{ikj} \nabla_k \nabla_j \Phi$. But this is only possible if the latter is identically zero. It follows that the curl of a gradient always vanishes.

The curl may be physically interpreted in terms of the circulation of a vector field. Consider, for example, the velocity field of a fluid. If an object is allowed to float on the surface of the fluid and is carried along by it without rotating, it follows that the net effect of the velocities or momenta of the various fluid particles around a closed path is zero. Now if this were the case everywhere, the velocity field would be said to have zero circulation. In general, however, the circulation of a vector field will have different values around different closed paths. If by the familiar limiting process we consider the circulation around an infinitesimal area, we arrive at the notion of the curl of a field. Intuitively, it is its circulation at a point and may be thought of as a minute vortex. A field whose curl is identically zero is said to be irrotational or to have zero vorticity. You may recall that a zero divergence is an indication of the absence of a so urce. The vanishing of the curl is to be similarly interpreted, although the source in the latter case would be of the vectorial rather than scalar variety.

Now that we have the rudiments of the mathematical theory of differentiable manifolds, we proceed to consider the nature of the entity, empty spacetime, with which it is to be coordinated. Now we must begin the speculative part of our venture. The points of M are associated with events or, perhaps, the loci of events. Similarly, the curves of M are the loci ofthe world-lines of particles. The importance of this is that matter and events are not required to "create" spacetime but are located in an ontologically pre-existing spacetime. Material events do not constitute the intervals of spacetime but the measure of such intervals. In the absence of events in the physical sense, spacetime may be assumed to be homogeneous and isotropic. Hence, although one might choose to ascribe a scalar field to it, such a field would be constant. Hence, spacetime has a zero gradient and a zero curl everywhere. In the absence of the derivatives of vectors, it follows that linear velocities and accelerations cannot exist in relation to spacetime but only in relation to matter. To that extent, the doctrine of the relativity of motion is vindicated.

On the other hand, the curl is closely associated with the notion of angular velocity or rotation. While a linear velocity is simply undefined for empty spacetime, an angular velocity may be defined in terms of the curl and even given the determinate value of zero. It should be appreciated the profound difference between the claim that a certain geometric object is undefined and the claim that such an object has an identically vanishing value. Analogously, in a flat affine space, the affinity vanishes, but that is not to say that flat space lacks an affinity. Accordingly, one may make an objective claim to the effect that spacetime does not rotate. Consequently, it is possible to assign an absolute significance to the rotation of matter——namely, that a material object may have a non-zero angular velocity with respect to irrotational spacetime. We see, therefore, that Newton's bucket experiment was not as ill-conceived as Mach claimed. More significant, however, is that a reason for the failure of general relativity to account fully for the effects of rotation along Machian, i.e. strictly relationalist, lines can be provided in terms of our spacetime ontology. By the same token, the invariant rest-mass of matter, which cannot be accounted for along relationalist lines, can be made sense of in the proposed ontological framework.

All of our discussions are donde—and will be done—presupposing classical logic and mathematics, and since they are timeless we should note that our attempt aims to capture an intuitive idea of time and space in order to finally achieve a proper formalism able to describe them in agreement with our present physical knowledge. We start with the basic structure of a topological differentiable manifold and gain insight into our quest for some mathematically refined counterparts for our primitive ideas of space and time. The minimally structured manifold we have at the begining can be endowed with additional features to arrive at some other kind of physical manifold; that is to say, the Newtonian absolute space and time, or the Minkowskian space-time for special relativity.

Thus, the ultimate question to be faced by a philosophy of spacetime—namely, whether spacetime is nothing but a system of relations among material events or whether such events presuppose spacetime for their very possibility—may finally, although not dogmatically, be resolved. Our picture of the world includes both relational and absolute aspects. In brief, all those properties which depend on the existence of a metric affinity may be deemed relational. The manifold properties of spacetime such as dimensionality, continuity and differentiability are absolute. The most relevant remark is obvious: spacetime exists.

