

HARE AND OTHERS ON THE PROPOSITION

JOHN CORCORAN

University of Buffalo

Abstract. History witnesses alternative approaches to “the proposition”. The proposition has been referred to as the object of belief, disbelief, and doubt: generally as the *object of propositional attitudes*, that which can be said to be believed, disbelieved, understood, etc. It has also been taken to be the object of grasping, judging, assuming, affirming, denying, and inquiring: generally as the *object of propositional actions*, that which can be said to be grasped, judged true or false, assumed for reasoning purposes, etc. The proposition has also been taken to be the subject of truth and falsity: generally as the *subject of propositional properties*, that which can be said to be true, false, tautological, informative, inconsistent, etc. It has also been taken as the *subject and object of logical relations*, e.g. that which can be said to imply, be implied, contradict, be contradicted, etc. *Prima facie*, such properties and relations are non-mental and objective. It has also been taken to be the *resultants or products of propositional operations*, usually mental or linguistic; e.g. judging, affirming, and denying have been held to produce propositions called judgments, affirmations, and negations, respectively. Propositions have also been taken to be certain *declarative sentences*. Finally, propositions have been taken to be *meanings* of certain declarative sentences. This essay is an informal, selective, and incomplete survey of alternative approaches to “the proposition” with special attention to the views of the late American philosopher Peter Hare (1935–2008) and of those who influenced him.

Keywords: Propositional actions; attitudes; operations; properties; relations.

That all sound philosophy should begin with an analysis of propositions is a truth too evident, perhaps, to demand a proof.

BERTRAND RUSSELL, 1900, Ch. II, p. 8.

Prologue

The English word ‘proposition’ is ambiguous in that people use it with multiple normal meanings. Several of its meanings are vague in the sense of admitting borderline cases. Moreover, there are cases in which it is difficult to determine whether a passage using it should be interpreted as *definitional* or as *informational*: that is, whether the passage is stipulating how the word is to be used or whether the passage is making an informative statement about propositions in a sense already thought to have been established. This situation can be especially troublesome since, as Frango Nabrasa pointed out (per. comm.), the fact that a person defines a word in a certain

Principia 15(1): 51–76 (2011).

Published by NEL —Epistemology and Logic Research Group, Federal University of Santa Catarina (UFSC), Brazil.

sense is no evidence that they always, or ever, use it in that sense. A passage allegedly containing Aristotle's "definition" of proposition (*protasis*)¹ might exemplify both of the last two points. And it is worth reminding oneself that stating a necessary and sufficient condition for being a proposition need not be a definition.

Let us begin by reviewing some of the main alternative approaches to "the proposition". The proposition has been referred to as the object of belief, disbelief, and doubt: generally as the *object of propositional attitudes*, that which can be said to be believed, disbelieved, understood, etc. It has also been taken to be the object of grasping, judging, assuming, affirming, denying, and inquiring: generally as the *object of propositional actions*, that which can be said to be grasped, judged true or false, assumed for reasoning purposes, etc. Each propositional attitude and each propositional action is mental and subjective: each is a certain person's attitude or action. A person begins, performs, and ends an action in a limited time interval; a person's attitude is established at a time but then persists. Attitudes are sometimes established through actions: judging a proposition establishes an attitude of believing or disbelieving, grasping a proposition establishes an attitude of understanding—in one sense of 'grasp' and one sense of 'understand'.

The proposition has also been taken to be the subject of truth and falsity: generally as the *subject of propositional properties*, that which can be said to be true, false, tautological, informative, inconsistent, etc. It has also been taken as the *subject and object of logical relations*, e.g. that which can be said to imply, be implied, contradict, be contradicted, etc. *Prima facie*, such properties and relations are non-mental and objective. Certain forms of the attitude, action, property, and relation approaches complement one another. Each of the above construals admits of variations: e.g., in some cases a propositional attitude is thought to *presuppose* prior existence of its proposition—its object, while in other cases propositions are thought to *be produced* by attitudes or in some other way to depend for their existence on attitudes. More generally, in some cases the proposition is thought to be ontologically prior to any attitude toward it and, conversely, in some cases the propositional attitude is thought to be ontologically prior to or concurrent with the proposition that is its object.

Propositions have also been taken to be certain declarative sentences. In fact, in some contemporary literature of logic, the word 'sentence'—originally a technical term of grammar—occurs repeatedly in contexts that would have called for 'proposition' in former times (Church 1956b: 6). Of course, in such contexts the word 'sentence' is not taken to denote uninterpreted strings of sounds or characters² per se. Rather it is intended to denote composites "containing" both strings *per se* and their "meanings" or "senses".³ In the composite sense, the word 'sentence' in certain contemporary literature is reminiscent of—if not synonymous with—the word 'proposition' as it was used in a broad segment of older literature. Finally, propositions have been taken to be "meanings" of certain declarative sentences. In some

cases a meaning of such a sentence was assumed to be a structured entity so that e.g. ‘No square is a circle’ has a different meaning than its converse and different than its double negation. In some cases a meaning of such a sentence was assumed to be an unstructured or amorphous entity so that e.g. ‘No square is a circle’ has the same meaning as its converse and its double negation. For more on such amorphous entities devoid of logical form see my 1998 Santiago paper (Corcoran 1998).

Introduction

The reader who is not a historian of logic might appreciate being brought up to date somewhat. Although there are scattered passages in Plato (427–347 BCE) and Aristotle (384–322 BCE)⁴ that deserve mention in any comprehensive discussion of the proposition, our topic concerns more recent developments. The Latin word *propositio* from which we get *proposition* was used by Boethius (c. 475–526 CE) to translate Aristotle’s Greek word *protasis*.⁵ In some passages in Aristotle, it makes sense today to translate *protasis* by the *relational* word *premise* as in ‘premise of an argument’, but in other passages a *non-relational* word such as *sentence* or *proposition* makes sense. In the middle ages, the word *propositio* was used non-relationally as we do today, but it was also used relationally: in a broad sense for premise of an argument and in a narrow sense for the so-called major premise of a two-premise syllogism (Gracia 1975: 546).

It would be easy to overlook the logically important distinction between the relational and non-relational senses: every argument’s premises are all propositions and conversely every proposition is a premise of an argument. The word *pro-tasis* is etymologically a near equivalent of *pre-mise*, *pro-position*, and *ante-cedent*—all having positional, relational connotations now totally absent in contemporary use of *proposition*. Taking *premise* for *protasis*, Aristotle’s statement (24a16)

A *protasis* is a sentence affirming or denying something of something . . .

is not a definition of *premise*—intensionally: the relational feature is absent. Likewise, taking *proposition* for *protasis*, it is not a general definition of *proposition*—extensionally: it is too narrow (Corcoran and Boger 2010).

Some more recent history will help to set the scene. Charles Sanders Peirce (1839–1914) endorsed a view of the proposition in the context of a view of logic he attributed to Thomas Aquinas (1225–1274): logic concerns three “operations of the Understanding”—Simple Apprehension, Judgment, and Reasoning (CP 4.38).

According to Peirce (CP 4.39), Apprehension *produces* concepts, expressed by names; Judgment *produces* judgments, which are true or false and expressed by sentences; and Reasoning *produces* inferences, which are expressed by argumentations.

His examples include “man” for a name expressing a concept, “Man is mortal” for a sentence expressing a judgment, and “I think, therefore I must exist” for an argumentation expressing an inference.⁶ Central to the Aquinas-Peirce view is that logic is about mental operations that produce the entities of interest in logical investigations.

Thomas’s three-operation view of logic brings to mind the first three of the six parts of the *Organon*, Aristotle’s collected logical work: *Categories*, *On Interpretation*, and *Prior Analytics*, the third of which Aristotle referred to as *On Syllogism*. In the present context, it is suggestive to think of *Categories* as concerning concepts, *On Interpretation* as concerning judgments, and *Prior Analytics* as concerning inferences. Whether Thomas arrived at his view this way, Peirce does not say.⁷ Justin Legault (per. comm.) believes that Aquinas made the association of the three operations with the three Aristotelian works but only after arriving at his three-operation view of logic. However, it is unclear whether such a mentalist view can be fairly attributed to Aristotle or even whether it is consistent with Aristotle’s explicit statements.

