doi: 10.5007/1808-1711.2011v15n2p333

On the Principle of Excluded Middle

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Abstract. I carry out in this paper a philosophical analysis of the principle of excluded middle (or, as it is often called in the version I favor here, principle of bivalence: any meaningful assertion is either true or false). This principle has been criticized, and sometimes rejected, on the charge that its validity depends on presuppositions that are not, some believe, universally obtainable; in particular, that any well-posed problem is solvable. My goal here is to show that, although excluded middle does indeed rest on certain presuppositions, they do not have the character of hypotheses that may or may not be true, or matters of fact that may or may not be the case. These presuppositions have, I claim, a transcendental character. Hence, the acceptance of excluded middle does not necessarily require, as some have claimed, an allegiance to ontological realism or some sort of cognitive optimism, construed as factual theses concerning the ontological status of domains of objects and our capability of accessing them cognitively.

Keywords: Excluded-middle; logical principles; transcendental logic; phenomenology.

Introduction

Tertium non-datur — excluded middle or bivalence[1] — is one of the fundamental principles of logic.[2] It asserts that meaningful assertions[3] always have a definite truth-value — the true or the false — regardless of whether we can actually tell which, now or ever. Of the three basic principles of logic (the others are identity and non-contradiction) it is the one more often contested. Intuitionists, for example, are well known for denying the universal validity of excluded middle. The reason is that they believe mathematical assertions express mental experiences (or constructions), not transcendent facts, and consequently no mathematical assertion can have a definite truth-value independently of us being able to tell which, if we cared to. If decision procedures are not available for any given assertion (procedures that do not involve the disputed principle, of course[4]), it lacks, they say, a definite truth-value. To accept excluded middle unconditionally, intuitionists claim, amounts to presupposing the solvability of any well-posed problem. For making an assertion, they say, amounts to posing a problem, that of verifying the assertion; for intuitionists, asserting commits one to the task of providing intuitionistically valid proofs of one's assertion whenever asked.

Aristotle, who thought the validity of the principle went along with the ontological definiteness of reality,[5] also believed excluded middle was not valid in general,

Published by NEL — Epistemology and Logic Research Group, Federal University of Santa Catarina (UFSC), Brazil.

for it failed for future contingencies, that is, assertions about future events that are not necessarily predetermined at the moment of assertion (for the world of the future is not a definite reality today).

In short, for intuitionists, the unrestricted validity of excluded middle requires decision procedures; for Aristotle, ontological definiteness of reality. The difference of approach has to do with different conceptions of truth. For Aristotle, assertions are true if they conform to the facts, independently of whether we have the means for witnessing this conformity (hence, no assertion can have a definite truth-value if the factuality of what it asserts remains indeterminate); for Brouwer and the intuitionists, assertions are true only if they are verifiably true (so, no assertion can have a definite truth-value if its verification is not within our reach by "constructive" means). On the one hand, truth as an *objective* correspondence between what is said and what is; on the other, truth as the *subjective* experience of such a correspondence.

So, if one endorses excluded middle one has to offer some reason for so doing, at least if one does not take the formalist or syntactic stand in logic, for which a principle of logic is not a constraint for *correct* reasoning, but simply a matter of choice.[6] But how can a logical principle, which is prior to any justification (since logical principles, together with logical rules of inference, constitute the foundation of any justification), be justified? A logical principle cannot be proven, otherwise it wouldn't be a principle; if a logical principle were to rest on other logical principles it would be reducible to them. So, by justification I cannot obviously mean a logically valid proof, but simply some ground or rationale for accepting the principle, which of course must be more compelling than personal taste or the exercising of free will. I will try here to make clear on which bases the validity of excluded middle can be put to rest, which presuppositions it requires and the nature of these presuppositions. More than a justification in the strict sense my goal is philosophical clarification.