2. The birth of a non-mechanical "mechanics"

It could be thought that some of the paradoxes typical in non-relativistic quantum physics perhaps have their origin in the specific role of time as a physical quantity in the quantum theory. In fact this is a very debated matter. It was even pointed out by Von Neumann (Von Neumann 1955, p. 354):

> (...) an essential weakness that is, in fact, the main weakness of the quantum mechanics: its non-relativistic character, which distinguishes time t of the three space coordinates x, y, z, and presupposes a concept of objective simultaneity. In fact, while all the other quantities (especially those x, y, z closely connected with t by Lorentz transformations) are represented by operators, an ordinary numeric parameter corresponds to the time t, just as in the classic mechanics.

There is no doubt that the elementary quantum theory was elaborated in a non -concordant format with the einsteinian relativity, but it is erroneous to suppose that the space coordinates are represented by means of operators. The truth is that t is given the same treatment as space coordinates (x, y, z); none of those magnitudes possesses a functional operator associated to them. Let us see the reason of this.

Quantum theory was developed following the same recipes as hamiltonian mechanics because of two crucial reasons, partly historical and partly logical. Since the victory in the XVIII century of the newtonian natural philosophy, most of the scientists had the conviction that any fundamental theory of matter—and, generally speaking, of the whole physics—had to be some kind of "mechanics"; that is to say, a group of equations of motion for particles that interact among them obeying some law of forces that could be more or less complex. The field notion introduced later on by Faraday in the XIX century did not substantially change the situation. In fact, the author who mathematically elaborated Faraday's ideas, the famous James Clerk Maxwell, obtained his equations imagining a mechanical ether subjected to Newtonian laws. It was natural, therefore, that the new theory of quanta were called "quantum mechanics", a not very fortunate name for a parcel of physics born with the purpose of explaining spectroscopic series, distributions of electromagnetic frequencies, specific heats, and a diversity of quantities deeply far away from the genuine mechanical magnitudes.

On the other hand, and also in the XIX century, the mathematical methods of Hamilton demonstrated to embrace the mechanical description of particles as much as that of waves too. This supposed an absolute novelty in the traditional procedures of classic mechanics. Many authors have often speculated if with something more than perspicacity—as though Hamilton had lacked it—the brilliant Irishman had been able to take a step more and to discover himself the mathematical framework of the future quantum theory. It does not seem that the state of things were so simple (Goldstein 1990, p. 596):

> (...) It has been said that if Hamilton has advanced a little more, he would have discovered the equation of Schroedinger. It is not this way; he lacked experimental authority to give that jump. In the days of Hamilton it was considered that classical Mechanics was rigorously true and justifications based on the experience to consider that just an approach to a broader theory did not exist. (...)

Neverthless, that experimental arguments did exist in the begining of the XXth century. So, it was almost unavoidable to appeal to hamiltonian formalism in the nascent quantum physics, as long as its physical referents—the quantum objects—manifested as much a corpuscular as a wave-like behaviour. And it was made this way, decorating the analytic mechanics of Hamilton with Von Neumann's algebra of operators on the functional space of Hilbert. This cocktail of famous names often darkens the relative simplicity of the situation.

In the analytic mechanics of Hamilton, a system of n particles is described by means of $3n$ couples of conjugated dynamic variables that are usually represented as q_k and p_k in its canonical form. These variables obey the relationships expressed in the Poisson brackets:

$$\{q_k, p_l\} = \delta_{kl}; \{q_k, q_l\} = \{p_k, p_l\} = 0.$$

We define with it a $6n$-dimensional point in the space of phases of the system. And evolution in time is characterized by means of the hamiltonian functional of these dynamic variables, $H = H(q_k, p_l)$ that are assumed to be not explicitly dependent of t:

$$dq_k/dt = \{q_k, H\}, dp_l/dt = \{p_l, H\}.$$

In all the physical theories, except in General Relativity, it is supposed that space and time (or the space-time if we speak about Special Relativity) constitute a merely passive stage in which the natural processes occur, a sort of indispensable inert background to describe the physical phenomena. This makes necessary to distinguish among the space-time coordinates (t, x, y, z) and the dynamical variables q_n and p_n corresponding to the space of phases. The first ones, on their own, are good to mathematically label the different points of space and time; this is the reason why they could be denominated "field variables", although they were only acting with respect to a metric field that define distances among these points. On the contrary, q_n and p_n are dynamical variables associated, for example, to the position and the impulse of a certain physical object. They do not label in a generic way all the points of a continuous manifold (as space-time) used as basic framework to formalize our theories; they only refer to the points that an specific physical object occupies in fact. Otherwise, q_n and p_n specify the states of specific material systems, while coordinates t, x, y, z characterize the continuous space-time adopted as our background manifold where we can immerse this specific material systems.

Now we find easier to distinguish, firstly, among the variable of position for a point-like particle, q_x, and the space coordinate of the point that this particle occupies in a certain instant, x. It is true that we have the algebraic relationship $q_x = x$ (and similarly for the rest of coordinates), but we must make a difference between the point-like particle (as a physical entity endowed with mass-energy, position, speed and acceleration) and the geometric coordinate x of a fixed point in a preexistent empty space.

We cannot forget the essential role carried out by symmetries in our understanding of physical laws. This laws are not modified when we change the position of the origin of our reference system (space-time symmetry of displacements), neither when we rotate their axes a certain angle (space symmetry of rotations). The Lorentz transformations add the equivalence of systems in relative inertial motion, what is put forwards in the symmetry of space-time rotations.

But again we must underline an outstanding distinction: the mandatory fulfilment of certain symmetry requirements in nature, only concerns to the physical laws (that is to say, to the formal representation of the entirety of phenomena and physically permissible processes), not necessarily to the individual and specific physical systems. Material systems will show a lot of situations that, because of the asymmetry of their configuration, for instance, will not be rotationally symmetrical. And it does not mean that the rotational symmetry of natural laws has been infringed.

The symmetries of spatial displacements are generated by means of the total impulse, P, time symetries depend on total energy (although we should not assume that H always stands for the total energy of a physical system), H, and the generator of rotational symmetries is the total angular momentum, J. If all these symmetries are on an equal foot, we may wonder about the priority usually given to hamiltonian functional, H, representing the time evolution of the physical systems. In our description of nature, what is the priority of time displacements upon the spatial ones owed to? The answer must be sought in the historical tradition of analytical mechanics, mostly engaged to the study of point-like masses and rigid bodies, all which trivially transform under spatial displacements. However, the case of classical fields (electromagnetic, distribution of speeds or densities in a fluid, etc.) is very different, because those displacements in space are anything but trivial. In such situations, P and H get the same importance; so, in Special Relativity energy and linear momentum constitute the components of a tetravector in space-time.

The strong formal similarity among the q_x behavior and of x under time displacements and spatial rotations, has notably darkened the background differences between both magnitudes. And the use of the notation x for the particles position (equally for the other coordinates) has still carried bigger confusion to such an extent that we will hardly find many textbooks where the distinction among both variables is explicitly pointed out. Even more, the bold efforts of some authors to include the time coordinate, t, as conjugated canonical variable of H, were bound to be a failure (provided that we stay inside the orthodox hamiltonian scheme). The hamiltonian functional H depends on the original canonical variables, q_n and p_n (and sometimes also on t); therefore time cannot be itself an independent canonical variable. The mistake is consequence, again, of confusing space-time coordinates (a mathematical label assigned to the points of the space-time) with dynamical variables (states that characterize the physical systems located in space-time).