Views resembling the above have reverberated throughout the history of mainstream logic until the end of the 1800s, when they mysteriously vanished almost without a trace. In his 1874 *Logik*, Frege’s teacher Lotze mentions “concept, judgment, and syllogism” as “forms of thought” and accordingly he divides Book I into three chapters devoted respectively to them.⁸ There is only a faint trace in the highly-regarded Keynes book *Formal Logic* (1884, Part I, Chapter I, §6) and there is no trace whatever in the 1934 Cohen–Nagel *Introduction* or the 1936 Tarski *Introduction in German*.⁹

Such mentalist views dominated Medieval and Renaissance textbooks (Ashworth 1974: 26–36). And, with an interesting innovation due to the Cartesian philosopher Arnauld (1611–1694) writing in the 1662 *Port Royal Logic*, these mentalist views continued to be accepted without criticism (Kneale and Kneale 1962: 315–8). The innovation was to add a fourth operation variously called *ordering* and *method*. It is tempting to speculate that this was related to the *Posterior Analytics*, the fourth book of the *Organon*, which concerns demonstration and axiomatic method. However, the four-operation view might well be the result of dividing the third (“reasoning”) into two: (1) grasping immediate implications and (2) chaining them together directly and *indirectly* as in *Prior Analytics*.¹⁰

The hypothesis of four mental operations seems reasonable. It seems clear that deduction requires the ability (1) to deduce conclusions immediately from finitely many premises and another ability (2) to chain immediate deductions to deduce a conclusion mediately. According to Henri Poincaré (1854–1912):

Imagine a long series of syllogisms [...] between the moment in which we first meet a proposition as a conclusion and that in which we reencounter

it as a premise [...] some time will have elapsed [and] several links of the chain will have unrolled [...]. A mathematical demonstration is not a simple juxtaposition of syllogisms, it is syllogisms placed in certain order, and the order in which these elements are placed is much more important than the elements themselves. (Newman 1956: 2041–3)

Poincaré’s use of ‘order’ echoes the word ‘ordering’ in the *Port Royal Logic*.

The four-operation view is found on pages 1–6 in the 1724 *Logick* by Isaac Watts (1674–1748), which became the standard logic text at Oxford, Cambridge, Harvard and Yale for well over 100 years (Kennedy 1995: 134). Peirce wrote favorably of it when preparing his own logic text; he found the Watts book “far superior” to the books then used in colleges. Only Watts’ view of propositions is relevant here (1724: 142).

When the Mind has got Acquaintance with Things by framing Ideas of them, it proceeds to the next Operation, and that is to compare these ideas together, and to join them by Affirmation or disjoin them by Negation, according as we find them to agree or disagree. This Act of the Mind is called Judgment [...]. As an Idea is the Result of our Conception [...], so a Proposition is the Effect of Judgment.

For Watts, the “joining” of concepts by an act of Affirmation produces as effect a proposition—not a ‘judgment’ as Peirce would have it. Moreover, the “disjoining” of concepts by an act of Negation also produces as effect a proposition: “disjoining” seems to be a kind of negative “joining”; perhaps just as disbelief is a kind of negative belief.

These metaphors are reminiscent of Aristotle’s *Prior Analytics* 24a16 juxtaposed with 24b16. At 24a16, the *protaseis* (“propositions”)—not people—“affirm” or “deny” something (the predicate) of something else (the subject). At 24b16, the copula “joins” or “divides” the subject and predicate. It is useful to recall the two main ways of “joining”—“belongs-to-every” and “belongs-to-some”: the universal affirmative copula and the particular affirmative copula. Likewise, there were two main ways of “disjoining”—“belongs-to-none” and “does-not-belong-to-every”: the universal negative copula and the particular negative copula.¹¹ According to Paolo Crivelli (2004: 154): “Aristotle thinks that in every affirmative predicative belief one item is joined with one item, and that in every negative predicative belief one item is separated from one item”. A related passage from Watts is quoted by Alonzo Church (1903–1995) on page 26 of his 1956a *Introduction to Mathematical Logic* in connection with an explanation of Church’s usage of the word ‘proposition’.

To anticipate later developments, it could be said that more recent writers seem to use the words ‘affirm’ and ‘negate’ (or ‘deny’) not for private acts that produce propositions but for public acts that purport to reveal the speaker’s attitudes of belief

or disbelief in propositions whose prior existence is taken for granted. In an ideal case, a speaker is asked a yes-no question to which an affirmation purports to reveal belief and a negation or denial purports to reveal disbelief (cf. the first page of Frege's "Negation", 1997: 347). One might speculate that Watts intends Affirmation and Negation not merely to produce propositions but also to produce attitudes of belief or disbelief in those propositions. However, his statement that we "[...] join them by Affirmation or disjoin them by Negation, according as we find them to agree or disagree" suggests that the attitudes of belief and disbelief somehow precede the production of the propositions—a rather puzzling if not paradoxical view. This issue will receive further discussion below.

Despite the plausibility and fresh insight of the four-operation view, it is the three-operation view that is found, almost in Peirce's wording, in Book II, Chapter I of Richard Whately's influential 1826 *Elements of Logic*. Whately begins the Chapter "Of the Operations of the Mind and of Terms" with the following sentence: "There are three operations of the mind which are immediately concerned in argument; 1st. Simple Apprehension, 2nd. Judgment, 3rd. Reasoning". This book is widely credited with reviving logic in England. It was read by most logicians writing in the 1800s: Mill, Boole, De Morgan, Jevons, and Peirce to name five. Interestingly, Peirce repeatedly claimed to have read Whately's book at the age of 12.¹²

There is evident justification in calling these views *mentalist*, but there is no reason in sight for calling any of them *psychologistic* if this epithet implies the view that laws of logic are referred to the pre-conscious, behavioral, or physiological nature of the mind. For example, one psychologistic view entertained by Peirce is that any proposition a person's mind compels assent to once assent has been given to another is thereby a consequence of the other (Corcoran 2007).

Background on Hare

Peter H. Hare (1935–2008) developed informed, original views about the proposition: some published (Hare 1969 and Hare and Madden 1975); some expressed in conversations I participated in at scores of meetings of the Buffalo Logic Colloquium and at dinners following. The published views were expository and critical responses to publications by Curt J. Ducasse (1881–1969), a well-known presence in American logic, a founder of the Association for Symbolic Logic and its President for one term.¹³

Hare was already prominent in the University of Buffalo's Philosophy Department in 1969 when I was appointed. Soon after, he became Chair. As his Associate Chair 1971–1975, I spent many hours with him in Buffalo and on professional trips (Corcoran et. al. 2008: 50).

Without realizing it at the time, I assimilated many of his philosophical attitudes, interests, distinctions, and notational stipulations—and much of his naturalistic philosophical framework—despite his unfailingly respectful leeriness of my frank and unstinting Platonism. Even though my critical Platonism was as tough-minded and non-religious as Hare’s philosophy, it never became a “live option” for him. He *knew* of some of the arguments for the Platonist hypostatization of propositions (Hare and Madden 1975: 90), but—at least once—he dismissed them unceremoniously and without demonstrating awareness of their force (loc. cit.). His Platonistic tendencies were muted and restrained, always diluted or reinterpreted naturalistically. What he later insightfully called his “irenic impulse” fit well with his inclination to integrate and conciliate conflicting philosophies (Hare 2008: 357–8). His respectful and non-confrontational style made it easy for me to selectively incorporate his ideas.

I thank Hare for my understanding of the philosophical centrality of the proposition as *the object of belief, disbelief, and doubt*. My previous education and research along paths set out by Aristotle, Boole, Tarski, and Quine had not prepared me for what Hare brought me to see: that the problem of the ontic and epistemic nature of propositions must be confronted by any comprehensive philosophy. Since learning from Hare, I have come to see the history of logic in a new perspective. Among the first things I now look for in historical logicians is awareness of the proposition as the object of the “propositional attitudes”,¹⁴ the most important being belief, disbelief, and doubt—not ‘doubt’ in the ordinary sense of incipient disbelief, but in the philosophical sense of suspended judgment or, as Hare put it, “unconsummated judgment”.¹⁵ Hare wrote that a theory of propositions should respond to “the many epistemological and metaphysical . . . questions about the nature and status of the entities which serve as the objects of believing, [*sc.* disbelieving], doubting, etc”. (1969: 268).¹⁶

Framework

Not only is this article about Hare’s views, it uses a framework of terminology and notation adapted from his. He used single quotes and double quotes to mark a distinction: single quotes for words and other strings; double quotes for meanings¹⁷ (1969: 267). Thus he notated a variant of the sense-referent distinction he called ‘connotation-denotation’ (1969: 270).¹⁸ E.g., in some contexts the five-letter word ‘truth’ *connotes* the meaning “truth” and *denotes* the object truth. More saliently, for Hare (loc. cit.), the three-word *sentence* ‘Hare admired Whitehead’ *connotes* the proposition “Hare admired Whitehead” and *denotes* the fact that Hare admired Whitehead.¹⁹ The true proposition *corresponds* to the fact. The string, connota-

tion, and denotation can be thought of as the vertexes of a triangle: the relations of connoting, denoting, and corresponding as the sides, in *this* case. Other cases of “the semiotic triangle” or “semantic triad” are considered below.

