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MODELS & PROOFS: LFIS WITHOUT A CANONICAL INTERPRETATION

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Abstract. In different papers, Carnielli, W. & Rodrigues, A. (2012), Carnielli, W. Coniglio, M. & Rodrigues, A. (2017) and Rodrigues & Carnielli, (2016) present two logics motivated by the idea of capturing contradictions as conflicting evidence. The first logic is called *BLE* (the Basic Logic of Evidence) and the second—that is a conservative extension of *BLE*—is named LET_J (the Logic of Evidence and Truth). Roughly, *BLE* and LET_J are two non-classical (paraconsistent and paracomplete) logics in which the Laws of Explosion (EXP) and Excluded Middle (PEM) are not admissible. LET_J is built on top of *BLE*. Moreover, LET_J is a *Logic of Formal Inconsistency* (an *LFI*). This means that there is an operator that, roughly speaking, identifies a formula as having classical behavior. Both systems are motivated by the idea that there are different conditions for accepting or rejecting a sentence of our natural language. So, there are some special introduction and elimination rules in the theory that are capturing different conditions of use. Rodrigues & Carnielli's paper has an interesting and challenging idea. According to them, *BLE* and LET_J are incompatible with dialetheia. It seems to show that these paraconsistent logics cannot be interpreted using truth-conditions that allow true contradictions. In short, *BLE* and LET_J talk about conflicting evidence avoiding to talk about gluts. I am going to argue against this point of view. Basically, I will firstly offer a new interpretation of *BLE* and LET_J that is compatible with dialetheia. The background of my position is to reject the one canonical *interpretation thesis*: the idea according to which a logical system has one standard interpretation. Then, I will secondly show that there is no logical basis to fix that Rodrigues & Carnielli's interpretation is the canonical way to establish the content of logical notions of *BLE* and LET_J. Furthermore, the system LET_J captures inside classical logic. Then, I am also going to use this technical result to offer some further doubts about the one canonical interpretation thesis.

Keywords: Philosophical interpretations • paraconsistency • LFIs • conflicting evidence.

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1. Introduction

In this paper, I am interested in analyzing the relationship between pure logics and their interpretations. For example, it could be stimulating to discuss if there are some intrinsic characteristic in pure modal logic *S5* (presented by axioms or by Kripke-models) and the well-known David Lewis' interpretation using actual possible worlds and accessibility relations. Is there a single canonical interpretation for *S5*? Are there any central features *S5* sufficient to determine a single canonical interpretation? Questions like those raised for all kinds of logics. In particular, non-classical logics have been motivated considering specific Interpretations that serve as reasons for challenging classical logic. In the context of paraconsistent logics, logicians have provided different ways of interpreting how we should reason in the presence of contradictions. In this way, in a recently work, Rodrigues & Carnielli (2016) present two logics motivated by the idea of capturing contradictions as conflicting evidence. The first logic is called *BLE* (the Basic Logic of Evidence) and the second—that is a conservative extension of *BLE*—is named LET_J (the Logic of Evidence and Truth). Roughly, *BLE* and LET_J are two non-classical (paraconsistent and paracomplete) logics in which the Laws of Explosion (EXP) and Excluded Middle (PEM) are not admissible. LET_J is built on top of BLE. Moreover, LET_J is a Logic of Formal Inconsistency (an *LFI*). This means that there is an operator that, roughly speaking, identifies a formula as having classical behavior. In particular, there are instances of EXP that can be captured inside LET_J using a circle operator. Both formal theories are presented by a natural deduction system (a set of introduction and elimination rules for the conjunction, disjunction and the material conditional), But, they focus on the negation and introduce special rules for refutability. The main idea is that there are different conditions for accepting or rejecting a sentence of our natural language. So, there are some special introduction and elimination rules in the theory that are capturing different conditions of use. As I said before, the underlying motivation provided for *BLE* and LET_J is that rules should preserve evidence for an assertion rather than its truth. Rodrigues & Carnielli's paper has an interesting and challenging idea. *BLE* and LET_J are incompatible with dialetheia. It seems to show that these paraconsistent logics cannot be interpreted using truth-conditions that allow true contradictions. In short, *BLE* and LET_J talk about conflicting evidence avoiding to talk about gluts. I am going to argue against this point of view. Basically, I will firstly offer a new interpretation of *BLE* and LET_J that is compatible with dialetheia. The background of my position is to reject the one canonical *interpretation thesis*: the idea according to which a logical system has one standard interpretation. Then, I will secondly show that there is no logical basis to fix that Rodrigues & Carnielli's interpretation is the canonical way to establish the content of logical notions of *BLE* and LET_J. Furthermore, the system LET_J captures inside classical logic. Then, I am also going to use this technical result to offer some further doubts about the one canonical interpretation thesis.

The paper is structured as follows. I first provide a quick presentation of *BLE* and LET_J. Then, I distinguish between pure and applied logics. I show both levels are relatively independent and with different proposals. I describe some aspects of the relationship of these planes. There are important results that seem to show that the relationship between both levels is not so direct as Rodrigues & Carnielli seem to hold: there are some pure systems with multiple interpretations, and there are also pure logics without interpretation at all. My proposal is to show that pure theories are not directly connected with some particular interpretation. In next section, I offer some arguments against Rodrigues & Carnielli's point of view on the standard interpretation of *BLE* and LET_J. The first argument tries to reject the thesis according to which these systems are incompatible with dialetheia. The next step is to show that it is not possible to find a logical grounding to consider that Rodrigues & Carnielli's interpretation is the standard interpretation. Finally, the last argument tries to offer additional reasons to reject the thesis according to which pure systems have canonical interpretations. In last part of the paper, I try to avoid some possible misunderstandings connected with my ideas.

2. The pure logics *BLE* and LET_J

In circumstances where evidence is incomplete or contradictory, the inferences that are right are different from classical logic. In particular, there are good reasons to support the idea according to which classical principles of Explosion and Excluded Middle are not valid. Rodrigues & Carnielli share this idea. Then, they present two justification logics that try to deal with the way in which we should reason in everyday situations; contexts with 'excess of information' and 'lack of information'. *BLE* and LET_J are logics motivated by these epistemic constraints.

2.1. A system of proof for BLE

Now, it is important to show which are the main characteristics of *BLE* and LET_J. Let L_0 be a language with a denumerable set of basic formulas $\{p_0, p_1, p_2, \ldots, p_n\}$, parentheses, and closed under the connectives in the set $\{\neg, \wedge, \vee, \rightarrow\}$. The total set s_0 of formulas of L_0 is obtained recursively in the usual way. Roman capitals stand for meta-variables for formulas of L_0. The definition of a derivation D of A from a set Γ of premises is the usual one for natural deduction systems. With the proposal to show which are the assertion conditions of a formula in the system, Rodrigues & Carnielli present the system *PIL* (positive intuitionistic propositional logic) as a starting point. One gets *PIL* adopting the following rules:

$$\frac{A \quad B}{A \wedge B}\ \mathrm{I}\wedge \qquad\qquad \frac{A \wedge B}{A}\ \mathrm{E}\wedge \qquad\qquad \frac{A \wedge B}{B}\ \mathrm{E}\wedge$$

$$\frac{A}{A \vee B}\ \mathrm{I}\vee \qquad \frac{B}{A \vee B}\ \mathrm{I}\vee \qquad \frac{A \vee B \quad \begin{array}{c}[A]\\ \vdots \\ C\end{array} \quad \begin{array}{c}[B]\\ \vdots \\ C\end{array}}{C}\ \mathrm{E}\vee$$

$$\frac{\begin{array}{c}[A]\\ \vdots \\ B\end{array}}{A \to B}\ \mathrm{I}\to \qquad \frac{\begin{array}{c}A \to B\\ A\end{array}}{B}\ \mathrm{E}\to$$