Presuppositions behind excluded middle usually involve different ways of understanding the notions of reference, meaning and truth. For the Aristotelian minded (call him "classicist"), assertions are meaningful provided they represent situations that may or may not subsist in independently existing "worlds out there" (empirical reality, the world of mathematics, or any other). So, the validity of excluded middle in the logic underlying a rational discourse depends on the presupposition that the reality this discourse purports to represent is completely determined in-itself (or ontologically complete)[7]. For those sympathetic to intuitionism or similar perspectives (call them "intuitionists") statements are meaningful if and only if they can be verified in some appropriate, "constructive" way; i.e. we have the means for bringing about an experience of verification, if we cared to. It follows that any "intuitionistically" meaningful assertion is either true or false (but not all "classically" meaningful assertions are also, necessarily, "intuitionistically" meaningful). "Classicists", if they accept excluded-middle, are clearly committed to the ontological

completeness of domains of reference whereas "intuitionists", if they do not chose to restrict the validity of the principle to "intuitionistically" meaningful assertions (as real intuitionists do), are clearly committed to a form of epistemological optimism.[8]

Upon closer consideration, however, there is something odd about all this. Both presuppositions involve matters of *fact*, concerning either how the world is or the range of our cognitive abilities (we, the cognitively severely limited beings we are). But a principle of logic, taken as a canon of *reason*, cannot depend on matters of fact; a principle, if on anything at all, can depend solely on matters of *principle*.

An obvious alternative would be to simply move from the sphere of fact to that of right; that is, to substitute the "is" of these presuppositions by a "must be". If the relevant reality (the domain of interest) were, *by necessity*, ontologically complete or proper assertions, as a matter of principle, *necessarily* verifiable, then, it seems, any assertion would necessarily be either true or false, even if we were not able to actually tell which.

Along this line of validation, however, excluded middle is made to depend on presuppositions involving *necessities*. This, at first sight, seems odd; how can necessities be a matter of presupposition? Our problem then shifts to the nature of presuppositions that cannot be construed as hypotheses or matters of convenience or taste; presuppositions that, like the principle they justify, must also be a matter of principle, which cannot involve simply what *is* the case, but what *must be* the case. Only when the nature of these presuppositions is clarified the principle of excluded middle can claim the dignity of a real logical principle. There is then no alternative but going beyond the facts to where necessity lies, the transcendental realm; i.e. the intentional life of a cognitively oriented transcendental subject, concerned, in particular, with the transcendental task of constituting reality and which experiences concerning reality are in principle available to it (given *only* how reality is conceived or intentionally constituted). The validity of *tertium non datur* will be shown to depend on how the field of possible experiences of the subject and its objective counterpart, the "world" the subject strives to know, are intentionally constituted. In fewer words, this paper intends to be an analysis of the transcendental sources of validity of the principle of excluded middle.

1. Excluded Middle

Consider the following versions of excluded middle:

A) Meaningful assertions are either *truth in-themselves* or *false in-themselves*, tertium non-datur. Husserl called this the *objective* version of the principle of excluded middle; it asserts that a definite truth-value belongs to meaningful assertions by right (whether or not we can tell which, now or ever).

B) Meaningful assertions can in principle be verified. If we drop the qualifying "in principle", this is the "intuitionist" characterization of meaningfulness. Husserl called this the *subjective* version of excluded middle. The important point is that only verifiability *in principle*, not in actual fact, is required here (we may not be able to actually carry out verifications that are granted in principle). The difference is immense, but since some idea of verification is present, if only in principle, it is not inappropriate to use Husserl's terminology.

An experience of verification is either an experience of harmony or one of conflict. In the first case the situation represented by a meaningful assertion is experienced as a fact of reality; that which the assertion asserts to be the case *is experienced* as being in fact the case. In the second, instead of harmony we have conflict; the subject experiences the inadequacy of the situation represented vis-à-vis the facts of reality. If *either* experience is actually lived through, we say the assertion is actually verified (by an actual evidential experience); if either experience is within our reach, even if not actually lived through, we say the assertion is actually verifiable. Our problem is to find out what verifiability "in principle" can possibly mean. Given this understanding of verifications, (C) below is only a restatement of (B):

C) Meaningful assertions represent situations that can in principle be checked against the facts; that is, either the harmony or the conflict of the content of a meaningful assertion with reality constitutes a possible experience of a subject in general.

Of course, there are so many undefined terms in (A), (B) and (C) that we do not really know what these sentences are expressing. Before any clear sense is given to them we must clarify what *meaningful* assertions and *intrinsic* truth-values are and what it means to say that assertions (resp. evidential experiences or experiences of verification) are *in principle verifiable* (resp. *in principle possible*).

