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On the Philosophical Import of Some Accomplishments of Newton da Costa

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Abstract. From Newton da Costa's works, many people in France know only the revival of paraconsistency. We give some reasons in defence of investigations in this part of logic. But above all we recall one of the major contributions of Newton da Costa: his proof, in 1991, in collaboration with Doria, of the gödelian undecidability of motion in mathematical physics, a result which was somewhat foreseen on other grounds by Duhem in 1906.

Keywords: Paraconsistency; mathematical physics; undecidability of ergodicity.

I related in Guillaume 1996 how I was induced to become the first European scholar to come to Brazil to meet Newton da Costa when his paraconsistent systems began to come out. I am delighted with the thought that in this journey, and in the ones that followed over several years, may be found the first distant and not at all exclusive origin of the recognition which was awarded him by his election as a member of the International Institute of Philosophy just twenty years ago.

Nevertheless, a recent polemic in France has revealed a persistent misunderstanding of Newton's accomplishments among the general public, as well among scholars, including some of the highest level.

First, the purpose of paraconsistency is misunderstood as simply "allowing contradictions", without at all referring to the fact that our ordinary life (and even, since the twentieth century, our scientific life) is surrounded by contradictions.

It may be conceded that there are contradictions which come from differences in the ways in which such-and-such a question is tackled, and that in such cases a contradiction can be reduced to as many incompatible consistent treatments. It may also happen that a contradiction finds its origin in incomplete or, worse, inaccurate information. In this case, it may be that the time needed for acquiring the indispensable complementary amount of information is unknown, or known to be much greater than the time required for reaching some urgent decision. How can one reason safely in such cases? Abstracting somewhat further, this question leads one to ask, *what are the laws which remain open to reasoning, in the presence of contradictions?* Such problems were stated long before da Costa,[1] and even studied before him,[2] by scholars of levels as high as those of his critics. His contribution was to propose some concrete modern formal systems as a first, and not definitive, approach. He

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himself is always the first to admit that many other and perhaps better views about paraconsistent logic can and must be investigated, and that corresponding formal systems can and should be studied thoroughly.

Thus the doors of philosophical speculation were opened wide to paraconsistency. *Without illusions*, for Newton, either with regard to the purely exploratory character of his work (which nevertheless shed some light on the links between meanings of connectives[3]), or with regard to its philosophical importance, which must neither be underestimated nor overestimated: there is much life outside paraconsistency.

This is precisely the *second point* on which Newton da Costa is misunderstood. He was incessantly widening the range of logical and computational problems in the progress of his research, and the philosophical impact of some of his results in these subject matters surpasses by far that of his work on paraconsistency.

One of the most noteworthy in my view, perhaps the most striking *of these results* of which many scholars and biographers are not sufficiently aware, is the Gödelian undecidability of some problems of classical mechanics and dynamical systems theory, as established in Da Costa and Doria 1991 with the help of references lavished by many guest speakers at a seminar held in São Paulo.

I shall now explain what is at stake in these matters by relating what was known in the past, and what the results of da Costa and Doria add to our former knowledge about the undecidability of a simple problem in physics related to some properties of the trajectories of material points.

Let us read together two passages written by Pierre Duhem, commenting on some contributions of Jacques Hadamard. These quotations are a bit long, but the first is one of the greatest literary texts ever written by a mathematician, on a par with some of the writings of men such as Blaise Pascal and Henri Poincaré. I borrow the quotations[4] from Bouligand 1936, who was interested in commenting on them for reasons that differ from my own. Here is the first of these texts:[5]

> Imagine the forehead of a bull, with the protuberances from which the horns and the ears start, and with the collars hollowed out between these protuberances; but elongate these horns and ears without limit so that they extend to infinity; then you will have one of the surfaces we wish to study.[6]
>
> On such a surface, geodesics may show many different aspects.
>
> There are, first of all, geodesics which close on themselves. There are some also which are never infinitely distant from their starting point even though they never exactly pass through it again; some turn continually around the right horn, others around the left horn, or right ear, or left ear; others, more complicated, alternate, in accordance with certain rules, the turns they describe around a horn with the turns they describe around the other horn, or around one of the ears. Finally on the forehead of our bull

with his unlimited horns and ears there will be geodesics going to infinity, some mounting the right horn, others mounting the left horn, and still others following the right or left ear.

Despite this complication, if we know with complete accuracy the initial position of a material point on this bull's forehead and the direction of the initial velocity, the geodesic line that this point will follow in its motion will be determined without any ambiguity. In particular, we shall know whether the moving point will always remain at a finite distance from its starting point or whether it will move away indefinitely so as never return.