As usual in natural deduction systems, $[A]$ means that hypothesis A has been discharged (or canceled). The elimination rules may be obtained from the introduction rules (Gentzen & Prawitz's Inversion Principle). Rules for $\wedge$, $\vee$ and $\to$ turn out to be intuitionistic. But, of course, an important issue in paraconsistent and paracomplete logics is to specify a negation without (some of) the properties of a classical or intuitionistic negation.[1] Up to this point, Rodrigues & Carnielli have given the sufficient conditions for assertions. Now, they introduce specific rules for a non-classical negation in formulas with the other logical expressions of L_0. The initial motivation at this point is to capture the conditions of refutability: for example, what would be sufficient conditions for refuting a conjunction or a conditional? In Fitting's words: "Negation is, so to speak, not treated negatively but positively" (2016, p.2). Then, instead of intuitionistic negation, *BLE* has the introduction and elimination rules for Nelson's strong negation. Following this idea, they introduce these rules for negations:

$$\frac{\neg A}{\neg(A \wedge B)}\ \neg\mathrm{I}\wedge \qquad \frac{\neg B}{\neg(A \wedge B)}\ \neg\mathrm{I}\wedge \qquad \frac{\neg(A \wedge B) \quad \begin{array}{c}[\neg A]\\ \vdots \\ C\end{array} \quad \begin{array}{c}[\neg B]\\ \vdots \\ C\end{array}}{C}\ \neg\mathrm{E}\wedge$$

$$\frac{\neg A \quad \neg B}{\neg(A \vee B)}\ \neg\mathrm{I}\vee \qquad \frac{\neg(A \vee B)}{\neg A}\ \neg\mathrm{E}\vee \qquad \frac{\neg(A \vee B)}{\neg B}\ \neg\mathrm{E}\vee$$

$$\frac{A \quad \neg B}{\neg(A \to B)}\ \neg\mathrm{I}\to \qquad \frac{\neg(A \to B)}{A}\ \neg\mathrm{E}\to \qquad \frac{\neg(A \to B)}{\neg B}\ \neg\mathrm{E}\to$$

$$\frac{A}{\neg\neg A}\ \mathrm{DN} \quad \text{(double line)}$$

The logic defined by the rules DN, introduction and elimination for $\wedge$, $\vee$ and $\to$, plus introduction and elimination for negated $\wedge$, $\vee$ and $\to$ is *BLE*. Rodrigues & Carnielli point out that the negation rules exhibit a symmetry on the corresponding assertion rules for the dual operators. As expected *BLE* does not have a way to prove the principles: $A \vee \neg A$ or $\neg(A \wedge \neg A)$. But *De Morgan* principles: $\neg(P \vee Q) \longleftrightarrow (\neg P \wedge \neg Q)$ and $\neg(P \wedge Q) \longleftrightarrow (\neg P \vee \neg Q)$ have a proof in this logic. It is important to note that $\neg\neg P \longleftrightarrow P$ also has a proof and It could be important to emphasize that If one adds to *BLE* the rules:

$$\frac{A \quad \neg A}{B}\ \mathrm{EXP} \qquad \frac{\begin{matrix}[A] \\ \vdots \\ B\end{matrix} \qquad \begin{matrix}[\neg A] \\ \vdots \\ B\end{matrix}}{B}\ \mathrm{PEM}$$

one gets the classical logic. But, obviously, these rules are not part of *BLE*.

The derivations of *BLE* have the following properties: Reflexivity, Monotonicity, Transitivity (cut), Deduction theorem and Compactness. Since its consequence relation is reflexive, monotonic and transitive, then *BLE* is a Tarskian logic.

The pure logic BLE is motivated by the idea according to which there are situations where one has "excess of evidence" or "lack of evidence". Evidence could be incomplete and contradictory in a lot of real life contexts. But, maybe there are some contexts where evidence is complete and consistent. In such contexts, reasoning becomes classical: Excluded Middle and Explosion should be valid. How to recover these contexts without reject BLE? Rodrigues & Carnielli give a response introducing the pure logic LET_J.

2.2. A system of proof for $\mathbf{LET}_J$

The *LFIs* are a family of paraconsistent logics able to express, inside the object language, the notions of 'consistency', or even 'inconsistency', as applied to formulas.

This is done employing a unary propositional connective '$\circ$' to the language, where '$\circ A$' is informally interpreted as *A is consistent*. *BLE* is a paraconsistent logic, but it is not an *LFI*. (*BLE* has not expressive power to recover classical validities.) But one might be tempted to get a tool for dealing with classical contexts of reasoning inside a paraconsistent frame. The idea is to use a consistency operator to be able to restore classical logic for some propositions. So, what we need is to add to *BLE* the means of recovering the properties of classical negation—or, more precisely, we need to restore the validity of explosion and excluded middle on those formulas for which we want to recover classical logic. This will be made clear in what follows.

Let L_1 be a language that is the result of adding to L_0 a consistency operator '$\circ$' . As before, L_1 is closed under the connectives in the set $\{\neg, \wedge, \vee, \rightarrow, \circ\}$. The total set s_1 of formulas of L_1 is obtained recursively in the usual way. And again roman capitals stand for meta-variables for formulas of L_1 and the definition of a derivation D of A from a set Γ of premises is the usual one for natural deduction systems. Intuitively one has to describe how to use the consistency operator in order to capture the classical behavior of propositions.

So, LET_J is the logic defined by the addition of EXP$\circ$ and PEM$\circ$ to *BLE*.

$$\frac{\circ A \quad A \quad \neg A}{B}\ \text{EXP}\circ \qquad\qquad \frac{\circ A \quad \begin{matrix}[A]\\ \vdots \\ B\end{matrix} \quad \begin{matrix}[\neg A]\\ \vdots \\ B\end{matrix}}{B}\ \text{PEM}\circ$$

Paraconsistent logics can deal with contradictory scenarios, avoiding triviality using the rejection of the Principle of Explosion. It is in the sense that these theories do not trivialize in the presence of (at least some) contradictory sentences. *LFIs* are able to capture inside a paraconsistent logic classical scenarios where accepting and rejection the same sentence is not allowed.

2.3. A model theory for *BLE* and LET_J

Rodrigues & Carnielli also offer a semantic theory for *BLE*. According to them, these models lack an intuitive appeal independent of the corresponding deductive system. They emphasize that "(r)ather, such semantics should be seen as a mathematical tool capable of representing the inference rules in such a way that some technical results may be proved (...)" (Rodrigues & Carnielli 2016, p.10). So, Model theory for *BLE* and LET_J should not be considered as a way to offer actual meanings to logic constants of these systems. In this deflationary sense, the following semantics presented below is sound, complete, and yields a decision procedure for *BLE*.