Let us begin with the notion of intrinsic truth-values. Consider the following definition (or characterization):

Definition 1. Assertions have definite *intrinsic truth-values* or *truth-values in-themselves* (the true or the false) if and only if they are in principle verifiable (expressions in *italic* are those being defined). Of course, this is so far only a dictionary definition, for we do not yet know what to be in principle verifiable means.

This way of understanding what it means to say that an assertion possesses a logical value by right, even though we may not tell which, is, I think, completely natural: the intrinsic character of logical values goes together with the possibility in principle, *but only in principle*, of verifying which value this is; "intrinsic" or "in-itself" are to assertions what "in principle" is to our possibilities of verification. If we

eliminate "in-itself" from the left-hand side of the equivalence and, in the right-hand side, substitute "in principle" by "actual", we have the "intuitionist" characterization of truth-value possession: an assertion has a definite truth-value if and only if it is actually verifiable (i.e. we can tell which value this is by appropriate, "constructive" means).

Notice that, given definition 1, A and B are — as they should be — equivalent (both are, after all, statements of the same principle).

Let us now consider the criterion of possibility in principle for evidential experiences of a subject in general. The field of possible evidential experiences of a subject contains three distinct regions: 1) experiences it has already had; for, after all, actuality implies possibility, for any characterization of possibility; 2) experiences actually within its reach (the actually possible or adequately implementable experiences), and 3) experiences that are possible only in principle. Of course, what is or is not possible in principle cannot be a matter of experience but must be determined *a priori*. Hence, in determining the scope of its field of possible (evidential) experiences the subject is clearly fulfilling a transcendental task, which is invariably expressed as a principle. The principle below has then a transcendental character:

Principle. Evidential experiences *possible in principle* are those that cannot be ruled out a priori as actually obtainable (emphasis on *a priori*); all the others are, *as a matter of principle*, impossible.

Of course, it remains for us to tell when such an experience cannot be ruled out a priori as actually obtainable. As is clear, this principle extends the limits of evidential experiences available to the subject (at least in principle) well beyond the actually possible experiences.

Now, as we have already seen, an assertion is in principle verifiable if and only if either the harmony or the conflict between the situation it represents (its content) and the relevant facts of the domain of reference (the relevant reality) is an evidential experience possible in principle. So, we have the following:

Corollary 1. *Assertions are verifiable in principle if and only if the situations they represent cannot be ruled out a priori from being checked against the facts.*

Hence we have the following equivalent of the subjective version of excluded middle: situations represented by meaningful assertions cannot be ruled out a priori from being confronted with reality (call this *).

Therefore, by definition 1 and the corollary above:

Corollary 2. *Assertions have definite intrinsic truth-values if and only if what they assert (their content) cannot be ruled out a priori from being checked against the facts.*

It is easy to see that * and corollary 2 together imply the objective version of excluded middle: meaningful assertions have definite intrinsic truth-values.

We must now investigate what the conditions are for *evidential experiences* (resp. *contents of assertions*) to be unable of being ruled out a priori *as obtainable* (resp. *from being confronted with reality*). But first we must find an independent characterization of meaningfulness.

2. Meaningfulness

There are many theories of meaning on the philosophical shelves. For one of them, meaning has to do with truth conditions; to know the meaning of an assertion is to know what has to be the case for it to be true. Meaning is what assertions express; what one grasps when one understands them; what connects assertions to the reality they purport to describe; arrows pointing towards reality, if assertions are true, away from it, if they are false. Intuitionists, M. Dummet in particular, believe that meaning has to do with conditions of verification, instead of conditions of truth. To know the meaning of an assertion, they claim, is to know how to verify it. To grasp the meaning of assertions is to be able to move along the arrows of a previous metaphor towards or away from reality, to be able to confirm or disconfirm them. Husserl, on his turn, believed meanings are objective, albeit non-independent (i.e. abstract) entities, analogous to Frege's *Sinne*, constituted in intentional acts of consciousness by means of which sounds and symbols are given a representational role.