It will be quite a different matter if the initial conditions are not mathematically but practically given: the initial position of our material point will no longer be a determinate point on the surface, but some point taken inside a small spot; the direction of the initial velocity will no longer be a straight line defined without ambiguity, but some one of the lines included in a narrow bundle connected by the contour of the small spot; and our practically determined initial conditions will, for the geometer, correspond to an infinite multiplicity of different initial conditions.

Let us imagine certain of these geometrical data corresponding to a geodesic line that does not go to infinity; for example, a geodesic line that turns continually around the right horn. Geometry permits us to assert the following: Among the innumerable mathematical data corresponding to the same practical data, there are some which determine a geodesic moving indefinitely away from its starting point; after turning a certain number of times around the right horn, this geodesic will go to infinity on the right horn, or on the left horn, or on the right or left ear. More than that: despite the narrow limits which restrict the geometrical data capable of representing the given practical data, we can always take these geometrical data in such a way that the geodesic will go off on that one of the infinite folds which we have chosen in advance.

It will do no good to increase the precision with which the practical data are determined, to diminish the spot where the initial position of the material point is, to tighten the bundle which includes the initial direction of the velocity, for the geodesic which remains at a finite distance while turning continually around the right horn will not be able to get rid of these unfaithful companions who, after turning like itself around the right horn, will go off indefinitely. The only effect of this greater precision in the fixing of the initial data will be to oblige these geodesics to describe a greater number of turns embracing the right horn before producing their infinite branch; but this infinite branch will never be suppressed.

If, therefore, a material point is thrown on the surface studied starting from a geometrically given position with a geometrically given velocity, mathematical deduction can determine the trajectory of this point and tell whether this path goes to infinity or not. But for the physicist, this deduction is forever unutilizable. When, indeed, the data are no longer known geometrically, but are determined by physical procedures as precise as we may suppose, the question put remains and will always remain unanswered.

Bouligand then skips a bit more than two pages before quoting another passage. I shall do likewise, because the parts in question shed more light on the point, from mathematics to physics, than I wish to draw attention to here. The text continues as follows:

> … a mathematical deduction is of no use to the physicist as long as it is limited to asserting that a given *rigorously*[7] true proposition has for its consequence the *rigorous*[7] accuracy of some such other proposition. *To be useful for the physicist, it must still be proved that the second proposition remains* APPROXIMATELY *exact when the first is only* APPROXIMATELY *true*.[8] And even that does not suffice. The range of these two approximations must be delimited; it is necessary to fix the limits of error which can be made in the result when the degree of precision of the methods of measuring the data is known; …[9]

Here appears with greatest clarity the fundamental reason why the aspects of trajectories which are *mathematically* decided are undecidable *for physics*: a physical measure never has the *absolute precision* assumed by the mathematical datum of a real number; such a daydream is given from an imagined infinite sequence of integers, and by various methods of handling the terms of the sequence in an imagined uniform way; an approximative measure furnishes only an initial segment of the sequence, and there will always be a continuum of infinite sequences sharing this segment; it is easy to prove that the length of the segment will not change the characteristics which the members of the bundle of infinite sequences extending it can possess.

Be it under this form or under another, this reasoning did not take into account Heisenberg's uncertainty principle, since the lines quoted above were written in *Duhem 1906*, commenting on *Hadamard 1898*! What does the principle of Heisenberg add to this earlier reasoning? Since the principle *entails* that the precision with which the set of required data can be given has a greater than zero constant inferior bound, it paves the way for replacing by a formal proof the previous informal proof of physical[10] undecidability, which appealed to some non-axiomatized notions.

Now, what does Da Costa and Doria 1991 add to this matter? In their own words, they prove[11] that

> If ZFC is arithmetically consistent[12] … there is a motion $m(t)$ on $\mathbb{R}^2$ of which it is true[13] in **M** that $m(t)$ is ergodic in $\mathbb{R}^2$, but such that its ergodicity cannot be proved from the axioms of the theory.

The proof of this result pertains to *mathematical* physics, and here is precisely the point: this time, the undecidability is algorithmic; even *with the absolute precision* assumed to be at our disposal, we do not escape undecidability.

Appendix: The French original of the two passages from Duhem 1906 quoted above

Imaginons le front d'un taureau, avec les éminences d'où partent les cornes et les oreilles, et les cols qui se creusent entre ces éminences, mais allongeons sans limite ces cornes et ces oreilles, de telle façon qu'elles s'étalent à l'infini ; nous aurons une des surfaces que nous voulons étudier.