A semivaluation s for *BLE* is a function from the set S of formulas to $\{0, 1\}$ such that:

1. if $s(A) = 1$ and $s(B) = 0$, then $s(A \to B) = 0$;
2. if $s(B) = 1$, then $s(A \to B) = 1$;
3. $s(A \wedge B) = 1$ iff $s(A) = 1$ and $s(B) = 1$;
4. $s(A \vee B) = 1$ iff $s(A) = 1$ or $s(B) = 1$;
5. $s(A) = 1$ iff $s(\neg\neg A) = 1$;
6. $s(\neg(A \wedge B)) = 1$ iff $s(\neg A) = 1$ or $s(\neg B) = 1$;
7. $s(\neg(A \vee B)) = 1$ iff $s(\neg A) = 1$ and $s(\neg B) = 1$;
8. $s(\neg(A \to B)) = 1$ iff $s(A) = 1$ and $s(\neg B) = 1$.

A valuation for BLE is a semivaluation for which the condition below holds:

(Val) For all formulas of the form $A_1 \to (A_2 \to \ldots \to (A_n \to B)\ldots)$ with B not of the form $C \to D$:
if $s(A_1 \to (A_2 \to \ldots \to (A_n \to B)\ldots)) = 0$, then there is a semivaluation s' such that for every i, $1 \leq i \leq n$, $s(A_i) = 1$ and $s(B) = 0$.

One says that a valuation v is a model of Γ ($v \models \Gamma$) if for all B $\in \Gamma$, $v(B) = 1$; $v \models A$ means that $v(A) = 1$.

Now, the notion of *logical consequence* in *BLE* is defined as follows:

$\Gamma \models$ A if and only if for every valuation v, if v is a model of Γ, then $v(A) = 1$.

While it is true that logical consequence of *BLE* is defined as truth preservation, this should be seen only as a way of talk. The actual meaning of logical expressions of *BLE* should be looked for the inferential rules. Rodrigues & Carnielli seem to think that there is something about *BLE* that forces us to adopt a proof-theoretic semantics to explain the content of logical constants of this system. A similar strategy is adopted by them in the case of LET_J.

Now, in order to extend the semantics presented before to LET_J we need only to add the clause:

9. $s(\circ A) = 1$ implies $[s(\neg A) = 1$ iff $s(A) = 0]$

The clause 9 above says that if $\circ A$ holds, we secure classical conditions for negation, but not the converse. Indeed, there may be a valuation such that $s(\neg A) = 1$ and $s(A) = 0$ (or vice-versa) but $\circ A$ still does not hold. But again, according to Rodrigues & Carnielli, the actual meaning of circle operator should be looked for the rules EXP$\circ$ and PEM$\circ$.

3. Introducing what is a logic: pure and applied formal systems

As I remarked, Rodrigues & Carnielli (2016, p.10) argue that *BLE* and LET_J are logics that are capturing different conditions for accepting or rejecting an assertion based on evidence. In its treatment, evidence might be incomplete or contradictory. Then, there are an introduction and an elimination rule in these theories that are capturing different conditions of use of assertions in contexts in which one has conflicting information. So, the underlying motivation provided for *BLE* and LET_J is that rules should preserve evidence for an assertion. For example, the interpretation of the positive introduction-$\wedge$ is if k and k' are evidence, respectively, for A and B, k and k' together constitute evidence for $A \wedge B$. Instead, they don't think of this rule as saying if A and B are true, so is $A \wedge B$.

Now let me distinguish between *pure* and *applied logics*. The main proposal to develop a logic is to offer a formal theory about consequences. Uncontroversially, logic is the study of reasoning. Nevertheless, logic is not about the way that people actually think. People frequently reason making mistakes. Logic does not tell us how people *do* reason, but how they *ought* to reason. So, logic is normative.[2] The study of reasoning, in the sense in which logic is interested, concerns the issue of what follows from what. And this issue can be analyzed at two different levels.

At the pure level, given a set of sentences, logic should show us what follows from these sentences. A good argument is one whose premises entails its conclusion; its conclusion is a consequence of its premises. Logic should be very general, abstract and topic neutral. One should pay no attention to the subject matter of the sentences: when one is interested in logical properties, it does not matter what about the sentences are talking. It does not matter either what is known or the epistemic source of the reasons to support a premise. What is only important is to pay attention to the logical relationship between them. With the proposal of explaining this notion, one develops several approaches to give a theory of what is entailment from what. Then, proof-theoretical account tries to explain how to proof something from something. They can be presented by sequent calculus, natural deduction, axiomatics systems. These theories have been developed to focus on different aspect of proofs. Pure logics also can be presented by models. Model theoretical approach tries to explain how to preserve semantics designated values from semantics values. Set-theoretical models, plurals models, non-deterministic semantics, partial valuations, Kripke's models, etc. have been developed to illuminate the notion of *validity*. Models and proof are mathematical instruments to understand the concept of *consequence*. Generally speaking, soundness and completeness are properties that pure systems should have. In this sense, any evaluation of a pure logic is based on the fulfillment of those properties.

Generally speaking, logics should be neutral about what there is or what one knows that there is. It should not talk about anything. Pure systems help us to understand better theories about consequences and their properties. The search for results of soundness and completeness is a central goal at this level. The deduction and the cut-elimination theorems, the interpolation results, the compactness of models and the decidability of a system of proof are fundamentals elements to evaluate a pure logic.

In Priest's words:

> First, there are numerous pure logics. This point I take to be relatively uncontentious. There are the many-valued logics that Łukasiewicz invented, not to mention others such as intuitionism, quantum logic, and paraconsistent logic (one of which, LP, we met in the preceding sections). Possibly, a purist might say that they are not logics since they are not the *real* logic. But that would be like saying that non-Euclidean geometries are not geometries since they are not the *real* geometry. In both cases we have a family of structures (logics or geometries) that are perfectly well-defined mathematical structures; and, as far as that goes, all on a par. (2005, p.164–5)

In contrast, these formal systems can be applied to bear on a particular problem in a specific area. Of course, one can use a pure logical theory (from a proof-theoretical or model-theoretical point of view) to get a reliable grasp of some (philosophical-scientific) concepts. This is what makes useful logic in philosophy reasoning about time, process, modalities. This is also useful to computer science when one can apply logic to computational process, belief revision, database, etc. Or even in linguistic when one can apply logic to grammar structures. Logic can be the result of our legitimate interest in some philosophical and scientific concepts. For example, modal logics have been motivated by our interest in explaining what is metaphysically necessary or possible. Then, one can propose interpreting *true in w* as *true in a possible world*. But our interest could be another. For example, one can be interested in knowledge. And we can use Kripke-models as talking about epistemic states o informational scenarios. Or one can be interested in what is obligatory and permitted and uses Kripke-models as talking about acceptable worlds. Similar structures can be used to give the models for Intuitionistic logic. Here one can understand this structures as talking about constructive proofs. Applied logic is about interpretations: inferential rules can be used as a model for mathematical reasoning, truth values can be interpreted as gaps, gluts, meaningless or as evidence to support beliefs. Let me emphases the point with another example: It is well known that truth values can be interpreted in different ways in the system Strong Kleene *K3*. In particular, the value .5 can be interpreted as a gap or glut. A similar consideration can be made about Weak Kleene *WK3*: the value .5 can be interpreted as being meaningless (Bochvar's interpretation (Bochvar 1981)) or as being off-topic (Beall's interpretation(Beall 2016)). Moreover recently several works about how to analyze paraconsistent Weak Kleene have been published. All of them are examples that involve the pure systems shows how these features could determine one or more interpretations.[3]

Nowadays, there are a lot of pure logics. Classical logic is one of them. Some pure systems extend classical logic (as modal and epistemic logic), and there are a wide variety of non-classical systems: intuitionistic logic, minimal logic, etc. Paracomplete and paraconsistent logics are pure theories about consequences that are in this last group. Some of these systems have interpretations, some of them have different interpretations, and some of them do not have interpretation at all. Obviously, some of these systems were motivated by some interpretation at the moment of their development. Sometimes is really important to have in mind some informal reading of the inferential rules or models of a pure system to propose a logic. I am not rejecting this idea at all. Simply, I am saying that the question of giving a reading of a pure system of logic is not a natural consequence of this pure system. And it is an interesting task to find new interpretations for pure systems. But one has to have in mind that pure logics usually have multiple interpretations. And one also has to admit that pure systems are important beyond interpretations. For example, it seems intuitively clear that multiple conclusion sequent calculus of Gentzen does not applicate to our actual inferential practice, but it can be considered part of theoretical reflexions about logical consequences.