To further set the scene, I quote from page 12 of Ralph Eaton’s well-known 1931 textbook *General Logic*, owned by Hare and known to Ducasse: it was listed in the “Bibliography of Symbolic Logic” published by Ducasse’s ASL in the first year of his presidency.

The *proposition* must be distinguished from the *sentence*, the combination of words or signs through which it is expressed; from the *fact*, the actual complex situation whose existence renders it true or false; and from the *judgment*, which affirms or denies the proposition.

Hare could have accepted this, but only as an approximation needing supplementation and clarification.

Concerning its supplementation, the four concepts—proposition, sentence, fact, and judgment—are not enough for Hare: he also distinguished the proposition from the private *belief* that it is true and from the *statement* making the belief public. During the 1960s and 1970s, Hare favored a six-sided framework recognizing propositions, sentences, facts, judgments, propositional attitudes, and speech acts—although he did not use the expression ‘speech act’. Years later, I adopted much of his approach without however sharing its naturalistic metaphysics and epistemology (Corcoran 2009). It might be misleadingly general to say ‘naturalistic’—the word ‘mentalist’ would be more specific: he preferred to think of propositions as “accusatives” of certain “mental activities” (1969: 269), the latter an expression from the 1600s (Arnauld 1662/1964: 111) that he later updated to “psychological activities” (1975: 88). To reconcile this view with traditional principles he accepted, he resorted to the lame and hackneyed dodge of taking each proposition to be an equivalence class of such mental accusatives (1969: 270).²⁰

Concerning its clarification, Hare would have objected to Eaton’s improper use of the overworked verbs ‘affirm’ and ‘deny’. In one *proper* sense, *people*, not *judgments*, affirm or deny [*sc.* negate] propositions and they do so by making affirmative or negative statements. In a derived sense, a *statement* can be said to affirm or deny [*sc.* negate] the proposition. But there are other contexts into which philosophers force the words: for Aristotle, an affirmative *proposition* affirms the predicate of the subject (*On Interpretation*, VI; *Prior Analytics* A1). Eaton’s quoted usage is not even grammatically the same as Aristotle’s. Eaton used a two-place or transitive verb taking a subject and a direct object; Aristotle used a three-place or hypertransitive verb taking a subject, a direct object, *and* an indirect object.

Hare would have noticed that Eaton’s misuse suggests failure to distinguish the judgment from the assertion or statement. The judgment is always private and silent,

and never done in writing; the assertion is normally public and done in writing or speaking. Hare and I discussed that contrast. Frege never explicitly treats it in work he published. In fact in 1879 he refers to the sign of assertion as ‘the judgment stroke’ (Frege 1997: 52, 198). Throughout his published work, the necessarily private mental act of judging is often conflated with the possibly public act of asserting. Nevertheless, in his posthumous writings we find the following forthright statement *italicized*, but unaccompanied by any retraction (Frege 1997: 239).

When we inwardly recognize that a thought is true, we are making a judgment; when we communicate this recognition, we are making an assertion.²¹

However, Hare never notes that propositional *attitudes* such as doubting and believing are grounded in propositional *actions* such as grasping and judging, which seem to presuppose prior existence of propositions. He seemed to sense the “chicken-egg” problem (1969: 269): do propositions exist through belief or does belief presuppose prior existence of propositions? Hare’s peculiar use of ‘mental activity’—instead of say, ‘mental attitude’—for believing and disbelieving, which has the proposition as its accusative, suggests inattention to the attitude/action distinction. Attitudes and actions are mental and subjective: each is a certain person’s attitude or action. However, an attitude persists after being established. In contrast, an action is completed or aborted in a limited time interval. My judging that Peter Hare had died began and ended in a limited time interval on one sad day in January 2008; but the belief, the attitude established at that time, persists and is recalled almost every day. Attitudes are sometimes established through actions: judging a proposition establishes an attitude of belief or disbelief. But attitudes do not seem to require action in order to maintain their existence.

Hare’s writings about propositions are sensitive to several contexts: to ordinary “man-in-the-street” usage, to then-current views in the logic community, and to history of logic. But he rarely gives specific bibliographic references for his normally accurate observations. E.g., he wrote the following (1969: 268).

Indeed, the venerable doctrine that a proposition is “the verbal expression of a judgment”, unpopular as this view is among modern logicians, probably is more in accord with both man-in-the-street usage and the history of logic than Ducasse’s account.

This doctrine is found almost in this wording on page 75 in Richard Whately’s influential 1826 *Elements of Logic*. The view goes back at least to the famous *Port Royal Logic* (1662/1964: 99, 111, 114).

There can be no discussion without a shared vocabulary. If one person says ‘Every proposition is either true or false’—as Hare and I do—and certain others say ‘Not every proposition is either true or false’ (Mill 1843/1879, V. I, Bk. II, Ch. VII, §5),

Principia 15(1): 51–76 (2011).

there need not be disagreement. The second sentence can but need not be used to deny what the first was used to affirm.²² In order for a second speaker to contradict a first by uttering the negation of a given *sentence* uttered by the first, it is sufficient for the second to have used the negation to deny the very same *proposition* the first used the given sentence to affirm. If “certain others” use the word ‘proposition’ for something that might change or even lose its truth-value or that has no truth-value until conclusively tested, there is no contradiction. Likewise, there is no contradiction if “certain others” use ‘true’ in a coherence sense, in a pragmatist sense, or in an epistemic sense. Of course, one person might share another’s vocabulary for purposes of discussion without adopting it.

Hare’s writings on propositions used a vocabulary he shared with his intended “audience”, which included C. J. Ducasse, E. H. Madden (1930-2006), and R. E. Santoni—to mention three. Moreover, being a historian, he was aware of writing for a specific, limited audience that shared certain presuppositions and for which certain theses were uncontested. The richer the class of shared presuppositions and uncontested theses the more fruitful a dialogue can be. These uncontested theses can serve to characterize meanings of words such as ‘proposition’ occurring in them—in much the same way that the axioms of geometry have sometimes been regarded as characterizing meanings of geometrical words.²³ Perhaps more aptly, holding that the word ‘true’ is indefinable, Frege thought its meaning was explained by the laws of logic (1918: 290).²⁴

Uncontested Theses

Among Hare’s uncontested theses was the traditional law of *excluded middle*—“every proposition is either true or false”—and also the law of *non-contradiction*—“no proposition is both true and false”. He was generally careful to point out a “venerable doctrine” he took to be contested (1969: 268).

He also subscribed to the *propositional attitude thesis*: every object of belief, disbelief, or doubt is a proposition (1969: 268; 1975: 80). Instead of the ambiguous word ‘object’—which he used roughly in the sense of “patient”—he preferred ‘accusative’, which, though less familiar, lacks ambiguities of ‘object’. His ontological use of this normally grammatical expression is carefully chosen. He seemed to characterize or locate propositions as accusatives of the three traditional propositional attitudes.

The propositional attitude thesis, so prominent in Hare and in Ducasse, is not even mentioned by Eaton 1931, whose index omits the crucial words ‘attitude’, ‘belief’, ‘disbelief’, and ‘doubt’. It would be interesting to know who first proposed it, what its historical origin is. Mill (1843/1879, V. I, Bk. I, Ch. I, §2) wrote:

Whatever can be the object of belief, or even disbelief, must, when put into words, assume the form of a proposition.

Mill's qualification would be absurd, then, unless the object of belief or disbelief not expressed in words is *not* a proposition: our unspoken beliefs and disbeliefs are not propositions nor are they *of* propositions—in Mill's sense of *proposition*. For the record, Frege (1997: 52–4) was moving in the right direction when he discussed “the content of the judgment” in 1879, separating the judgment *per se* from its object. Even though Frege continued to give insufficient attention to the traditional propositional attitudes, by about 1918 he had distinguished the act of grasping a proposition from the act of judging it, and he distinguished the two both from the proposition—which is not an act but the object thereof—and from the act of asserting the proposition (1997: 329).