However, I am not concerned here with meaning, but meaningfulness, the property of being meaningful or having a meaning. I will admit, as already established, that meaningful assertions represent situations that *can in principle* be confronted with reality, in some sense of "can". The question is how to interpret this modality. Since the situations that can and those that cannot in principle be confronted with the facts must be determined *a priori* (we are, after all, dealing with matters of principle), this "can in principle be" can only, I claim, be read as "cannot be ruled out a priori from being". Our problem now is to find necessary and sufficient conditions for assertions to be unable to be ruled out a priori from being confronted with reality; once this is accomplished we will have a characterization of meaningfulness.

These conditions must, of course, be independent of what the facts are. One alternative is to search for them in the system of language itself. The conditions for meaningfulness must, I suppose, be related to those for the *correct* use of language, since these are the conditions that guarantee that language can indeed play a representational role. To tell meaningfulness from meaninglessness or, equivalently, whether a combination of signs of the system in which assertions are expressed (the language) represents a situation that cannot be ruled out a priori from being verified

must, I claim, be within the reach of *any* competent user of this system. And to be a competent user of a language is to use it according to (largely implicit) rules of grammatical and semantic correctness.

The most basic claim of this paper is then that assertions express situations that can in principle be checked against the facts (or, equivalently, are meaningful) if and only if they accord to rules of *syntactic* and *semantic* correctness; rules defining, respectively, syntactic and semantic sense, and whose mastery constitutes linguistic competence.

An assertion has syntactic sense if it conforms to the syntactic rules of the representational system in which the assertion is expressed; that is, rules regulating the lawful combination of *logical types*. It has semantic sense if it conforms to rules governing the lawful combination of *semantic types*. These rules, as we will soon see, are closely related to how the domain of reference *is meant*. The task of semantic rules is to avoid *material counter-sense* (such as "large fat numbers tend to snore below the visible spectrum"); that of syntactic rules, merely *formal nonsense*.

Some examples: "Pegasus is flying" is semantically meaningful, for "Pegasus" purports to refer to a particular horse that is not *a priori* forbidden to fly in the real world, the only where things can *really* fly. Since Pegasus *in fact* (*but not in principle*) does not exist, the judgment is false. (This assertion can also be meant to refer to a fictional or mythological reality where things can fly-in-the-fiction, in which case the assertion can very well be true.) "The number π is flying", on the other hand, is semantically meaningless (albeit syntactically meaningful), for numbers are a priori forbidden to fly. Relational terms, on their turn, must refer to relations that are not ruled out a priori from subsisting in the domain, *considering only the meaning attributed to it*. For example, "prime numbers are lonelier than composite numbers" is semantically meaningless; for, giving the meaning we attach to the numerical domain, "lonelier than" is not a numerical relation, and so cannot appear, as a matter of principle, in arithmetical assertions. The mathematical assertion "175 is a prime number", on the other hand, is meaningful, and then, according to our characterizations, either true in-itself or false in-itself. A quick verification shows that it is actually false (and so, verifiably false). [9]

Based on these considerations I suggest the following definitions:

Definition 2. An assertion is *meaningful* if and only if it has both syntactic and semantic sense.

Definition 3. A situation *cannot be ruled out a priori* from being confronted with the facts if and only if it is represented by a meaningful assertion. An experience of either harmony or conflict of the content of an assertion with the facts (i.e. an experience of verification) *cannot be ruled out a priori* from belonging to the field of

(evidential) experiences of a subject in general if and only if the assertion in question is meaningful.

There is a last strand we have to pull, a further question we must address in order to drive excluded middle back to its origins: what are, after all, the sources of linguistic correction? Where does the semantic rule that tells, for instance, that a given predicate P is in principle predicable of a given subject S — and so "S is P" is meaningful — take its authority from? Obviously, linguistic rules are not arbitrarily set; rather, they are designed to serve the main purposes of languages, especially scientific languages, of *describing* facts and *representing* situations (in whatever domain or reality we may be interested on). Syntactic rules deal with logical types (or syntactic categories) and tell us when a determinate combination of types is well formed so as to fulfill a representational role. But they are not sufficient; combinations of semantic types must also obey a priori laws. Semantic types are the linguistic counterparts of ontological types; and so semantic rules, those governing the lawful combination of semantic types, must reflect the legality built into the domain of reference itself.