Sur une telle surface, les géodésiques peuvent présenter bien des aspects différents.

Il est, d'abord, des géodésiques qui se ferment sur elles-mêmes. Il en est aussi qui, sans jamais repasser exactement par leur point de départ, ne s'en éloignent jamais infiniment ; les unes tournent sans cesse autour de la corne droite, les autres autour de la corne gauche, ou de l'oreille droite, ou de l'oreille gauche ; d'autres, plus compliquées, font alterner suivant certaines règles les tours qu'elles décrivent autour d'une corne avec les tours qu'elles décrivent autour de l'autre corne, ou de l'une des oreilles. Enfin, sur le front de notre taureau aux cornes et aux oreilles illimitées, il y aura des géodésiques qui s'en iront à l'infini, les unes en gravissant la corne droite, les autres en gravissant la corne gauche, d'autres encore en suivant l'oreille droite ou l'oreille gauche.

Malgré cette complication, si l'on connaît avec une entière exactitude la position initiale d'un point matériel sur ce front de taureau et la direction de la vitesse initiale, la ligne géodésique que ce point suivra dans son mouvement sera déterminée sans aucune ambigüité. On saura très certainement, en particulier, si le mobile doit demeurer toujours à distance finie ou s'il s'éloignera indéfiniment pour ne plus jamais revenir.

Il en sera tout autrement si les conditions initiales sont données non point mathématiquement, mais pratiquement ; la position initiale de notre point matériel sera non plus un point déterminé sur la surface, mais un point quelconque pris à l'intérieur d'une petite tache ; la direction de la vitesse initiale ne sera plus une droite définie sans ambigüité, mais une quelconque des droites que comprend un étroit faisceau dont le contour de la petite tache forme le lien ;[14] à nos données initiales pratiquement déterminées correspondra pour le géomètre une infinie multiplicité de données initiales différentes.

Imaginons que certaines de ces données géométriques correspondent à une ligne géodésique qui ne s'éloigne pas à l'infini, par exemple, à une ligne géodésique qui tourne sans cesse autour de la corne droite. La Géométrie nous permet d'affirmer ceci : Parmi les données innombrables qui correspondent aux mêmes données pratiques, il en est qui déterminent une géodésique s'éloignant indéfiniment de son point de départ ; après avoir tourné un certain nombre de fois autour de la corne droite, cette géodésique s'en ira a l'infini soit sur la corne droite, soit sur la corne gauche, soit sur l'oreille droite, soit sur l'oreille gauche. Il y a plus : malgré les limites étroites qui resserrent les données géométriques capables de représenter nos données pratiques, on peut toujours prendre ces données géométriques de telle sorte que la géodésique s'éloigne sur celle des nappes infinies que l'on aura choisie d'avance.

On aura beau augmenter la précision avec laquelle sont déterminées les données pratiques, rendre plus petite la tache où se trouve la position initiale du point matériel, resserrer le faisceau qui comprend la direction initiale de la vitesse, jamais la géodésique qui demeure à distance finie en tournant sans cesse autour de la corne droite ne pourra être débarrassée de ces compagnes infidèles qui, après avoir tourné comme elle autour de la même corne, s'écarteront indéfiniment. Le seul effet de cette plus grande précision dans la fixation des données initiales sera d'obliger ces géodésiques à décrire un plus grand nombre de tours embrassant la corne droite avant de produire leur branche infinie ; mais cette branche infinie ne pourra jamais être supprimée.

Si donc un point matériel est lancé sur la surface étudiée à partir d'une position géométriquement donnée, avec une vitesse géométiquement donnée, la déduction mathématique peut déterminer la trajectoire de ce point et dire si cette trajectoire s'éloigne ou non à l'infini. Mais, pour le physicien, cette déduction est à tout jamais inutilisable. Lorsqu'en effet les données ne sont plus connues géométriquement, mais sont déterminées par des procédés physiques, si précis qu'on les suppose, la question posée demeure et demeurera toujours sans réponse.

(. . .)