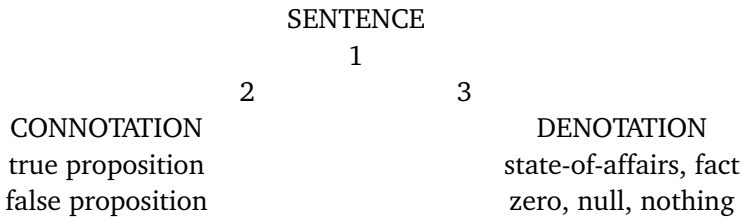
Believing a proposition is holding it to be true; disbelieving is holding it to be false. However, as mentioned, there are two senses of the transitive verb ‘to doubt’ relevant here. Doubting a proposition, in the first sense intended, is not simply neither believing nor disbelieving: it is impossible to doubt a proposition that one does not understand. Doubting is an attitude that requires an object, an accusative, toward which it is directed. Once a proposition has been *grasped*, the propositional attitude of *understanding* it has been established, and only then can we begin the process of

judging.²⁵ When the process is completed, or consummated, to use Hare's term, the judge has a belief or disbelief. Judging the proposition to be true produces belief of it; judging the proposition to be false produces disbelief of it. But, if the process has not started or has not come to a conclusion, the judging person has doubt—in this mainly philosophical sense.

The converse of the propositional attitude thesis is that every proposition is the object of a definite “opinion”—held at a certain time by a certain person.²⁶ Hare certainly accepted this, but he seemed reluctant to assert it (1969: 271). I put the word ‘opinion’ in double quotes when I use it in the broad sense: roughly, for a proposition toward which one has one of the three classical propositional attitudes, following Ducasse and Hare (1975: 88). In this technical sense, every proposition *known* to be true or *known* to be false is an “opinion” of the knower. Moreover, among a person's “opinions” are the propositions the person grasped but did not judge. In this sense, an “opinion” is a belief, disbelief, or doubt, and conversely.²⁷ The question is not limited to traditional propositional attitudes. It concerns understanding: are there propositions that have never been understood by anyone? The “venerable doctrine” construed literally would yield a resounding “Of course not, judgments are human creations!” Arnauld (1662/1964: 111) wrote, “. . . all mental activity can be reduced to conceiving, judging, reasoning, and ordering . . .”.

Hare also accepted the *truth-value coherence thesis*: propositions are the things that can literally and coherently be said to be true or be said to be false. He cheered Ducasse's insistence that an "opinion"—someone's belief, disbelief, or doubt—should never be said to be true or false. A person's belief or disbelief is "correct" if the proposition believed is true or the proposition disbelieved is false but "erroneous" if the proposition believed is false or the proposition disbelieved is true. And I applaud Hare, who would have been amused to know that a similar terminological nicety had been anticipated over three centuries earlier in the 1662 *Port Royal Logic*, which—in the 1964 Dickoff–James translation—called a judgment "correct" or "incorrect" according as the proposition involved was true or false—evidently ignoring negative judgments (Arnauld 1662/1964: 111). Although Mill praised the *Port Royal Logic*, he did not always learn its lessons: his stated view was that "errors are false propositions" (1843/1879, V. I, Bk. I, Ch. I, §2). In a later paper I hope to discuss whether Aristotle deals with the twin distinctions of true propositions from correct beliefs and of false propositions from erroneous beliefs.

Respecting the truth-value coherence thesis, Ducasse does not apply "true" or "false" to sentences or to facts. Hare suggests that Ducasse's theory could accept interrelating or tying together the sentence, the proposition, and the fact in a form of "the semiotic triangle".²⁸ The sentence is at the vertex; the connotation and the denotation are at opposite ends of the base. The sentence, composed of words, connotes the proposition, composed of meanings. If the proposition connoted is true, the sentence or propositional expression denotes the fact—composed of the things the proposition is about. Hare wrote that "the expression can connote a proposition while having zero denotation (i.e., the proposition expressed [*sc.* connoted] is false)" (1969: 269–70).^{29,30}



Hare and Madden criticized Ducasse for identifying the true proposition with the fact it *corresponds to* (Hare and Madden 1975: 89). However, Hare and Madden never show the slightest understanding of why an intelligent person would be inclined to make this mistake nor do they ever admit that other important figures such as Frege (1918/1956) might have made it.³¹ In 1918 Frege wrote (1997: 342): "What is a fact? A fact is a thought [*sc.* proposition] that is true". In a spirit similarly lacking sympathy and respect, Austin (1961: 91) disapproves of taking 'fact' as synonymous with 'true statement'.

Hare and Madden never mention the apparent interchangeability of expressions such as ‘it is true that’ and ‘it is a fact that’, or ‘is true’ and ‘is a fact’. In particular, they never mention contexts like the following that make it appear that certain expressions, e.g., ‘that zero is even’, apparently denoting facts also appear to denote true propositions and conversely.

Zero is even.
 It is a fact that zero is even.
 It is true that zero is even.
 The proposition that zero is even is true.
 It is a true fact that zero is even.
 The fact is that zero is even is a known truth.
 The proposition that zero is even is true if and only if it is a fact that zero is even.

It is remarkable, then, that they did not think to mention the many cases where ‘fact’ is not replaceable by ‘true proposition’.

The untimely death of Peter Hare is a sad fact.
 No fact can be deleted from the past.
 Facts can be hidden but they cannot be destroyed.
 Histories do not present all the facts and not all things they do present are facts.

Neither Frege nor Ducasse felt any need to explain such contexts.³² Do Frege and Ducasse agree? Ducasse might have been hypostasizing propositions while Frege might have been doing the reverse to facts. Either way, Hare and Madden’s criticisms seem decisive. The thought that the universe is exhaustively composed of timeless abstractions such as true propositions seems wildly implausible. But neither Frege nor Ducasse, as far as can be determined from the Hare and Madden discussion, seemed to accept this interpretation. Moreover, as far as I can tell, there is no discussion by Frege³³ or by Ducasse of the ambiguity of the word ‘fact’. Ducasse implied that Mont Blanc is a constituent of “Mont Blanc is cold” (Hare 1969: 271), whereas in his posthumous writings Frege implied the contrary (1997: 293). It might seem to follow that for Frege Mont Blanc is *not* a constituent of “the fact” that Mont Blanc is cold: he might take the individual concept “Mont Blanc” to be a constituent of the proposition he takes to be a fact. In 1892 Frege wrote (1997: 158): “A truth-value cannot be part of a thought [*sc.* proposition] any more than, say, the Sun can, for it is an object not a sense”. Frege does not reveal awareness of ambiguities of the word ‘fact’.

A more plausible interpretation of Frege is that he was simply using the word ‘fact’ as synonymous with ‘true proposition’, not that he was taking facts in the sense of Eaton (*supra*), the things true propositions were true in virtue of to be the very

things that were true. My guess is that Frege never took “truth-makers”, Eaton’s and Hare’s facts, to be “truth-bearers”, things that are true in the most basic sense.³⁴ Although Frege probably meant his sentence ‘A fact is a true proposition’ informatively, he would have forestalled uncertainty if he had indicated that it could have been meant definitionally (see above).

It is a sad irony that Hare’s beloved Peirce uses the word ‘fact’ as a synonym for ‘proposition’—not just ‘true proposition’—in his famous 1877 article “The Fixation of Belief” (Peirce 1992: 109–123, esp. 113). Hare must have read that article dozens of times. But Peirce is not the only accomplished logician to do this. Tarski also uses ‘fact’ where ‘proposition’ would be called for. But it would be misleading to say that he uses it as a *synonym* for ‘proposition’, a word he studiously avoids: he even calls propositional logic ‘sentential calculus’. For occurrences of ‘fact’ used where ‘proposition’ would be called for, see Tarski (1941/1946: 122 and 1956/1983: 146, 158–9, 249, 385, 449). For example, in both works “expressions” are said to “express” “facts”, where is clear that the “facts” are not all true, so to speak (1941/1946: 122 and 1956/1983: 385).

Ambiguity of ‘not’

To deny a statement is to affirm another statement, known as the *negation* or *contradictory* of the first.—W.V. Quine, *Methods of Logic*, 1959: 1.