Ontological rules are constitutive of how domains are *meant*, and so point back to intentional acts of constitution. We are again on transcendental grounds. The subject conceives, a priori, the domain of knowledge as endowed with a certain meaning, and it is this meaning that in the end determines the laws regulating the combination of ontological types. So, meaningfulness of assertions depends basically on how the domains to which assertions refer are intentionally constituted, the meaning intentionally attached to them previously to the subject actually experiencing or describing them, or even referring to them.

Of course, conceiving a world, or, equivalently, endowing it with a meaning, does not mean to predetermine what is and is not the case in this world, but only what can and cannot in *principle* be represented as being the case. The world, any world, is a totality of things falling into categories and modes of being obeying a priori "cans" and "cannots"; i.e. there is a "grammar" of categories and modes of being imposed by how any given domain of being is conceived (a moral value, for instance, cannot *in principle* be green; nor a rational number be mildly annoyed). Assertions can only be meaningful if they obey, besides purely syntactic rules regulating the combination of symbols (an assembling of symbols is not always an assertion), semantic rules reflecting at the linguistic level the ontological rules of the grammar of being of the domain in question. In short, meaningfulness of assertions depends ultimately on how the domains assertions refer to are conceived. And endowing things with meaning is a typical act of transcendental subjectivity. In the end, an assertion is meaningful if (1) it is well-formed according to the syntactic rules of formation of the language in question; (2) its terms denote entities (objects, relations, etc.) that

are not a priori seen, given how the domain is conceived, not to exist or subsist in the domain (i.e. the terms must denote possibly existing entities); and (3) the semantic types occurring in it "have to do" with each other (considering only a priori compatibilities and incompatibilities internal to the domain).

No "justification" of *tertium non-datur* can avoid Aristotle's future contingencies such as "there will be a sea battle tomorrow". Let us see how they can be handled.

The logical form of the assertion "there will be a sea battle tomorrow" is this: there is an x, such that x is a sea-battle-between-instants-t_1-and-t_2. Now, the question is: to which reality this assertion refers; what is its scope? If it is the world as given *today*, i.e. before instant t_1, the predicate "sea-battle-between-instants-t_1-and-t_2" is a priori seeing as not applicable to the world; therefore, the assertion is semantically meaningless, and so does *not* have a definite intrinsic truth-value (in conformity with Aristotle's conclusion). If, on the other hand, we mean it to refer to the world of today *and* tomorrow taken together, then it is not only meaningful, but actually verifiable; the verification procedure being: wait until tomorrow and check. Since, however, the domain of reference is not time-invariant definiteness of intrinsic truth-value is also time-dependent: it *will* be determined at instant t_2, but not before. We cannot expect truth-value to be determined *sub species aeternitates* when reality is not.

3. Completeness

It is constitutive of our conception of the field of *actual* evidential experiences of a subject in general that it constitutes a *consistent* whole (this finds logical expression in the principle of non-contradiction). The principle of excluded middle sets an ideal: the field of actual evidential experiences of the subject is in the limit a *maximally* consistent totality; *either* the harmony *or* the conflict with the facts of any adequately expressible situation (but not both) can ideally be experienced by the subject. This translates into an *ideal* of science: to produce maximally consistent, that is, complete systems of assertions (theories) concerning domains of scientific interest (Husserl explicitly stated this desideratum in his logical writings). In case of formally axiomatized theories this translates into *logical completeness*, i.e. the requirement that any sentence of the language of the theory, or its negation (but not both) be a theorem of the theory[10].

When the *theory* (not necessarily the formalized theory) of a domain is complete I say this domain is *epistemologically complete*. Epistemological completeness entails ontological completeness, but not conversely. It may be the case that a domain is "out there" entirely constituted and complete for us to know but we, subjects of knowledge, do not yet have the means for carrying out this task to completion.

Since *tertium non-datur* poses epistemological completeness as an *ideal*, it implicitly *presupposes* the ontological completeness of the domain the subject strives to know. This presupposition, however, as sufficiently stressed, does not have the status of a *factual* hypothesis; it enjoys instead the transcendental status the intentional constitution of the domain of knowledge confers to it. The presupposition of ontological completeness rests on how the subject *conceives* a priori the domain of knowledge to be, be it physical nature itself, the realms of mathematics or any other.