. . . une déduction mathématique n'est pas utile au physicien tant qu'elle se borne à affirmer que telle proposition, rigoureusement vraie, a pour conséquence l'exactitude rigoureuse de telle autre proposition. Pour être utile au physicien, il lui faut encore prouver que la seconde proposition reste à peu près exacte lorsque la première est seulement à peu près vraie. Et cela ne suffit pas encore ; il lui faut délimiter l'amplitude de ces deux à peu près ; il lui faut fixer les bornes de l'erreur qui peut être commise sur le résultat, lorsque l'on connaît le degré de précision des méthodes qui ont servi à mesurer les données ; . . .[15]

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Resumo. Das obras de Newton da Costa, muitas pessoas na França conhecem apenas o renascimento da paraconsistência. Apresentamos algumas razões em defesa de investigações nessa parte da lógica. Acima de tudo, porém, relembramos uma das maiores contribuições de Newton da Costa: sua demonstração, em 1991, em colaboração com Doria, da indecidibilidade gödeliana do movimento na física matemátcia, um resultado que foi de certa forma previsto, por outras razões, por Duhem em 1906.

Palavras-chave: Paraconsistência; física matemática; indecidibilidade da ergodicidade.

Notes

[1] For example by Wittgenstein 1964, which consists of notes from the *Nachlass* dated 1929-1930 by the *Stanford Encyclopedia of Philosophy*. Arruda 1980, a review of contributions to paraconsistent logic, refers specifically to page 332 of this text.

[2] By Jaśkowski 1948 certainly (following a suggestion of Łukasiewicz, according to Arruda 1980), and perhaps, but this time in a non-formalized way, by Vasili'ev some years before World War I. What Vasili'ev did from 1910 to 1931, in particular by laying philosophical and axiomatic foundations for some non-formalized logical systems and by deriving their first developments, had in 2003 not yet been completely investigated—this according to Moretti 2007, who mentions one of these systems which does not meet the propositional spirit in which many other paraconsistent systems—in particular the C_n of da Costa—have been conceived.

[3] In Guillaume 2007 I was able to establish for some subsystems of C_n ($n \leq \omega$), where the principle *ex falso sequitur quodlibet* (where 'false' means 'asserted and denied') is put for 'well-behaved' formulas, an equivalence between the acceptance of some halves of De Morgan's laws for well-behaved conjuncts and the corresponding da Costa axioms of 'propagation of well-behavior' from these conjuncts to their conjunction—and the same for disjunction. I also established some analogues for implication as well as for negation, under the condition in this last case that $n < \omega$ holds. Each of these equivalences is *independent* of the others.

[4] However, I reinsert three passages, each of about two lines, left out by Bouligand.

[5] When first presenting this article, I used a translation that I myself had made some days earlier. At the time I mentioned my suspicions of the existence of an earlier translation by some author of whom English was the mother tongue. On the grounds of Pierre Duhem's

reputation as a philosopher of science, I thought that such a translation should date from the two decades following the publication of the original. I found that in fact the translation, Duhem 1982 (first published in 1954) by Philip P. Wiener, which I substitute here for my own, had been produced in the middle of the twentieth century. However, Bouligand quotes Duhem 1906, pp.226–8, whereas Wiener has translated Duhem 1914, pp.209–11. In the later edition of the original there are a few small changes in the use of words which do not at all change the meaning of the earlier text, so that the translation conserves the whole of its value with regard to the earlier edition as well. The strength in French literary expression in the first edition is, in my eyes, a bit affected in the places where these changes have been made.

[6] [Bouligand's footnote] To be understood thus: a surface with opposite curvatures, endowed with four infinite folds.

[7] This emphasis is introduced in Duhem 1914 and appears neither in Duhem 1906 nor in Bouligand 1936.

[8] In this phrase, the emphases *in italics* and *in small capitals* have been added by Bouligand. Duhem 1906, Duhem 1914, and Wiener 1982 emphasize (in italics) only the words found here in small capitals.

[9] Duhem 1906, p.231. I give the original of the quoted French passages from Duhem 1906 in an appendix to this article.

[10] To wit, belonging to a theory of physics.

[11] Da Costa and Doria 1991, proposition 5.3, 1., p.1069.

[12] This means that all arithmetical sentences that are provable in ZFC are 'satisfied in the standard model of the axioms of arithmetic'—a condition of coherence.

[13] Here **M** is a model of ZFC which involves the standard model of the axioms of arithmetic.

[14] Writing in the France of the beginning of the twentieth century, where eighty per cent of the population were peasants and in the cities of which many houses were flanked by private gardens, Duhem had in mind the image of a bale of straw with its band, or that of a bunch of flowers, as well as, nearer to physics, that of a bundle of rays of light.

[15] Bouligand omitted, in error to my mind, to quote the last proposition of the phrase: "il lui faut définir le degré d'incertitude que l'on pourra accorder aux données, lorsqu'on voudra connaître le résultat avec une approximation déterminée." *Wiener's translation has: "it is necessary to define the probable error that can be granted the data when we wish to know the result within a definite degree of approximation".*