Hare, attuned to the importance of ambiguity in philosophical dialogue, recognized two senses of the word ‘not’. In one sense ‘not’ is used for the familiar internal “truth-functional” negation as in ‘teaching is not just a job’. Sometimes ‘not’ in this sense is synonymously interchangeable with ‘it is not the case that’ as in ‘if not every teacher is happy, some teacher is unhappy’. When used in this sense to make a statement, it expresses part of the proposition stated, a part that is *internal* to the proposition while in no way indicating the speaker’s propositional attitude, which is *external*. In another sense it indicates that the speaker intends to deny something (1969: 267) and thus has the attitude of disbelief toward that something. The sentence ‘not every number is even’ might be used to affirm a negation—to affirm the negative proposition “it is not the case . . .”—or to deny a universal—to deny the general proposition “every number . . .”). In the first sense, which I call the *truth-functional* sense, the *constitutive* sense, or the *internal* sense, it indicates a feature of the logical form of the proposition expressed; it indicates a constituent internal to the proposition.³⁵ It is internal to the proposition.

In *some* simple cases, inserting or deleting an occurrence of such a ‘not’ reverses truth-value. Thinking that this is *always* the case leads to the *fallacy of single nega-*

tion: “some number is even” and “some number is not even” are both true. In the second sense, which I call the *attitudinal* sense, the *rejective* sense, or the *external* sense, it indicates a feature of the speaker’s attitude but has no bearing on logical form of the proposition expressed: it is external to the proposition.

The word ‘not’ is never used in the external sense when it occurs in a clause of a larger sentence as in the following.³⁶

If not every number is even, then some number is not even.
 Zero is not odd and one is not even.
 Every number that is not even is odd.

In all such occurrences, it contributes to expression of a part of the proposition that the sentence is used to affirm. This is so even if it is repeated as in ‘if two is even, it is not the case that it is not the case that two is even’. Of course, the proposition expressed has the same truth-value as the one expressed by deleting the repeated ‘it is not the case that’.

An occurrence of ‘not’ being used by a speaker to indicate denial of a proposition is used in what I called the *attitudinal* sense. However, in this case its meaning is not part of the proposition denied—any more than the meaning of ‘is it the case that’ is part of the proposition questioned using a sentence beginning therewith or that the meaning of the question mark is part of the proposition questioned. Hare thought that Ducasse meant to say that the attitudinal sense never occurs as part of a proposition (1969: 267).³⁷

Hare is the first person I know of to recognize the distinction between the truth-functional and attitudinal senses of negative expressions such as ‘not’ and ‘it is not the case that’. Along with this goes recognition that denying a proposition is not the same act as affirming the negation of a proposition. The truth-functional negation is not used to deny a proposition. As Aristotle first noted in Chapter VI of *On Interpretation*, whatever is the object of an act of affirming can be the object of an act of denying and whatever is the object of an act of denying can be the object of an act of affirming: affirming and denying apply to the same things—propositions.

In order to clarify the fact that I am not giving Hare too much credit, I should note that in 1879 Frege has a very closely related distinction: roughly, between negatively judging a proposition devoid of negations and affirmatively judging the negation of such a proposition (1997: 54–5). However, later he abrogates virtually all ground for credit by repeatedly and improperly using ‘is denied’ where no denying or judging is relevant (1997: 56, 60), as I note below.

The declarative³⁸ [sentence] type covers two subtypes: the *assentive* and the *dissentive*. A declarative sentence is one “characteristically” used to make a statement, an imperative sentence is one “characteristically” used to make a command,

and an interrogative sentence is one “characteristically” used to make an inquiry (Lyons 1977, vol.1: 30). It is useful to consider the interrogative type along with the declarative. Compare Lyons 1977, vol. 2: 802–3.

- Interrogative: Is it the case that $1000! + 1$ is prime?
 Assentive: It is the case that $1000! + 1$ is prime.
 Dissentive: It is not the case that $1000! + 1$ is prime.

The three speech acts that use these three sentences are performed on one and the same proposition.³⁹

The interrogative sentence can be used to express curiosity—or “philosophical doubt”—in or toward the same proposition that the assentive is used to express belief in and the dissentive is used to express disbelief in. Being an *inquiry* is clearly a property of a speech-act and not of the proposition that is the object of the act. Likewise, being an *acceptance* is primarily a property of a speech-act and not of the proposition that is the object of the act. A person may accept any proposition whether “affirmative” such as “Every prime exceeds one” or “negative” such as “No prime precedes two”. As Hare points out, being a *rejection* is primarily a property of a speech-act and not of the proposition that is rejected. A person may reject any proposition whether “affirmative” or “negative”, echoing Aristotle’s point alluded to above.

There are cases of course where the interrogative preamble ‘is it the case that’ is omitted and the inquiry is indicated simply by the question mark or, in speech, by intonation. At least as common are cases where the affirmative preamble ‘it is the case that’ is omitted and the acceptance is signaled in speech by intonation or in writing by punctuation. There are even cases of course where the negative preamble ‘it is not the case that’ is omitted and the rejection is indicated simply by the context or, in speech, by ironic intonation. Recently I overheard a colleague say ‘That’s a wonderful idea’ in such a tone that it was clear the opposite was meant. Are there cases where an acceptance is made using an interrogative type sentence? Of course. Some of these use what are known as “rhetorical questions”.⁴⁰

It goes without saying that the assentive preamble is subject to an ambiguity similar to that of the dissentive: instead of signaling assent it can be intended as empty rhetoric. This is especially common when it occurs inside the sentence as in ‘if $1000! + 1$ is not prime, then it is the case that a smaller number is a prime factor of $1000! + 1$ ’.

An important difference between the truth-functional and the attitudinal interpretations of the ambiguous negative preamble ‘it is not the case that’ concerns their roles in dialogue. If the first speaker makes a statement without preamble, the second can express disbelief using a dissentive, the same sentence with the negative preamble.

A: $1000! + 1$ is prime.

B: It is not the case that $1000! + 1$ is prime.

However, if the first speaker's statement was dissentive, it would not be open to the second speaker to simply affix the negative preamble to the second as follows.

A: It is not the case that $1000! + 1$ is prime.

B: It is not the case that it is not the case that $1000! + 1$ is prime.

The second speaker is not contradicting the first: the second is not implying that $1000! + 1$ is prime. The second's point has to do with the second's state of mind, not with the nature of $1000! + 1$.

It is clear that A's statement in the second example implicates that A disbelieves that $1000! + 1$ is prime. The question arises whether the same statement can be made using 'I disbelieve that $1000! + 1$ is prime'. More generally, the question is whether the preamble 'I disbelieve that' is exactly synonymous with the preamble 'It is not the case that' in the attitudinal sense.

Another important difference between the truth-functional and the attitudinal interpretations of the ambiguous negative preamble 'it is not the case that' concerns the "double negation", e.g., the following:

It is not the case that it is not the case that $1000! + 1$ is prime.

In the truth-functional sense, this could be used to assert a proposition having the same truth-value as the proposition that $1000! + 1$ is prime, as said above. But in the attitudinal sense, things are different. In the first place, it is not the case that denying that I am denying that $1000! + 1$ is prime is asserting that $1000! + 1$ is prime. Rather, denying that I am denying that $1000! + 1$ is prime is much weaker; it reveals very little about my attitude toward the proposition that $1000! + 1$ is prime. In fact, if someone were to ask me whether I deny that $1000! + 1$ is prime, I might well reply as follows.

I deny that I deny that that $1000! + 1$ is prime.

However, it is not clear to me whether the last sentence could be used as a "conversational equivalent" to the above "double negation" taken attitudinally. Anyway, asserting the double negation of a proposition is not achieved by "double denying" it or denying its "denial".

In the usual symbolic languages, there are no *attitudinal preambles* such as 'is it the case that', 'it is the case that', 'it is not the case that'. In such languages, absolutely every sentence is a component of larger sentences and therefore every

occurrence of a negation sign is truth-functional: no occurrence of a negation sign is attitudinal. However, in 1879 Frege did not always seem recognize the difference. He apparently thought that propositions involving truth-functional negation really involve nested denials. He writes as if his asserting “not every number is even” was his asserting that he was denying that for every number he was denying that it was even. He seemed deliberately and repeatedly to write ‘is denied’ where ‘it is not the case’ would have been more to the point and to write ‘is affirmed’ where ‘it is the case’ (or nothing) would have been more to the point (1997: 52–75).