3. Space and time in quantum physics

In the elementary quantum theory the situation is completely similar: the existence of a space-time background, continuous and inert, is presupposed, the points of which are specified by means of space-time coordinates that are classical variables without dispersion ("c-numbers" of Dirac). The symmetries and space-time transformations are expressed in terms of such coordinates. The dynamical variables, on the other hand, are indeed quantized, due to which they are substituted by self-adjoint operators in a Hilbert space. All the hamiltonian formulae conserve their validity by only replacing the Poisson brackets for quantum commutators, according to the well-known rule $\{A, G\} \rightarrow (i\hbar)^1[\hat{A}, \hat{G}]$. In particular, the canonical variables are substituted by operators that obey the following commutation relationships:

$$\lfloor \hat{q}_k, \hat{p}_l \rfloor = i\hbar\delta_{kl}; \lfloor \hat{q}_k, \hat{q}_l \rfloor = \lfloor \hat{p}_k, \hat{p}_l \rfloor = 0.$$

We arrive now at one of the basic keys of this controversy: the substitution of dynamical variables for operators and Poisson brackets for quantum commutators, expose the inherent limitations of representing quantum estates that are in no way associated to point-like objects by means of typical pointlike-particle magnitudes. In fact, we usually preserve the notation q_i for the cartesian component of the position of the quantum particle, considered as a material point, and similarly for the momentum components, p_j. But it happens to be that p_j has its corresponding differential operator, unlike the variable q_x, which is replaced by the so-called "multiplication operator", $x \cdot (\,)$. This last one is not a genuine operator because of its lack of true self-functions. The Dirac-deltas are not even authentic functions in a rigorous mathematical sense, for which no quantum state can be developed as linear superposition of self-functions of the operator position (Jammer 1966, 1974).

The linear momentum operator does not suffer from the precedent complications because the notion of speed, or the linear momentum, is compatible as much with ideally point-like objects (the point-like mass of the classical mechanics) as with extensive entities (an ideal plane wave). However, the dynamical variable q_i only applies to classical objects ideally reducible to a point, which is impossible for quantum ones. For that reason we speak about the propagation speed of a plane wave, but we do not speak about its point-like position; as we do know in geometric optics, the non-wave limit of a plane wave is a ray, not a point.

A big amount of texts on elementary quantum physics open the explanations considering only a mass-point, what implicates an error from the very begining. We know that quantum objects are spatially extensive entities (That is why the usual name is "electronic field"—insted of "electrons"—or "material fields" in general), the time-dependent wave function is not written $\Psi(q_x, t)$, as we could expect, but $\Psi(x, t)$, where x denotes not the instantaneous position of a pointlike corpuscle, but a geometric coordinate that embraces the whole space. And it must be so because a free quantum object is represented by means of an infinite plane wave. The function of quantum state, in fact, is a magnitude still located in a higher level of abstraction, whose formal characterization is given in a functional space of Hilbert with an specific algebra. Anyway, the usual notation, in which x and t appear in an equal foot as arguments of the function Ψ, incites us to wonder why t is not an operator like x. The answer, obviously, comes on remembering that neither t nor x are true operators.

By the way, just as they have been defined the operators associated to q_i and p_j are not enclosed, and their spectrum of allowed values extends to the whole real line. When periodic boundary conditions are imposed to the variable position, self-values of the linear momentum operator become discontinuous. And if the wave function must annul in the ends of a finite space interval (the ordinary example of the "particle in a box"), not even there exists then a self-adjoint operator for the linear momentum. The insistence of considering t as if it were a genuine operator, would take us to expect that it should obey the relationship $[t, H] = i\hbar$. Being this way, t should possess a continuous spectrum of self-values from $-\infty$ until$+\infty$, in so far it embraces all the moments in time. In consequence, the same behavior should be shown by the hamiltonian H, against the obvious evidence that there exist systems with quantized self-values for the energy.

This reasoning convinced to not few authors about the impossibility of building an "operator time", while the presumed existence of an operator position remarked the asymmetry between space and time moving away still more the quantum theory from a relativistic spirit. Whereas there is not an authentic problem; neither x nor t are operators, and the formal symmetry among both coordinates stays. An alternative, certainly, consists on formally defining an operator of time evolution (not an operator "time") that provides the transition from a particular state $\Psi_0(x)$ in an instant t_0 until another later state $\Psi_t(x)$ in an instant t.