Of course, one can ask whether claims about the connection between the formalism and the part of natural language modeled by the formalism are objective. In my view, pure logic theories should be judged independently of any applications to natural languages. The meta-logical properties are usually our criteria of adequacy. And what is even more important: It could be that disagreements about pure theories and disputes about pure logics do not imply anything about our ontological or epistemological commitments. It is evident to me that claims about a pure logic do not immediately imply claims about interpretations.

Nevertheless, some philosophers and logicians have adopted the opposite point of view. They have supported that there are ontological or epistemological consequences because of adopting some pure systems. For example, Timothy Williamson has supported classical Logic because of this system talks (canonically) about absolute generality. He thinks of a logical theory as a theory of unrestricted generalizations. These generalizations are not specifically about properties of arguments, sentences, propositions; they are generalizations about absolutely all things in the world (Williamson 2015). David Lewis is another example of one canonical thesis application. According to him, modal logics talk about actual possible worlds.[4] Michael Dummett has adopted a similar approach. From his point of view, classical logic has necessarily ontological commitments. This system is committed with metaphysical realism. The use of classical logic in a realm of discourse commits one to realism concerning that discourse (Dummett 1991). Another case in the same line is Prawitz. In a recently paper, he affirms:

> The term "proof-theoretic semantics" was introduced to stand for an approach to meaning based on what it is to have a proof of a sentence. (...) in contrast to a truth-conditional meaning theory, one should explain the meaning of a sentence in terms of what it is to know that the sentence is true, which in mathematics amounts to having a proof of the sentence. (Prawitz 2016)

Paraconsistent logics are not an exception. In general, these logics are motivated by our necessity to deal with contradictions without fall in trivialities. Even when inconsistencies are present in our premise set, we can sensibly distinguish between good and bad arguments relying on these premises. In classical logic (*CL*) a contradiction trivializes any premise set. Yet many have argued that we regularly face inconsistencies in our argumentative practices and it does not seem reasonable just stop arguing in front of inconsistencies. Rather, we reason on despite the presence of inconsistencies, relying on the remaining information at hand. A wide variety of logics has been devised for representing such reasoning in the presence of inconsistencies.[5] Obviously, from my point of view, paraconsistent logics have not a canonical or standard interpretation. But my position is rejecting by Priest. According to him, and connected with the discussion about one or many logics. He claims:

> (...) there are many pure logics. (...) Each is a well-defined mathematical structure with a proof-theory, model theory, etc. There is no question about rivalry between them at this level. This can occur when one requires a logic for application to some end. Then, the question of which logic is right arise. If one is asking about pure logic, then, pluralism is contentiously correct. Plurality is an issue of substance only if one is asking about applied logics. (2001, p.24)

And Priest also adds:

> As I argued there, each pure logic, when given its canonical interpretation, can be thought of as a theory concerning the behavior of certain notions; specifically, those notions that are standardly deployed in logic. Validity is undoubtedly the most important of these—to which all the others must relate in the end. (2005, p.176)

Priest accepts that pure logics can have different applications:

> Of course, a pure logic can have many applications (as may pure geometries and arithmetics). For example, standard propositional logic may be used to test inferences or simplify the design of electrical circuits; and for some of these applications (e.g. the latter) the question of which logic is correct may well be a theory-laden and corrigible matter. (2005, p.165)

Nevertheless, not all applications are at same level:

> But when talking of application here, what we are talking about is what one might call the canonical application. The canonical application of geometry is in physical geometry; the canonical application of arithmetic is to counting and measuring; the canonical application of logic is to reasoning. (2005, p.165)

It is well known that Priest affirms that the 'canonical application' of a logical theory is deductive reasoning. Of course, as Lavinia Picollo and Lucas Rosenblatt have pointed me out, the canonical application of a system of logic is not the same that their interpretations. But it is clear to me that how to apply a pure system of logic is connected with how to interpret it. Logics are canonically applied to reasoning. Logicians creatively design different systems of logic to explain argumentative practices. And for these pure designs to apply to that practice, they must model our real reasoning behaviors. For this objective to be fulfilled, the logicians must also provide interpretations that allow us to explain how we actually reason. If there are truths with true negations in our reasoning, what is it to be false. Then, logicians as Graham Priest designs pure paraconsistent logics to describe our reasoning giving a canonical application of these logic. To do it, they need also to tell us how to interpret these systems. Pure systems do not describe anything. But in conjunction with an interpretation, they could explicate our rational practice. In this way, Priest considers that paraconsistent Logic *LP*, that is the right logic, talks about dialetheia.[6] In Priest & Berto's words (2013): "A *dialetheia* is a sentence, *A*, such that both it and its negation, $\neg A$, are true (...) *Dialetheism* is the view that there are dialetheia. (...) dialetheism amounts to the claim that *there are true contradictions*." In other words, they add that "[T]he paradoxes of self-reference are not the only examples of dialetheia that have been mooted. Other cases involve contradictions affecting concrete objects and the empirical world" (Priest & Berto 2013).

There are other ways to interpret pure paraconsistent logics. In particular, there are different alternatives to interpret the pure system *LP*. For example, it's perfectly possible to avoid any kind of commitments with dialetheia adopting an interpretation that involves two forms of assertion: strict and tolerant (Ripley 2013 and 2015). According to this interpretation, strictly, the liar and other paradoxical sentences cannot be asserted; but, tolerantly, they can. The same goes for their negations. Since the truth predicate is fully intersubstitutable, if we speak strictly we do not claim either that these sentences are true or that they are not true; if we speak tolerantly, we happily claim both. Thus, I don't see any intrinsical reason in pure logic *LP* to select one of these interpretations as canonical. So, adopting *LP-valuations does not mean accepting dialetheia*.[7] It's well known that David Lewis has offered (1982, p.440) an alternative interpretation of three-valued semantics for *LP* in which sentences can be regarded as both true and false. But again, such interpretation avoids any commitments with dialetheia. Instead, this interpretation involves that we look to ambiguity.[8]

Obviously, there are even more ways to interpret pure paraconsistent logics. For example, one may think that we should treat a sentence or a theory consistently such as possible. However, once one encounters a contradiction in reasoning, one should adapt to the situation. *Adaptive logics*, developed by Diderik Batens and his collaborators in Belgium, are logics that 'adapt' themselves to the (in)consistency of a set of premises available at the time of application of inference rules. Adaptive logics model the dynamics of our reasoning as it may encounter contradictions in its temporal development. It is also well known that the Brazilian paraconsistent approach has adopted a different strategy. Roughly they have accepted that there is a standard interpretation of paraconsistent logic, but they differ about what is this interpretation. For example, Carnielli & Rodrigues claim about the paraconsistent pure logic *mbC*:

> We defend the view according to which logics of formal inconsistency may be interpreted as theories of logical consequence of an epistemological character." (...) Furthermore, we argue that an intuitive reading of the bivalued semantics for the logic *mbC*, a logic of formal inconsistency based on classical logic, fits in well with the basic ideas of an intuitive interpretation of contradictions. On this interpretation, the acceptance of a pair of propositions A and $\neg A$ does not mean that A is simultaneously true and false, but rather that there is conflicting evidence about the truth value of A." Conclusive evidence is tantamount to truth, and if there is conclusive evidence for A, it cancels any evidence for $\neg A$ (mutatis mutandis for $\neg A$ and A). Therefore, the acceptance of a pair of contradictory propositions A and $\neg A$ need not to be taken in the strong sense that both are true. (Carnielli & Rodrigues 2012, p.156)

Similar ideas can be found in Rahman & Carnielli:

> Actually there are two main interpretations possible [about paraconsistent logics]. The one, which we call the *compelling interpretation*, based on a naive correspondence theory, stresses that paraconsistent theories are ontologically committed to inconsistent objects. The other, which we call the *permissive interpretation* does not assume this ontological commitment of paraconsistent theories. In the permissive interpretation, (for example) lack of information prevents us from rejecting prima facie either A or $\neg A$. Such an interpretation is the underlying concept behind Carnielli's semantic formulation of Jaskowski's ideas". (Rahman & Carnielli 2000, p.202)

This is the general view adopted by Rodrigues & Carnielli on pure systems *BLE* and LET_J. They support the thesis *one pure logic one canonical interpretation* with the following words:

> The main idea is to present a paraconsistent formal system according to which *true contradictions are not tolerated*. Contradictions are, instead, epistemically understood as conflicting evidence, where evidence for a proposition A is understood as reasons for believing that A is true.

The authors support the idea according to which LET_J is anti-dialetheist in the sense that, according to the intuitive interpretation proposed by them, its consequence relation is trivial in the presence of any true contradiction. The inferential rules of LET_J and its semi-valuations offer some technical results that (according to them) fit the intended intuitive interpretation.

The Rodrigues & Carnielli understanding of evidence is informal. The rules of *BLE* and LET_J relate, in a perspicuous way, evidence that a proposition A is true with evidence that A is false. Suppose X is evidence that A is true. It is reasonable to see X as evidence that it is false that A is false. Now suppose, conversely, that X is evidence that it is false that A is false. Again, it is reasonable that X also constitutes evidence that A is true. Thereby, a contradiction is understood as saying that there may be evidence only for A, or only for $\neg A$, but such evidence is non-conclusive. On this way, an informal reading of *BLE*-semivaluation can be summarized as following:

> $v(A) = 1$ means 'there is evidence that A is true';
> $v(A) = 0$ means 'there is no evidence that A is true';
> $v(\neg A) = 1$ means 'there is evidence that A is false';
> $v(\neg A) = 0$ means 'there is no evidence that A is false'.

But it is clear that Rodrigues & Carnielli believe, at least for these systems, that meaning of logical expressions is defined by the roles they play in our reasoning. Where does meaning come from? The rules of logic alone determine the meanings of the logical connectives. The actual meaning of the sentences of *BLE* and LET_J is not in the semi-valuations. It is in the rule the express about the meanings of the logical symbols they govern: which sentences entail it and which ones it entails. In Rodrigues & Carnielli's words:

> the semantics to be presented here for *BLE* is not intended to have any intuitive appeal independent of the deductive system. Rather, such semantics should be seen as a mathematical tool capable of representing the inference rules in such a way that some technical results may be proved (by *BLE*) (...) (Rodrigues & Carnielli 2016.)

They seem to be supported by the idea according to which a particular interpretation of inferential rules of *BLE* and LET_J gives the actual meaning of logical expressions of these systems. The natural deduction rules of these systems are thought of as preserving evidence instead of truth.

For example, consider rule $\neg E \rightarrow$.

$$\frac{\neg(A \rightarrow B)}{A}\ \neg\mathrm{E}\rightarrow \qquad\qquad \frac{\neg(A \rightarrow B)}{\neg B}\ \neg\mathrm{E}\rightarrow$$

In the Carnielli & Rodrigues' interpretation, the rule says that when X is evidence that a formula $A \rightarrow B$ is false, X must also be evidence for the truth of the antecedent A and for the falsity of the consequent B. This reading that uses the notion of *evidence* constitutes (part of) the actual meaning of the conditional of *BLE* and LET_J. This reading does not appeal to dialetheia and contradictions are conflicting evidence.

4. There is no a canonical interpretation for pure logics *BLE* and LET_J

Generally speaking, there are no precise limits between pure and applied systems of logic. Many times logical systems are presented considering a particular interpretation, and our evaluations on these theories combine some meta-theoretical elements (as soundness and completeness) with questions about a specific interpretation. Evidently, I do not have any reservations about that. But this fact should not obscure the differences between both two cases. To bring this into the light, let me emphasize that the level of applications is relatively independent of the level of models and proofs. No semantic structure or inference rule system necessarily leads to a particular interpretation. There are usually multiple interpretations for each logical system. There is not a single privileged interpretation. To show this, I am going to analyze Rodrigues & Carnielli's interpretation for *BLE* and LET_J. I'll offer some arguments to establish some problems connected with the thesis one logic only one (standard) interpretation.

4.1. Dialetheism is compatible with *BLE* and LET_J

Now I want to argue that *BLE* and LET_J are compatible with dialetheia. This means that these systems can have different interpretations as pure logics, even an interpretation that is committed with true contradictions. There is nothing in these pure logics that allows avoiding the existence of truth-value gluts.

Maybe, one could find some fundamental or intrinsic characteristic in *BLE* and LET_J that fixes one canonical interpretation that is incompatible with dialetheia. In particular, it could be thought that the actual reason to avoid true contradictions is linked with *BLE-semivaluations*. Paraconsistent logics as *LP* have valuations that assign explicitly a value 0,5. Obviously, this pure value does not mean anything necessarily. But given a formula *A*, *LP-valuations* allow assigning 0,5 to *A* and ¬*A*. Then, one could believe that both formulas are gluts: true and false at the same time. But, nothing similar happens with *BLE-semivaluations*. So, maybe *BLE* could not allow the existence of gluts.

Nevertheless, It is not complicated to note the connexions between *BLE* and four-valued logics suggested by Belnap and Nelson's constructive logic with strong negation *N4*. As Melvin Fitting points out in a recently paper (Fitting 2016), the pure system *BLE* turns out to be equivalent to Nelson's paraconsistent logic *N4*, resulting from adding strong negation to Intuitionistic logic without Intuitionistic negation. This point is also presented and analyzed by Carnielli, Coniglio & Rodrigues (2017). As such, much is known about semantics and proof theory of *N4*. There is both algebraic and possible world semantics. Lots of technical details are analyzed by Carnielli & Coniglio (2016). For example, they show that *N4* is algebraizable (in the sense of Blok and Pigozzi) and complete on a class of algebras called *N4-lattices*. In 5.1.3 they also describe how to get a useful semantics for *N4* in terms of twist-structures, a general framework which was independently proposed by Fidel and Vakarelov. Carnielli, Coniglio & Rodrigues (2017) give Fidel structures for *BLE*. However, differently of *N4*/*BLE*, they show that it is not clear whether or not LET_J is algebraizable by Blok and Pigozzi's method (Carnielli, Coniglio & Rodrigues 2017, p.17). They also show how too get Fidel-structures semantics for LET_J. Additionally, Kripke semantics for *N4* is readily obtained from the usual Kripke semantics for intuitionistic logic by a mapping sending pairs of propositional variables and worlds into four-element Belnapian matrix with its truth values *True*, *False*, *Neither*, and *Both*. So, a way to get models for *BLE* is to use Kripke models with Belnap's truth values. In this point, It is worth noting that this semantics shows that *BLE* is not compositional, in the sense that the semantic value of a complex formula is not functionally determined by the semantic values of its component. In particular, *BLE-semivaluations* for negation can be represented by four initial scenarios:

A	$\neg A$	
1	1	Conflicting evidence about A: both A and $\neg A$ hold;
1	0	Only evidence that A is true: A holds and $\neg A$ does not hold;
0	1	Only evidence that A is false: A does not hold and $\neg A$ holds;
0	0	No evidence at all: both A and $\neg A$ do not hold.