One may argue that objectively independent domains, like empirical nature, are ontologically complete; but that man-made domains, such as those of mathematics (supposing one is a mathematical constructivist) are not necessarily so and therefore *tertium non-datur* is not in general valid in mathematics. But, as already stressed, ontological completeness is not a character some domains have and others do not as a matter of *fact* (if it were, to *presuppose* ontological completeness would be an unjustifiable *metaphysical* presupposition), but a character *intentionally attributed* to them by cognitively motivated transcendental consciousness, and so a matter of principle. Ontological completeness is not the exclusive hallmark of ontologically independent domains, and so excluded middle is not, as Dummett claims, the hallmark of realism (Dummett 1978, pp. 145–65).

Hence, any theoretical domain can be metaphorically conceived as being completely given "out there" for us to investigate making full use of the principle of excluded middle and striving for epistemological (or maybe even logical) completeness. *The ontological completeness of scientific domains is a fundamental transcendental presupposition of science*.[11]

4. Final remarks

As I have argued here the validity of excluded middle does not to depend on factual hypotheses, but presuppositions of transcendental nature. This, of course, cannot be taken as a "proof", in the sense that the *truth* of the principle was established; a principle cannot be proven, only justified, that is, philosophically clarified. It is not even completely appropriate to attribute truth to logical principles. Adequacy would be better (adequacy to intentional acts, of course, not practical purposes). Now, can the subject refuse to endorse these presuppositions? Given that transcendental presuppositions are part of intentional acts of constitution, the subject can very well refuse them, if the intentional object (the domain) is posited as an ontologically incomplete totality, an object "in the making", as intuitionists conceive mathematical realms. Some presuppositions would then be immediately cancelled, in particular, obviously, that the domain so constituted is complete in-itself. Other presuppositions might still hold; for instance, that assertions correctly constructed (i.e. according to

the rules of the language) represent situations that are in principle verifiable — unless, of course, the notions of meaningfulness or verifiability in principle, or both, are also modified. Our subject may not equate possible evidential experiences with experiences that simply cannot be ruled out a priori, or verifiable assertions with assertions that are merely correctly constructed, in favor of much stricter (for example, effective) conditions for the possibility of experiences and the verifiability of assertions (the meaning attached to ontologically incomplete domains seems in fact to require these more stringent conditions).

The adherents to excluded middle (the "classicists") might reply that effective possibility cannot be granted a priori and requiring it would undermine the *a priori* character of logic (something that would not, as we know, trouble the intuitionist mind). To those who refuse to grant meaningful assertions the power of determining per se the scope of verifiability, or impose more severe conditions for meaningfulness (as Dummett did) the "classicist" might reply that, in the first case, it would become impossible to determine *a priori* which assertions are verifiable and, in the second, that meaningfulness would not be at reach of any competent user of the language who happens to know what he is talking about (the meaning attributed to the domain of reference). Anyway, principles cannot be proven, but they can be disputed. In the end, the true conflict is one between "world visions", or better, conceptions, ways in which intentional consciousness constitute different "realities". What does not seem right to me is the trade-off approach of the sort Quine and others have proposed (Quine 1981).

Even if the domain of knowledge does not exist in-itself as an independent reality, or, in other words, even if it is *metaphysically* unreal, to posit excluded middle as a valid principle of reasoning concerning this domain is tantamount to constituting it as an *objectively* real *transcendent* domain, completely determinate in-itself, extending maybe *beyond the horizon* of the field of actual possibilities of experience of the subject[12]. By directing its interests towards a domain so constituted, the subject of knowledge has the right, guaranteed a priori, *in principle*, to use excluded middle as a valid rule of reasoning, and expect to eventually obtain a logically complete theory in which each objective determination of the domain is established.[13]

Now, do we necessarily know how to put ourselves in a position to experience the facts proven? No, not necessarily. A non-contructive proof does not in general allow us to actually experience the fact established. This, however, does not mean that the "non-constructive" proof is irrelevant from the perspective of those who expect proofs to provide truth epiphanies: it tells us, at least, that the fact represented by the *negation* of the assertion proved cannot be actually experienced. The principle of excluded middle can then be used, *even by those who do not endorse it*, to downsize the set of candidates for the living experience of truth (or, in less metaphorical terms, to select the assertions for which constructive proofs — those which do not involve

excluded middle or any principle equivalent to it, such as the principle of double negation — can at least be expected). We cannot ask more of an *a priori* theory of truth.