We would have this way:

$$\Psi_t(x) = \hat{U}(t)\Psi_0(x),$$

where the operator $\hat{U}(t)$ is equal to an exponential function $\exp_e[-iHt]$. It is easy to prove that the time-evolution operator, although linear, is not hermitian; its self-values, $\exp_e[-iE_n t]$, are not real. For that reason, $\hat{U}(t)$ cannot be considered but a purely formal artifice conceived to express the transition from an initial state to another final one by means of a linear operator that only depends on H and t, in a mathematical language comparable to that of other authentic physical operators. The time-evolution operator plays an outstanding role in higher study of quantum systems depending on time. But it does not change its non-physical condition.

The situation is still more delicate when incorporating the Special Relativity requirements in the elementary quantum theory, because then we are even deprived of the position pseudo-operator at our disposal until that moment. In a relativistic quantum theory the notion of a particle with a "definite location"—and, besides, the notion of wave function as a carrier of a probability density—is still more controversial than in a non-relativistic situation. A good discusion could be found in Malament (1996) and Halvorson & Clifton (2002)

In 1949, T. D. Newton and E. P. Wigner published a well-known paper (Newton & Wigner 1949) in which they showed an almost univoc characterization of an operator called "of position" by means of its behavior under displacements and spatial rotations. However, the operator defined this way turns out to be non-covariant in the relativistic sense. Even more, due to the positive sign of the energy in the ordinary physical systems, if in a certain moment we have a self-state of this operator (a "located state", in Newton-Wigner terminology), after an interval of infinitely brief time the subsequent state is extended over all the space. So unpleasant behavior has been a source of plentiful literature around the discussion about the meaning and real utility of the concept of "localizable particle" in the framework of a consistent quantum-relativistic theory. For the Dirac spinors of quantum objects with spin $= \pm\frac{1}{2}$, the Newton-Wigner operator is equivalent to the Foldy-Wouthuysen "position average" operator (Foldy & Wouthuysen 1950).

The truth is that in the usual relativistic versions of quantum physics, neither position nor duration are counted among the basic notions. The main role in this context is played by the operator of quantum field parametrized by means of the space-time coordinates considered as classic magnitudes without dispersion (again, those "c-numbers" of Dirac).

4. Heisenberg inequalities

Another source of confusion is what we can denominate the "Schroedinger approach": the idea that elementary particles are no more than ultramicroscopic wave packets (which is equivalent to accept a wave ontology as the ultimate one for the quantum realm and, in general, for the whole physical reality). If a wave packet is a localized disturbance that results from the sum of many different wave forms, the more strongly localized the packet is, the more frequencies (or linear momenta) are needed to allow the constructive superposition in the region of localization and destructive superposition outside that region. So, representing an arbitrary wave as a superposition of plane waves:

$$\Psi(x,0) = \int_{\infty} d^3(k) A(k) \exp(ix \cdot k).$$

The amplitudes $A(k)$ can be expressed in turn as a function of $\Psi(x,0)$ evaluated at $t = 0$ by inverting the Fourier transform above:

$$A(k) = (2\pi)^{-3} \int_{\infty} d^3(x)\Psi(x,0) \exp(-ix. \cdot k).$$

Taking into account that $p = \hbar k$, many authors deduce the Heisenberg relation of indeterminacy $\Delta x \cdot \Delta p_x \geq \hbar$.

The same procedure applied to variables as time and angular frequency leads to similar conclusions. If we regard a wave as a function of time, we can write:

$$f(t) = \int_{\infty} d\omega \cdot g(\omega) \cdot \exp(-i\omega t),$$

where $g(\omega)$ is given by

$$g(\omega) = (2\pi)^{-1} \int_{\infty} dt \cdot f(t) \cdot \exp(i\omega t).$$

Correspondingly, from the features of a wave packet it could be deduced that $\Delta t \cdot \Delta\omega \geq 1$. And if we remember that $E = \hbar\omega$, we arrive at the "fourth relation of indeterminacy" $\Delta t \cdot \Delta E \geq \hbar$.