It is clear that the truth-value of a formula A does not determine the truth-value of $\neg A$. A and $\neg A$ could be both true, both false and one of them true and the other false. *BLE*-semivaluations only use two values to express every possibility. But using Belnap's tables allows understanding that the same information can be expressed by four truth-values. In this way, (only) true and (only) false are a part of the information that the formulas can express. Of course, from my point of view, there is no standard interpretation for these 4 truth-values. But there is a way of reading the 4-value matrix as (only) truth, (only) false, gluts and gaps.

In sum, *BLE* and *N4* are pure logics that are equivalent. Kripke models with Belnap's truth values is a well-known strategy to get adequate models for *N4*. These 4-value models can be understood in different ways. But, as well the paraconsistent logic *LP*, it is perfectly right to interpret pure systems *BLE* and *N4* as allowing true contradictions.

Let me note that *N4* is not an *LFI*. But, it is possible to define a consistency operator in *N4* by adding a bottom $\perp$. The pure logic $N4^{\perp}$ is a conservative extension of *N4* obtained by adding a bottom. Depending on how to get a consistency operator from a bottom, it could be shown that the system LET_J is equivalent to $N4^{\perp}$. Carnielli & Coniglio (2016) show how to obtain with slight modifications the twist structures associated with $N4^{\perp}$ from twist structures associated with *N4*. So, similar considerations can be formulated to reject the idea according to which LET_J is not compatible with dialetheia. A way to interpreting the circle operator of pure logic LET_J is the following: formulas that are sung with circle operator have classical behavior. Then, '$\circ A$' could mean "*A* is not a dialetheia". Because of the rule EXP$\circ$, accepting that *A* is not a dialetheia and *A* and $\neg A$, it is trivial. But this is compatible with accepting that there are formulas that are dialetheia.

The last point shows that *BLE* and LET_J are not incompatible with dialetheia. Both logics do not exclude dialetheia on logical ground. Moreover, this also exhibits that both systems have two different interpretations. But it could be that Rodrigues & Carnielli's interpretation is the canonical. And it might be that the alternative 4-value interpretation is a not standard one. Maybe *BLE* and LET_J are compatible with true contradictions. Perhaps these systems could be interpreted by gluts, but the standard way of understanding the meaning of their logical constants does not involve dialetheia. The point is connected with the relationship between pure systems and canonical interpretations. Maybe some pure systems are intrinsically related with one interpretation. It could be true that *BLE* and LET_J can be interpreted in different ways, even with a model theory that is compatible with true contradictions. But, it could reply to my point that Rodrigues & Carnielli's interpretation is the canonical interpretation at the end of the day.

Perhaps it is possible to find a logical grounding to consider that Rodrigues & Carnielli's interpretation is the standard interpretation. It is clear that *BLE* and LET_J are motivated by the idea according to which reasoning in situations of conflicting evidence requires to depart from the classical logic. I am not rejecting this point. But something more is needed to establish that one interpretation is canonical and another is not. As I suggested above, it should find certain intrinsic features in *BLE* and LET_J that determines which is the canonical way of understanding their logical notions.

For example, consider the case of classical logic vs intuitionistic logic. Perhaps some logicians think that classical logic has a canonical interpretation because of being intrinsically connected with *true preservation*. Moreover, because of this standard interpretation, classical logic could be closely linked with *ontological commitments*. Principles as *Excluded Middle* and *Double Negation Elimination* could be motivating the metaphysical realism. Maybe other logicians think that *intuitionistic logic* has a standard interpretation that involves essentially a type of constructivism. Rejecting such principles the intuitionism leads to anti-realism. Then, only epistemic interpretations could be allowed in this case. For example, It is well-known that Michael Dummett (1991) has characterized a debate between realism and anti-realism regarding a debate about how language in a realm of discourse gets its meaning. Claiming that the use of *CL* in a realm of discourse commits one to realism with respect to that discourse, he has challenged realists to show how they manifest their 'realist' understanding of their language. Perhaps classicalist's use of *excluded middle* and *double negation elimination* is a logical ground to pick up an interpretation. And maybe intuitionist's use of the negations based on deferents inference rules gives intrinsic reasons to select an interpretation. This kind of characteristic could be a grounding to fix different canonical interpretations to some pure logical theories.[9] Nevertheless, the key point is this case is that *CL* and intuitionist logic are not equivalent. Then, differences based on logical aspects could show us a logical ground to select a interpretation. Maybe, as I said before, the use of the negation could determinate in each case which interpretation is the standard and which is not. Nevertheless, *BLE* and *N4* are equivalent. And, as Carnielli and Coniglio show (2016, p.230), the logic *N4* can be now recast as an *LFI*. With the proposal to define a consistency operator in N4, one must observe that the very definition of *LFIs* (by means of a consistency connective o) needs the existence of finite trivial theories (namely, theories of the form $\{A, \neg A, \circ A\}$). Because of this, the logic to be regarded as an *LFI* is $N4^{\perp}$, the conservative extension of N4 obtained by adding a bottom $\perp$ with the following the axiom schema:

($\perp_1$) $\perp \rightarrow A$
($\perp_2$) $A \rightarrow \neg \perp$

Although the relationship between LET_J and $N4^{\perp}$ is an open question,[10] nothing indicates that as an LFI that is a conservative extension of BLE that is equivalent to N4, X can not also have an interpretation concerning dialetheia. So, there is no logical ground to choose Rodrigues & Carnielli's interpretation or an interpretation that admits dialetheia. The case of CL and intuitionistic logic is not the same case that LET_J and $N4^{\perp}$.

According to my view, connections about pure logics and their interpretations are not necessarily objectives. Even if one fixes all facts about a pure system itself (facts about what is valid or demonstrable), one could disagree about which interpretation is better. This means that analyzing *BLE* and LET_J one is not going to able to take an objective decision about different interpretations. In other words, one could have a genuine disagreement about the existence of true contradiction, even if one accepted *BLE*. Accepting or rejection dialetheia depends on our philosophical opinions. Of course, some pure systems are better at achieving some goals, and some are better for others goals. Even about the same goals, two logicians could disagree about which interpretation is better to adopt. *BLE* is better as a model of paraconsistent reasoning adopting Rodrigues-Carnielli's interpretation or Priest's interpretation.