In the 1918 Frege papers, which Hare probably did not read, there are many points that touch on themes Hare wrote about. E.g., Frege (1918/1956: 293) noticed that the same proposition is asserted whether an assentive sentence is used with or without an assentive preamble: “it is the case that $1000! + 1$ is prime” is the same proposition as “ $1000! + 1$ is prime”. Unfortunately, the only assentive preamble Frege considered was the ambiguous ‘it is true that’, which can also be used not as a preamble but to ascribe truth to the proposition. And worse, he failed to notice the ambiguity and, perhaps as a result, he mistakenly thought that ascribing truth did not change the proposition either. After all, whereas ‘it is the case that $1000! + 1$ is prime’ can be used to assert a proposition about a certain number and not about a proposition, the sentence ‘the proposition that $1000! + 1$ is prime is true’ can also be used to make an assertion about a proposition and not about a number. The proposition about the number $1000! + 1$ is expressed in the object language. The proposition about the proposition is expressed in the metalanguage. Moreover, as I only recently noticed, Frege (1997: 355) comes close to Hare’s distinction between the truth-functional and the attitudinal “not”—even though he never explicitly discussed propositional attitudes and he rarely used the words ‘belief’ and ‘disbelief’ in the relevant senses.

Hare (1969: 267) considers a statement made using the ambiguous sentence ‘God does not exist’. On Hare’s view this might be construed in at least three ways: as (1) a rejection, denying the affirmative proposition that God exists or as acceptance of either of two propositions: (2) acceptance that it is not the case that God exists—affirming the negation of an affirmative—or (3) acceptance that God is non-existent—affirming an affirmative having a negative predicate adjective. The difference between 2 and 3 is analogous to one of the differences described using the expressions *de dicto* and *de re*. On one analysis, statement uses the attitudinal negation (1); use the truth-functional (2) and (3). Hare has taken a step past the position suggested by Santoni (1969: 258).⁴¹

Propositions as Meaning Sentences and Sentence Meanings

As said above, propositions have also been taken to be certain declarative sentences. In fact, there is a contemporary literature in which the word ‘sentence’ occurs repeatedly in contexts that would have called for ‘proposition’ in former times. Of course, in such contexts the word ‘sentence’ is not taken to denote strings of sounds or strings of characters *per se* but rather it is intended to denote composites “containing” both strings *per se* and their “meanings” or “sense”. In the composite sense, the word ‘sentence’ in certain contemporary literature is reminiscent of if not synonymous with the word ‘proposition’ in a broad segment of older literature.

In this particular respect, Lewis Carroll (1823–1899) is representative of the late part of that older literature. In his *Symbolic Logic* (1896/1977, Book II, Chapter I) he wrote the following.

The word “proposition”, as used in ordinary conversation, may be applied to *any* word, or phrase, which conveys any information whatever.

Carroll includes not only what would be regarded by grammarians as complete or elliptical sentences but also answers to questions both those requesting simply a ‘yes’ or a ‘no’, but also those answered by phrases such as ‘your brother’ or ‘behind the chair’.

Again as mentioned in the beginning of this paper, propositions have been taken to be meanings of certain declarative sentences. On page 26 of his influential 1956a *Introduction to Mathematical Logic*, Church wrote as follows.

But in non-technical English the word [*sc.* ‘proposition’] has long been used rather for the meaning (in our view the [*sc.* Fregean] sense) of a sentence, and logicians have latterly come to accept this as the technical meaning of “proposition”.

Church seems unaware of the fact that in one sense of ‘meaning’ the meanings of certain declarative sentences are less than complete propositions: e.g., elliptical sentences and those involving indexicals such as ‘here’, ‘now’, ‘this’ and personal pronouns. Another logician who takes propositions to be sentence meanings is Nicholas Rescher (1964: 14).

It is therefore important to distinguish between a sentence and the information it conveys . . . A sentence is a linguistic complex, but a proposition is the meaning of an informative sentence, the information that it presents.

Thus, for Rescher an informative sentence is what Carroll called a proposition, something that conveys information. But for Rescher, a proposition is the sentence meaning and not that which conveys the information.

Acknowledgements

It is a pleasure to acknowledge helpful discussion, suggestions, and criticisms. Douglas Anderson, Chris Buckman, Lynn Corcoran, Catarina Dutilh, Caroline Gould, Jorge Gracia, Forest Hansen, James Henderson, David Hollinger, Kenneth James, Mark Jensen, Calvin Jongsma, Jon Kreiss, Justin Legault, Ray Lucas, Timothy Madigan, Richard Main, Corey McGrath, James McNabb, Joaquin Miller, Frango Nabrasa, Joseph Palencik, Charles Pailthorp, Joseph Paterno, Carlo Penco, David Plache, Russell Pryba, José Miguel Sagüillo, Wagner Sanz, Mark Spencer, and Thomas Sullivan deserve special thanks. This paper borrows from, clarifies, corrects, and expands a shorter paper on a narrower subject (Corcoran 2010). I started on this immediately after finishing the previous paper. Versions of this paper have been presented at the Buffalo Logic Colloquium and have been discussed with several philosophers, logicians, and historians.

References

- Aristotle. *Categories, On Interpretation, Prior Analytics*. Trans. H. P. Cooke and H. Tredennick. 1938. Cambridge, MA: Harvard UP
- Ashworth, E. J. 1974. *Language and Logic in the Post-Medieval Period*. Dordrecht: Reidel.
- Arnauld, A. 1662/1964. *The Art of Thinking: Port Royal Logic*. Trans. and eds. J. Dickoff and P. James. New York: Bobbs-Merrill.
- Austin, J. L. 1961. *Philosophical Papers*. Eds. J. Urmson and G. Warnock. Oxford: Oxford UP
- Beth, E. 1965. *Mathematical Thought*. Dordrecht: Reidel.
- Bochenski, I. 1956/1961. *History of Formal Logic*. Trans. and ed. by I. Thomas. Notre Dame, IN: Notre Dame UP
- Carroll, L. 1896/1977. *Symbolic Logic*. Ed. W. Bartley III. New York: Clarkson Potter.
- Cohen, M.; Nagel, E. 1934/1962/1993. *Introduction to Logic*. Indianapolis: Hackett.
- Church, A. 1956a. *Introduction to Mathematical Logic*. Princeton: Princeton UP
- . 1956b. Propositions and Sentences. *The Problem of Universals*. Bochenski et al. 1956. Notre Dame, IN: Notre Dame UP
- Corcoran, J. 1989. Argumentations and Logic. *Argumentation* 3: 17–43.
- . 1998. Information-theoretic logic. In *Truth in Perspective* edited by C. Martínez, U. Rivas, L. Villegas-Forero. Aldershot, England: Ashgate Publishing Limited, pp.113–35.
- . 2007. Psychologism. *American Philosophy: an Encyclopedia*. Eds. John Lachs and Robert Talisse. New York: Routledge, pp.628–9.
- . 2009. Sentence, Proposition, Judgment, Statement, and Fact: Speaking about the Written English Used in Logic. *The Many Sides of Logic*. Editors: W. A. Carnielli, M. E. Coniglio, I. M. Loffredo D'Ottaviano. Series “Studies in Logic”. London: College Publications, pp.71–103.
- . 2010. Peter Hare on the Proposition. *Transactions of the C. S. Peirce Society* 46: 21–34.