There are, however, many reason to suspect of the previous results. The main one is that quantum objects are not wave packets. If it were that way, each individual quantum particle would produce a whole intereference pattern in the double-slit experiment. And this is not what we observe.

From the relationships of commutation, $p_n q_m - q_n p_m = -i\hbar\delta_{nm}$, that are postulated by the theory, the definition of quantum average, the definition of standard deviation coming from the mathematical statistics, and the inequality of Schwartz (1950–51) taken from mathematical analysis, it is obtained without difficulty (Kennard 1927; Robertson 1929 and Schroedinger 1930):

$$\Delta_\Psi p_n \Delta_\Psi q_m \geq (\hbar/2)\delta_{nm}$$

for the component p_n of the momentum and the component q_m of the position of the micro-object represented by Ψ. Note that for individual quantum objects, statistical dispersions cannot be regarded as those applied to sets of quantum particles (Uffink & Hilgevoord 1985; Hilgevoord & Uffink 1988).

It is possible, nevertheless, to define a "time of evolution", $\delta_\Psi t_A$, for any dynamical variable A in a state Ψ as the interval of necessary time so that the mean value change of A would be appreciable compared with their intrinsic dispersion $\Delta_\Psi A$; algebraically:

$$\delta_\Psi t_A \equiv \Delta_\Psi A/\{d\langle A\rangle/dt\}.$$

Then we have $\delta_\Psi t_A \cdot \Delta_\Psi A \geq \hbar$, that would get to the discussed case when A is the energy (Gillespie 1976, p. 74).

5. Conclusions

Most of the debates engendered about the role of space and the time in quantum physics, could have been dissipated distinguishing among the space-time coordinates (that are c-numbers) and the dynamical variables (an inheritance of the analytical mechanics by the hamiltonian formalism) that characterize the behavior of the physical systems in space-time. Since the quantum objects are not reducible—not even ideally—to point-like corpuscles, a real "position-operator" does not exist in quantum theory, what balances the situation, because neither is there a "time-operator". The opposite belief, so common as it is, happens to be founded in a double mistake: on the one hand, to confuse the dynamical variables of position, typical of the particles, with the coordinates of points in space; and on the other, to assign the dynamical variables of position to physical entities, as quantum ones, for which they are essentially inappropriate. The indeterminacy relations are not the same when applied to x or t. As t has no dispersion, the physical meaning of this "fourth Heisenberg's inequality" is different.

When we try to submerge the quantum theory in a relativistic formulation, the covariance requirements for space-time transformations become so demanding that we are even prevented from appealing to a so-called "position operator": the concept of pointlike object gets lost *ab initio*, still in a much more transparent way than it is in the non-relativistic quantum theory, and the whole controversy becomes obsolete. Finally we would arrive at the domain of the quantum field theory, conceived as the royal road to insert the relativistic covariance in the quantum world. That is, at least, the general consent of the scientific community; a consent that has their own—and in no way negligible—inconveniences. But this is another story.

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Resumo. Desde o começo da teoria quântica teve início um debate acerca do papel nela apropriado do espaço e do tempo. Alguns autores assumiram que espaço e tempo tem seus próprios operadores algébricos. Com base nisso, supuseram que partículas quânticas têm “coordenadas de posição”, mesmo se não fosse possível determinar tais coordenadas com precisão infinita. Além do mais, considerou-se o tempo na física quântica como estando em pé de igualdade, por meio de uma assim-chamada “quarta relação de indeterminação de Heisenberg” dizendo respeito a tempo e energia. Neste artigo, analisamos o papel apropriado do espaço e do tempo no núcleo da física quântica não relativística. Descobriremos que, rigorosamente, tal relação para tempo e energia apresenta uma raiz diferente. Para o papel do espaço, discutir-se-á que não há nenhuma “coordenada de posição” na estrutura conceitual da física quântica porque as partículas quânticas não são objetos similares a pontos.

Palavras-chave: Espaço, tempo, operadores quânticos, coordenadas geométricas, variáveis dinâmicas.