Both interpretations show that one could admit different ways of interpreting *BLE* and LET_J that were resulting in two truly and incompatible accounts of the same discourse. And there is some king of logical indeterminism at interpretational level. It could be two competing, equally fruitful ways of interpreting the same pure system. Instead, there could be two fruitful interpretations for the same pure system. And our preference on which interpretation is better will not depend on some properties of the pure system. Obviously, one could prefer Rodrigues-Carnielli's interpretation because of some philosophical views about evidence. Or one could be committed with dialetheia because of some philosophical ideas on the reality. But my main point is one does not take these type of decisions based on having adopted a pure system.

To sum up what we have accomplished so far: facts about pure logics do not imply facts about how to select one interpretation over other. In particular, If one chooses BLE to model evidence transmission, then one should be careful when drawing a conclusion about what is implied from adopting this pure system to avoid other interpretations.

4.2. The capture argument

We should be impressed by translations between logics. This is a general phenomenon. Modal translation of intuitionistic theories is just a case. And it is a remarkable result that *CL* can be expressed inside some non-classical logics: its rules and laws can be captured as acceptable under certain conditions. For example, consider *LP* using Kleene's and Łukasiewicz's three-valued logics with a notion of *consequence* that uses two designated values. Most paraconsistent logicians do not propose a wholesale rejection of *CL*. They usually accept the validity of classical inferences in consistent contexts. *CL* is recovered inside this system only taking into consideration the classical values and the classical definition of validity as truth preservation. In this way, one can capture *CL* inside *LP* for some contexts.

This type of result is notable for evaluating the thesis one logic only one a standard interpretation. In particular, it is an important point that one can use *BLE* and LET_J to capture *CL*. This logic can be embedded into LET_J. Rules as EXP$\circ$ allow recovering the idea according to which in some circumstances contradictions imply triviality. In particular, during philosophical discussions of the notion of *consistency*, one often encounters the suggestion that even if certain inferences are considered to be invalid universally (EXP is just a case), there may remain particular cases in which their application is admissible. This is the assumption that there are certain circumstances— consistent cases—in which the principle of explosion is *locally valid*. The consistency operator serves as a formal mark that the formula to which it applies is one about which one may reason classically. Connected with this issue, in a recently paper, Carnielli, Coniglio & Rodrigues point out:

> We propose to expand the basic idea of inferential semantics to the paraconsistent logics *BLE* and LET_J. On what regards *BLE*, the point is how we use the connectives in inferences that preserve evidence. So, the meanings of the logical connectives is also given by the inference rules, but now in a context where what is at stake is preservation of evidence. The same idea applies to LET_J, that is able to deal simultaneously with evidence and truth. In LET_J, classical logic holds for formulas marked with $\circ$. Thus, we can say that for such formulas the meaning of the connectives is classical. (Carnielli, Coniglio & Rodrigues 2017, p.12)

But doubts like those of Quine and his skepticism about the translation between logics might appear at this point. We should not forget that the classical rule of explosion is not exactly the same as the rule of EXP$\circ$. It could be that the interaction between the consistency operator and the other logical symbols of LET_J change the meaning of logical expressions. So, one might wonder if EXP$\circ$ capture really classical explosion. My point here it is that if the connexion between pure systems and their interpretations were so close (as Rodrigues & Carnielli's seem to think), the response would be negative.

Assume *for the sake of argument* that pure logics have only one standard interpretation. Assume that LET_J and *CL* have different canonical interpretations. Suppose that Rodrigues & Carnielli's interpretation for LET_J is its standard interpretation. Then, remember that this interpretation only uses the notion of *evidence* avoiding to use the notion of *truth*. Assume now that truth-tables and truth-preservation allow an intuitive and standard interpretation for classical logic. Or as Rodrigues & Carnielli affirm:

"Indeed, this semantics [truth-tables and possible-worlds semantics] do provide an insight into these logics [classical logic and alethic modal logic] because it really seems that the semantic clauses 'make sense' independently of the inference rules and/or axioms" (Rodrigues & Carnielli 2016, p.10). But, if LET_J recaptures *CL*, and *CL* is intrinsically connected with its canonical interpretation on logical grounds, LET_J also has to capture *CL*'s canonical interpretation. But, this fact is contrary to the initial idea according to which LET_J can only be interpreted using the notion of *evidence*.

My argument seems to produce a dilemma: or one cannot capture classical explosion using EXP∘ inside LET_J or logics do not have canonical applications.

In a different way, one could say that if pure logics had standard interpretations connected intrinsically with their formalisms, there would be a way of capturing the standard interpretation of classical logic inside LET_J. But this result would mean that the consistent formulas inside also can be interpreted in a standard way using the notion of *truth* instead of *evidence*. The recovery result shows that *CL* is inside LET_J. Adding the rules EXP∘ and PEM∘ to *BLE* allows recapture classical properties of some formulas inside *LETj* and specifying semantic semivaluation for the consistency operator allows analyzing classical properties inside LET_J. But, again if one could get only one interpretation connected with pure systems (given a pure logic only an interpretation is its canonical one), the capture would be only a fiction.

5. Avoiding misunderstandings

In the last section, I have argued that the pure systems *BLE* and LET_J are compatible with dialetheia. I have given a new interpretation of these systems based on connexions between these logics and the four-valued logic *N4*. Using these models, it is possible to give a new interpretation *BLE* and LET_J that admits the existence of truth-value gluts. I also argued against the thesis according to the pure systems *BLE* and LET_J have only a standard interpretation. This argument based on the fact of that *BLE* and LET_J allow capturing *CL* seems to show that the relationship between pure systems and their interpretations is so close as Rodrigues & Carnielli believe.

Now, I would like to try to avoid some misunderstandings. Firstly, pure logics have different interpretations. Obviously, I am not rejecting that it is an important activity to develop interpretations for pure systems. In this way, I consider that Rodrigues & Carnielli's interpretation for *BLE* and LET_J is philosophically interesting to understand how to reason in situations where we have conflicting evidence. My points are not against this interpretation. Lots of times evidence is partial or contradictory. It is really essential to know what is right and wrong in epistemic contexts in which one is reasoning. As Fitting's work shows, even Rodrigues & Carnielli's interpretation opens the possibility to develop a modal pure logic in which the modal operator captures "has evidence" (in the way that *S4* does for "has a formal proof" for example). Just to emphasize I am not arguing against Rodrigues & Carnielli's interpretation for *BLE* and LET_J. Noticeably I think that such interpretation is very helpful and interesting.

Secondly, I am not rejecting that one takes in consideration interpretations to develop a pure system. In particular, I am sure that pure systems *BLE* and LET_J were developed taking in consideration Rodrigues & Carnielli's interpretation. Evidently, sometimes it is important to have in mind a specific interpretation to create a pure system of logic. Our proposal could be to represent a specific context of reasoning. Then we could develop a proof-theoretical presentation of a logical system having in mind an informal interpretation. Or maybe we could start working out models and valuations inspired by a philosophical idea. I am taking pure logics as models of the logical properties of (some) expressions in contexts. But from my point of view, pure systems are independent of questions about how to use a logic or how to apply it. It is an important question which is the relationship between pure systems and their (multiple) interpretations. And it is also relevant what interpretations are correctly modeling an inferential practice. Are these problems conceptual or empirical? When one is interested in how to apply a pure system, should there be a reflexive equilibrium between applied systems and evidence (inferential behavior)? All these issues are very complex and exceed the scope of this work. Enumerating them is enough to show that giving an alternative interpretation for *BLE* and LET_J committed with true contradictions is not rejecting Rodrigues & Carnielli's interpretation. What I am rejecting is that there is a privilege between interpretations. There is no single canonical interpretation for logical systems.