- Corcoran, J.; Boger, G. 2011. Protasis in Prior Analytics: Proposition or Premise? *Bulletin of Symbolic Logic* 17: 151–2.
- Corcoran, J.; Frank, W.; Maloney, M. 1974. String Theory. *Journal of Symbolic Logic* 39: 625–37.
- Corcoran, J.; Madigan, T.; Razin, A. 2008. Remembering Peter Hare 1935–2008. *Philosophy Now* 66(March/April): 50–2.
- Crivelli, P. 2004. *Aristotle on Truth*. Cambridge: Cambridge University Press.
- Ducasse, C. J. 1944. Propositions, Truth, and the Ultimate Criterion of Truth. *Philosophy and Phenomenological Research* 4: 318–40.
- Eaton, R. 1931/1959. *General Logic*. New York: Charles Scribner's Sons.
- Frege, G. 1918/1956. The Thought: a Logical Inquiry. Trans. A. and M. Quinton. *Mind* 65: 289–311.
- . 1997. *The Frege Reader*. Ed. M. Beaney. Oxford: Blackwell.
- Godden, D.; Griffin, N. 2009. Psychologism and the Development of Russell's Account of Propositions. *History and Philosophy of Logic* 30: 171–86.
- Gracia, J. 1975. Propositions as Premises of Syllogisms in Medieval Logic. *Notre Dame Journal of Formal Logic* 16: 545–7.
- Hare, P. H. 1969. Propositions and Adverbial Metaphysics. *Southern Journal of Philosophy* 7: 267–71.
- . 2008. Hare, Peter H.: Autobiography. *American Philosophy: An Encyclopedia*. Edited by John Lachs and Robert B. Talisse. New York: Routledge, pp.357–8.
- Hare, P. H.; Madden, E. H. 1975. *Causing, Perceiving and Believing: An Examination of the Philosophy of C. J. Ducasse*. Dordrecht: D. Reidel, 1975.
- Kennedy, R. (ed.) 1995. *Aristotelian & Cartesian Logic at Harvard*. Boston: Colonial Society of Massachusetts.
- Keynes, J. N. 1884. *Studies and Exercises in Formal Logic*. 1st ed. Macmillan: London.
- . 1906. *Studies and Exercises in Formal Logic*. 4th ed. Macmillan: London.
- Kneale, W.; Kneale, M. 1962/1988. *Development of Logic*. Oxford: Oxford UP.
- Kretzmann, N. 1967. History of Semantics. *Encyclopedia of Philosophy*. Ed. P. Edwards. New York: Macmillan.
- Lachs, J.; Talisse, R. (eds.) 2008. *American Philosophy: An Encyclopedia*. New York: Routledge.
- Lyons, J. 1977. *Semantics*. 2 Vols. Cambridge: Cambridge UP.
- Mill, J. S. 1843/1879. *A System of Logic*. 10th ed. 2 Vols. London: Longmans, Green.
- Newman, J. (ed.) 1956. *The World of Mathematics*. 4 vols. New York: Simon and Schuster.
- Ogden, C.; Richards, I. 1923/1946. *The Meaning of Meaning*. New York: Harcourt.
- Peirce, C. S. 1992. *The Essential Peirce: Selected Philosophical Writings (1867-1893)*. Vol. I. Eds. N. Houser and C. Kloesel. Bloomington: Indiana UP.
- Rescher, N. 1964. *Introduction to Logic*. New York: St. Martin's Press.
- Rivas-Monroy, U. 2010. Epistemology and ontology in Frege and Peirce: on thoughts and generals. Unpublished conference presentation.
- Russell, B. 1900. *Critical Exposition of the Philosophy of Leibniz*. London: Allen & Unwin.
- Santoni, R. E. 1969. Problems Regarding C. J. Ducasse's Analysis of a Proposition. *Southern Journal of Philosophy* 7: 257–65.

- Schlotter, S. 2006. Frege's Anonymous Opponent. *Die Verneinung. History and Philosophy of Logic* 27: 43–58.
- Seibert, C. 2005. Peirce's Reading of Whately's *Elements of Logic*. *History and Philosophy of Logic* 26: 1–32.
- Stein, S. 1963. *Mathematics: the Man-made Universe*. San Francisco: W. H. Freeman.
- Tarski, A. 1941/1946/1995. *Introduction to Logic and to the Methodology of Deductive Sciences*. Trans. O. Helmer. New York: Dover.
- . 1956/1983. *Logic, Semantics, Metamathematics, papers from 1923 to 1938*, 2nd ed. Ed. with introd. and analytic index by J. Corcoran. 3rd printing 1997. Indianapolis: Hackett.
- Watts, I. 1725/1790. *Logick: or the Right Use of Reason in the Inquiry after Truth*. London: Buckland, et al.
- Whately, R. 1826/1988. *Elements of Logic*. Ed. P. Dessi. Bologna: Editrice CLLEB.

JOHN CORCORAN
 Department of Philosophy
 University of Buffalo
 Buffalo, NY 14260-4150
 USA
 corcoran@buffalo.edu

Resumo. A História atesta diferentes abordagens da “proposição”. A proposição tem sido considerada como objeto de crença, descrença e de dúvida: geralmente como *objeto de atitudes proposicionais*, aquilo do qual pode se dizer ser acreditado, desacreditado, entendido, etc. Também tem sido tomada como sendo o objeto de apreensão, julgamento, suposição, afirmação, denegação, e de investigação: geralmente como o *objeto das ações proposicionais*, aquilo que pode ser dito ser apreendido, ser julgado verdadeiro ou falso, ser assumido para fins de raciocínio, etc. A proposição tem sido tomada como sujeito da verdade e da falsidade: geralmente como o *sujeito de propriedades proposicionais*, aquilo que pode ser dito verdadeiro, falso, tautológico, informativo, inconsistente, etc. Ela também tem sido tomada como *sujeito e objeto das relações lógicas*, e.g., aquilo que pode ser dito implicar, ser implicado, contradizer, ser contradito, etc. *Prima facie*, tais propriedades e relações são não-mentais e objetivas. Também tem sido tomada como sendo a *resultante* ou o *produto* das *operações proposicionais*, usualmente mental ou linguística; e.g., julgar, afirmar, e denegar tem sido vistas como produtoras de proposições chamadas julgamentos, afirmações, e negações, respectivamente. As Proposições tem também sido tomadas como sendo certas *sentenças declarativas*. Finalmente, as proposições tem sido tomadas como sendo o *significado* de certas sentenças declarativas. Este ensaio é uma exame informal, seletivo, e incompleto de abordagens alternativas a “a proposição” com especial atenção aos pontos de vista do falecido filósofo americano Peter Hare (1935–2008) e daqueles que o influenciaram.

Palavras-chave: Ações proposicionais; atitudes; operações; propriedades; relações.

Notes

¹ This passage, *Prior Analytics* A1 (24a16), is discussed in Corcoran and Boger 2010.

² The ambiguous words ‘string’ and ‘character’ are both used here in the sense of string theory as in Corcoran, Frank, and Maloney 1974.

³ Use of the word ‘composite’ in this way is due to Church 1956a and 1956b: 6.

⁴ For example, Plato’s *Sophist* (esp. 259–263) contains relevant material as does Aristotle’s *On Interpretation*. Below I have occasion to mention Book A of Aristotle’s *Prior Analytics*.

⁵ This retracts part of what I said about Boethius in my 2010 Transactions article (Corcoran 2010). Justin Legault’s research (per. comm.) almost rules out any chance that Boethius coined the expression. It is more likely that Boethius got it from Cicero.

⁶ I cannot recall seeing ‘argumentation’ used for a sentence before or since. Moreover, Peirce might seem to be taking concepts, judgments, and inferences to be act-events, which—like goal-scorings in football or basketball, or run-scorings in baseball or softball—have only historical existence once brought into being by a person through performing the Operations of Apprehension, Judgment, and Reasoning, respectively. Frege is quite explicit about the act-event nature of judgments in his 1919 article “The Negation” (1997: 354, fn.D).

⁷ Ashworth wrote (1974: 28) that Aquinas “... held that the Organon, and hence logic as a whole, was organized according to the three operations of the human mind”. But, without explicitly saying so, she distinguishes the organization of logic from the subject-matter of logic. The view that the three mental operations form the subject-matter of logic she attributed to Martinus Smiglecius (Marcin Smiglecki, 1564–1618) writing in the 1600s (1974: 34).

⁸ Lotze is thought to have influenced Frege’s thinking about logic (Beth 1965: 23; Frege 1997: 307, 387; Schlotter 2006: 45).

⁹ The logical part of the 1934 Cohen–Nagel book is available as Cohen and Nagel 1934/1962/1993 and a version of the 1936 Tarski book is available in English as Tarski 1941/1946/1995. Both are well worth reading today not only for their contrasting conceptions of logic but also because each represents a transition from “traditional logic” to modern logic.

¹⁰ Today (1) grasping immediate implications would correspond to learning rules of deduction such as modus ponens and substitution of identities while (2) chaining them together directly and *indirectly* would correspond to “chaining principles” as discussed on pages 34 ff. in Corcoran 1989.

¹¹ Robin Smith (1989: 108–9) has some interesting alternative interpretations of these passages, but he does not discuss how they have been interpreted traditionally. The best source for these issues is Paolo Crivelli’s 2004 *Aristotle on Truth*, especially pages 67–71, 82–6, 87–9, and 154–5.

¹² I remember reading this claim myself but at the moment I cannot put my finger on the exact reference. Fortunately, it is well attested (Peirce 1992: xxix). In fact, there is a 2005 article by C. Seibert devoted to it: “Peirce’s Reading of Whately’s Elements of Logic”.

¹³ Ducasse served as president of the ASL (1936–38), the APA (1939), the American Society for Aesthetics (1945–46), and the Philosophy of Science Association (1958–61).