I would like to conclude by clarifying what I understand by "transcendental subjectivity". The allusion to any kind of subjectivity, in particular when logical matters are concerned, tends to be read as a sure sign of psychologism, the bête noire of certain philosophical circles (which blocks the correct understanding of logical principles in these circles). Transcendental subjectivity is understood here as nothing more than a community at large (which coincides, in case common scientific interests is what ties the community together, with the scientific community), spread in space and time, sharing the same conceptions about reality and using the same language for referring to it (and working together cooperatively, in the case of scientific communities). This community, of course, does not play a purely passive role vis-à-vis domains of reference and knowledge; it befalls on it the unavoidable (transcendental) task of predetermining them to a certain extent (as far as their *necessary* features are concerned).

Let me give a scientific example. Physical nature, which could not be more metaphysically real, is pre-endowed (by the scientific community) with characteristics that cannot be obtained by experience (the *ideal* characters of nature), but which help to predetermine the experiences that can in principle be expected. For instance, continuity of physical space, time, and other physical magnitudes can *never* be verified, but we, nonetheless, can (*as a matter of principle*) expect to be able to *indefinitely* improve our measurements. Continuity in empirical domains is then an *ideal*, and *natura non facit saltus* a transcendental principle (at least according to the classical conception of physical nature born with modern science).

Mathematical realms although not metaphysically real are objectively real. The mathematical community (playing the role of transcendental subjectivity) constitutes them as objectively existing domains determined in themselves, thus constituting, concomitantly, complete systems of truths-in-themselves, approachable by gradually improved methods (that cannot be determined a priori, being as they are products of historical developments) which will be, or so we hope, completely disclosed eventually. To confuse this view with ontological or epistemological realism, or worse, *metaphysical* idealism is a gross error (if anything, it is *transcendental* idealism).

Transcendental subjectivity is an active agent in the dynamics of knowledge; it cannot be put aside in favor of an erroneous picture of knowledge as a dipole having in one end a reality that is what it is, and, in the other, a passive subject that tries to uncover its secrets as well as it can, without preconceived presuppositions. Where necessity and principles are concerned, transcendental subjectivity is at work.

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Resumo. Meu objetivo aqui é levar a cabo uma análise filosófica do princípio do terceiro-excluído ou princípio de bivalência: toda asserção com significado é ou verdadeira, ou falsa. Esse princípio tem sido criticado, e às vezes rejeitado, com o argumento que sua validade depende de pressupostos que não são, crêem alguns, universalmente válidos. Em particular, que qualquer problema bem posto pode ser resolvido. Eu quero mostrar que, ainda que o princípio dependa realmente de pressupostos, eles não têm o caráter de hipóteses que podem ou não ser verdadeiras, ou fatos que podem ou não ser o caso. Esses pressupostos têm, eu afirmo, natureza transcendental. Portanto, a aceitação do princípio do terceiro-excluído não requer, como alguns acreditam, compromissos com o realismo ontológico ou alguma forma de otimismo epistemológico, entendidos como teses sobre domínios de objetos e nossa capacidade de acessá-los cognitivamente.

Palavras-chave: Princípio do terceiro-excluído; princípios lógicos; lógica transcendental; fenomenologia.

Notes

[1] In my paper "Husserl on the Principle of Excluded Middle" (da Silva 2005) I attributed to Husserl views that are very close to those I put forward here. This time, however, I let them stand for themselves. This paper intends to offer what I take as the *correct* interpretation of the logical principle of excluded middle and the role it plays in science.
The principle of excluded middle is sometimes taken as simply asserting the truth of any sentence of the form $p \vee \neg p$, p an arbitrary assertion, and the principle of bivalence as asserting that, for any p, either p is true or p is false.
[2] By "logic" I do not mean freely conceived formal-logical systems designed to represent arbitrary procedures of formal derivation (in contexts maybe where the notion of truth may not even be the relevant one). I have in mind something along Husserl's conception of logic as the theory of science or Frege's, as the theory of truth.