Finally, I have adopted a neutral view about proofs and models. Logic is independent of ontological and epistemological considerations. So, presenting a logic as a natural deduction system is not a reason to support a specific philosophical approach in ontology or epistemology. Or using the notion of *truth* with model-theoretical apparatus is not commented with a particular philosophical view. Sometimes, this deflationary approach on pure logic is rejected arguing that there are some cases (as *BLE* and LET_J) in which models should be seen as a mathematical tool capable of representing the inference rules. And there are another cases (as classical logic) in which models provide an insight into these logics because it really seems that the semantic clauses 'make sense' independently of the inference rules and/or axioms (Rodrigues & Carnielli 2016, p.10). I do not find a way to get precise boundaries between these cases. When do models give meanings? When are models only mathematical tools? Same reflexions could be made about proofs. When is a system of proof (system of rules, sequents, etc) only a mathematical tool to reflect actual meaning involved in models? Why do rules represent the use of logical expressions sometimes? and why sometimes not? Models and proofs are useful instruments to analyze logical properties as completeness, soundness, compactness, etc. Lots of times pure logics have interpretations. These systems can be applied to interesting areas. Sometimes pure logics do not have any interpretations. Or sometimes one finds an interpretation after having worked a considerable number of years.

It is also important to note that adopting a model-theoretical approach about the notion of *logical consequence* does not imply necessarily any ontological commitment. There are a lot of reasons that it could be given to support this idea: models are sets or mathematical structures, and "the world" does not play any role in the technical definition of validity. The idea of defining logical consequence as the preservation of truth *in all (2-valued) models* is that this is a set-theoretically defined surrogate for defining it as *logically necessary* truth-preservation. But there are different reasons to doubt about the collapse between models and "possible worlds". The move from possible worlds, whose universes may be too large to be a set, to these surrogates where the quantifiers range only over a set, is essential for carrying out the set-theoretic definition. The collapse defendant finds herself in serious trouble when she aims to express unrestricted quantification among sets. Such thesis seemingly leads to the idea of capturing interpretations in the universe of sets. Hence, domains of quantification must be sets. But since the universe of sets is iterative, when we talk about a universal domain we cannot be referring to a set that can be found in the hierarchy. Because of that, if it is to be assumed that interpretations are captured by sets, the idea of a universal set would lead to a contradiction. Therefore, every quantification presupposing domains to be set-like entities cannot be absolutely general. This seems to means that given models have a set as their domains, there is on this explication no possible world that corresponds to the actual world![11]

Now, two short comments about my recovery argument. In the first place, Walter Carnielli replied[12] that what recovery argument is showing is that *BLE* and LET_J are capturing is the notion of *the strongest kind of evidence*. In some contexts, the evidence is neither incomplete nor contradictory. Then, classical logic inside *BLE* and LET_J allows knowing how to reasoning about this assertions. Newly, I am not rejecting this point of view. *CL* as every pure logic can be interpreted in different ways and one of them could be using strong evidence. But this view seems to reinforce my point: *the strongest kind of evidence* and *truth* are different philosophical notions. Recovering *CL* inside LET_J produce a deviation in interpretations: *truth* is transformed into *conclusive evidence*. Then, If there were logical features in *CL* that intrinsically determinate a standard interpretation, then this would not be possible. Or maybe pure systems lack of standard interpretations and nothing is missing when *BLE* and LET_J capture *CL*. But this means that *CL* has not a standard interpretation in term of its models. In the second place, Abilio Rodrigues argues that Carnielli and he accept that pure logic can have different interpretations.[13] For example, they claim that *PIL* (positive intuitionistic logic) is appropriate for expressing both a notion stronger and a notion weaker of truth, although *PIL* is not able to express preservation of truth (for example, $A \vee A \to B$ does not hold). So, the same formal system fits two different interpretations. Good point, but my objection against Rodrigues & Carnielli's position is not that they claim that pure systems of logic as *PIL* can only have one interpretation. The point is if there is something in pure logic *PIL* that determines one interpretation as canonical. Particularly, in the *BLE*, LET_J and *CL* case, we have to consider how to interpret the capturing result: if the limit of the interpretations for *BLE* and LET_J is in term of evidence and *CL* must be interpreted in term of *truth*, how would it be possible to express strong truth without losing the epistemic canonical interpretation. Again, strong king of evidence is not the same that truth.

6. Conclusions

In this paper, I have offer some reason to reject the thesis according to *BLE* and LET_J talk necessary about preservation of evidence. According to my view, logics—and in particular paraconsistent logics—don't have a canonical interpretation. Using a type of model (or a family of models) or any kind of systems of rules, sequents, etc don't mean to give a canonical interpretation. I have offered several arguments for supporting my view. Firstly, I have shown that *BLE* and LET_J are compatible with dialetheia. In order to succinctly describe this point, based on the equivalency between *BLE* and *N4,* I have given a interpretation that is conciliable with a reading that involves gluts. Secondly, I have tried to show that there is no intrinsic features in *BLE* and LET_J that allow to establish Rodrigues & Carnielli's interpretation as the canonical one. In this point, I have shown that the case of CL and intuitionist logic is not the same as BLE and N4. Same point can be points out to LET_J *and* $N4^{\perp}$. Finally one uses the consistency operator to capture classicality. I used this technical result to offer some further doubts about the one canonical interpretation thesis.

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Notes

[1] For a discussion about the concept of paraconsistency, see: Barrio et al. 2018.
[2] Of course, the normatively of logic should be neutral about how to understand philosophically the notion of *logical consequence*. In particular, this *normativity* should not prejudge whether good inferences *preserve truth, information, or evidence.* Thanks to an anonymous referee for this comment.
[3] For a detailed overview and discussion, see Ciuni 2017, Ferguson 2017, Omori & Szmuc 2017 and Szmuc 2017.
[4] Lewis 1986. Nevertheless, for example, Barrio, Rosenblatt & Tajer (2016) use modal logic to capture the logical properties of the notion of *validity* inside a formal theory with self-reference.
[5] For example, one could use a paraconsistent logic to capture the notion of transparent truth. See Barrio et al. (2017).
[6] For example, Priest 2014 and 2016. But, some logicians have also rejected this point of view: for example, Omori 2016.
[7] For a discussion of this point, see: Barrio & Da Ré(2018).
[8] Thanks Dave Ripley for this suggestion.
[9] I don't think so. But for the sake of argument I can grant the point for this specific case.
[10] Thanks to Abilio Rodrigues for pointing me this result.
[11] Notice that an appeal to the Kreisel squeezing argument (see Kreisel 1967) would not be of much help for the Collapse defendant. According to the theorem, in the case of first order logical theories, it is enough (in the sense of an extensionally adequate definition of logical consequence) to consider set theoretic interpretations. Nonetheless, the proof of the theorem relies on the completeness of the theory. That is why the theorem cannot be applied to a theory which complies with the conditions of Gödel's Theorems. In particular, it does not apply to higher order languages. For these cases, there is the possibility of an extensional divergence arising between set theoretic and non set theoretic ways of interpreting the expressions.
[12] Walter Carnielli suggested this strategy of response during my talk in the *IV Conference of the Brazilian Society for Analytic Philosophy*, Campinas, Brazil, Julio de 2016.
[13] This point has been suggested to me by Abilio Rodrigues in a private exchange.

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