¹⁴ According to Godden and Griffin (2009: 177), the expression ‘propositional attitude’ in this sense was coined by Bertrand Russell in 1918 although in 1910 G.E. Moore had written of belief, disbelief, and understanding as attitudes one can have toward a proposition.

¹⁵ Mill was one of the first logicians to emphasize this aspect of the proposition (1843/1879, V. I, Bk. I, Ch. I, §2). Mill uses the expressions ‘object of belief’ and ‘object of disbelief’ prominent in Hare’s writings. This section of Mill’s book deserves study by anyone interested

in this topic. In his brief §2, Mill combines wisdom and insightful originality with sloppiness and slavery to confused tradition.

¹⁶ Unfortunately, Hare's ideas about the ontics and epistemics of propositional relations such as consequence, contradiction, and consistency cannot be treated in the space available.

¹⁷ I did not adopt this until the 1980s. The following passage was written during that period (1989: 38). Hare attended the colloquium meetings where the paper was discussed.

Some sentences express propositions and some do not. The sentence 'Two exceeds one' expresses the true proposition "Two exceeds one". The sentence 'One exceeds two' expresses the false proposition "one exceeds two". The properties "true" and "false" have as their range of applicability the class of propositions. Any attempt to affirm or to deny "true" or "false" of a non-proposition results in gibberish, incoherence, category error, nonsense. The sentences 'One is true' and 'One is false' do not express propositions at all.

¹⁸ I do not know whether Hare got the terminology of 'connotation' and 'denotation' from Mill, but his usage of these words is not Mill's. See Kretzmann 1967.

¹⁹ Hare's undergraduate thesis was on Whitehead (2008: 357).

²⁰ I think he would regret this ill-advised expression of his naturalistic temperament. His heart was not in it: he did not pursue the project.

²¹ Frege still does not have this quite straight. For 'recognize' he needs 'observe' or some other word that does not connote reliance on previous judging. Besides, when I judged, say, that I missed Peter, I was not observing a proposition but an emotion. This substitution of a proposition for what the proposition is about occurs often in Frege's writings.

²² It will become increasingly important to continue to note ambiguities, particularly the various senses normally attached to the verbs *to affirm* and *to deny*.

²³ The word 'proposition' has been assigned many meanings, sometimes two or more by the same writer in the same work. Hare and Ducasse wrote as if 'proposition' had one meaning, something they do not believe (1969: 267–269).

²⁴ Frege's misnamed 1918 paper "The Thought" is becoming the historical *locus classicus* of the subject.

²⁵ Related points were made repeatedly by Frege in his 1919 "Negation" using essentially this terminology (1997: 349ff.). However, it is not always clear that Frege respects the difference between the act of grasping and the attitude of understanding which is established by, and persists after, the act. After all, it would be perfectly natural to use 'grasp' ambiguously for the act and the attitude. And likewise *mutatis mutandis* for 'understand'. It would be a little awkward to use 'understand' for the act and 'grasp' for the attitude, but no more awkward than usages Frege actually adopted.

²⁶ Were more space available, I would discuss the *propositional action thesis*—every object of a propositional action is a proposition—and its converse, of course.

²⁷ Hare and Madden say that an opinion is "a proposition plus some attitude toward it" (1975: 88). This seems to allow the possibility of opinions being accusatives of attitudes other than belief, disbelief, and doubt.

²⁸ This is my expression not Hare's or Ducasse's. If I remember correctly, it is suggested somewhere by Peirce. Kretzmann's 1967 "History of Semantics" uses 'semantic triad' for a similar constellation of ideas attributed to the Stoics on pages 364–5, but at least implicitly traced to Aristotle's *Categories* 1a1. In apparent ignorance of its history a triangular treatment of

meaning complete with a diagram is found in the Ogden-Richards classic *The Meaning of Meaning* (1923/1946: 11).

²⁹ Hare slipped into using the misleading ‘express’ where he preferred ‘connote’: strictly, propositions are expressed by people not sentences; propositions are connoted by sentences not people. But, we can cut ourselves and others some slack on such niceties.

³⁰ As Corey McGrath observed (per comm.), it should not be said that for Hare *every* two false propositions correspond to the same thing—unless it is added that for Hare *no* two false propositions correspond to the same thing. The point is that a false proposition corresponds to nothing, i.e., it is not the case that a false proposition corresponds. The word ‘nothing’ denotes nothing in the same sense that the word ‘something’ denotes nothing; neither are names and neither denote—in the 1956 Church sense of ‘denote’.

³¹ In conversations and colloquium meetings, Hare often emphasized two of his own hermeneutical principles, both violated in the chapter under discussion. Both concern obligations incurred by charging a respected and accomplished scholar of error. The first is to make it clear how such an error could have been made by such a scholar *and* then how the scholar could have overlooked it. The second is to explain how the mistake may have seemed to advance the scholar’s agenda or otherwise provided the scholar gratification.

³² At least Frege should have known of such contexts (1997: 93).

³³ Frege mentions this point only twice in his voluminous writings (1997: 93, 342).

³⁴ For another view, see Rivas-Monroy (2010), which compares Frege and Peirce on these and related issues.

³⁵ Use of the expression ‘truth-functional’ requires qualification and justification which will not be given in this article. This negation applies to conditions such as “ $x < 2$ ” and to other meanings such as “truth believes money”, which have no truth-value to reverse.

³⁶ To simplify the exposition, I am ignoring at least two other structural, non-attitudinal senses of negative particles such as ‘no’, ‘not’, and ‘non’: one adverbial as in ‘no worse’ and ‘not equal’; one adjectival as in ‘not men’ or ‘non-man’. Anyway, the truth-functional negation is non-adverbial and non-adjectival. This might be the place to express my doubt whether ‘truth-functional’ is the best term to use here. It would be better to have a word whose sense is to “sentence” as “adjective” is to “noun” and as “adverb” is to “verb”.

³⁷ On the face of it, this might seem to be a sensible view for Ducasse to take; the counter-examples—if any—would probably be “exceptions that prove the rule”. However, the passages Hare quoted to support his interpretation seem to suggest a different view often attributed to De Morgan and Jevons: that in a logically perfect language there are no negations of any kind: all terms come in complementary pairs neither of which contains any negative feature, “odd” and “even” in number theory would be an example.

³⁸ Other words such as ‘indicative’, ‘assertoric’, and ‘assertory’ have also been used for sentences that are characteristically used to assent or dissent (Keynes 1884, Part II, Chapter I). However, all three of these words have other meanings. In fact, ‘indicative’ is now so entrenched as a species of verb mood opposed to subjunctive and imperative that its use as a species of sentence is considered erroneous by purists such as Lyons.

³⁹ In 1963, the proposition that $1000! + 1$ is prime was a famous unsolved problem of number theory (Stein 1963: 27), and might still be as far as I know.

⁴⁰ This important topic, which deserves a separate work, has been treated by many logicians down through the ages. There have been three main sub-issues, one for each of the three

traditional propositional attitudes. E.g., in a given assertion, what indicates the acceptive attitude? What makes a speech-act an affirmation and not a denial or an inquiry? Aristotle started the discussion with the claim that the verb “always indicates that something is said or asserted of something” (*On Int.*, III). *The Port Royal Logic* expands on Aristotle (Arnauld 1662/1964, Part II, Ch.2, 104–7). On 107, it says: “The essence of a verb is to indicate the activity of assertion”. Mill said similar things (1843, V. I, Bk. I, Ch. IV, p. 85). And, perhaps surprisingly, Frege seemed to join the parade when he said that assertoric force is bound up with the predicate (1997: 54, 324). Although he broke ranks announcing his obscure but oft repeated view that the assertoric force is supplied by “the form of the assertoric sentence” (1997: 158, 330, 356)—writing as if all assertoric sentences have the same “form”, he never gave up trying to find assertoric force in the sentence as opposed to the speech act—unless he was taking the sentence to be the act and not the string of characters or sounds. Hare seemed to think that some verb forms indicate assertoric force and some do not (1969: 267).⁴¹ James McNabb (per comm.) pointed out that despite the similarity between attitudinal negation and “intuitionist negation”, if the intuitionists can be relied on for accurate reporting of their own meanings, the two cannot be the same since for the intuitionists the statement “it is not the case that $1000! + 1$ is prime” is an affirmation or acceptance, not a denial or rejection. Intuitionists who state that it is not the case that $1000! + 1$ is prime affirm that they have derived a contradiction from “ $1000! + 1$ is prime”.