[3] Or propositions, judgments, thoughts, sentences or any truth-bearer, I will not particularly care which.
[4] Whenever intuitionist proofs or decision procedures are meant, I presuppose them not to involve excluded middle. An intuitionist proof establishes truth in a "constructive" way, which counts as an *actual experience* of the fact proven. Proofs involving excluded middle (or principles equivalent to it), on the other hand, are considered non-constructive idealizations of the actual experience of the facts proven.
[5] This is how Aristotle justifies excluded middle in his *Metaphysics* (book Gamma, 1011b): "And the possibility of a middle between contradictories is excluded; for it is necessary either to assert or to deny one thing of another. This is clear from the definition of truth and falsity; for to deny what is or to affirm what is not is false, whereas to affirm what is or to deny what is not are true; so that any judgment that anything is or is not states either what is true or what is false." Implicit in this definition of truth is the *metaphysical* presupposition that reality is definite (or complete) and always one out of two contradictory situations is a fact. Since truth is adequacy to the facts, the validity of excluded middle follows from the ontological completeness of reality.
[6] Those who take this line of "justification" for logical principles, such as Church with respect to excluded middle (Church 1929), are in general inspired by the development of non-Euclidean geometries, in which geometrical axioms are seen as purely formal stipulations devoid of any material (intuitive) content. Similarly, a principle such as excluded middle is not supposed to express any truth; accepting it amounts to simply adding all assertions of the form $p \vee \neg p$, where p denotes an arbitrary assertion, to the stock of logical axioms.
[7] Ontological determinateness or completeness reflects the fullness of being (and I will not attempt to characterize it any further). Fantasies or fictions (a novel, for instance) need not be, and usually are not ontologically complete (it is not determinate in-itself whether a situation not explicitly contemplated by the author of a novel actually occurred or not), although readers usually prefer to think, for the fun of it, that they are.
[8] Brouwer, the staunchest critic of the unconditional validity of excluded middle in mathematics believed that to accept the principle is tantamount to indorsing Hilbert's optimism expressed in the belief that any well-posed mathematical problem has a solution. Hilbert, at least at some point in life, believed that this "principle of rational optimism" was provable; presupposing, of course, a non-constructive, mathematical sense of existence (maybe, I conjecture, by formalizing the whole of mathematics in a formal system that is provably syntactically complete, a hope Gödel utterly destroyed).
[9] More, it is logically necessarily false. The falsity of this assertion follows logically (by specification and modus ponens) from the *definition* of prime number *and* the *fact* that 175 has proper divisors. Since this is a *necessary* fact, the falsity of the assertion is a necessary consequence of the definition of prime number (since logical consequences of necessary facts are necessary). If we take our characterization of meaningfulness (semantic and syntactic correction) as defining a sense of possibility, weaker than even logical possibility we can say meaningful assertions represent *possible* facts. The assertion "175 is a prime number" would then be possibly true (in this weaker sense), even though necessarily false (in a logical sense).
[10] This is, I repeat, an ideal; we cannot guarantee that it is an attainable ideal for all possible theories.

[11] "The possibility of sciences depends entirely on this certainty that their provinces exist in truth, and that, concerning their provinces, theoretical truths-in-themselves exist, as actualizable by following explorable and gradually actualizable ways of cognition." (Husserl 1969, p. 199)

[12] The positing of a transcendent reality (or better, a reality *intentionally constituted as transcendent*) cannot be mistaken as a sign of commitment to ontological realism, which is a metaphysical perspective, involving metaphysical instead of transcendental presuppositions. The confusion of objective reality with metaphysically real reality (a stronger condition) must also be avoided (many mathematicians — and some philosophers of mathematics — cannot see the difference; this is why they are so often prone to ontological realism).

[13] Gödel's theorem is often raised as a caveat to such hopes, but Gödel only bars the possibility in general of systems of truths to be constituted as actually complete, not only as logical systems indefinitely under completion.