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ANALYTIC AND SYNTHETIC BASED ON THE PARADOX OF KNOWABILITY

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Abstract. The purpose of this paper is to show how the paradox of knowability loses its paradoxical character when we correctly interpret one of its premises. It is then shown how this new interpretation can be used to logically define analytical and synthetic truths. In this way, the paradox of knowability is traced back to the harmless affirmation that, in order to know every proposition with certainty, there must be no propositions whose truth is synthetic.

Keywords: Paradox of knowability • synthetic truths • analytical truths • principle of the factivity • knowledge • certain knowledge.

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1. Introduction

In this paper I will first make a brief summary of the paradox of knowability, the problematic aspects related to it and the way it has been dealt with in the literature. Then I will show the kind of strategy I have followed to solve it, how I have overcome its paradoxical aspects and how I have used it to define the synthetic and analytic truths.

2. The paradox of knowability

The paradox of knowability (Hart 1979) is a theorem from which two important conclusions can be drawn. The first conclusion states that if every truth is knowable then every truth is known. The second is that if there are unknown truths, then it is no longer true that every truth is knowable.

To understand the theorem we must know that Kp is a knowledge operator and stands for "someone knows that p", with p that is a proposition.

The theorem also makes use of the following premises.

The principle of the factivity of knowledge according to which if someone knows that p, then p:

(K-FATT) $Kp \rightarrow p$

The principle that knowledge distributes over conjunction according to which if someone knows that $p \wedge q$, then knows that p and knows that q:

(K-DIST) $K(p \wedge q) \rightarrow Kp \wedge Kq$

The modal rule according to which if a proposition p is the result of a demonstration then it is necessary that p:

(NEC) if $\vdash p$ then $\Box p$

And finally the modal rule according to which if it is necessarily false that p then it is not possible that:

(MOE) $\neg\Box p \vdash \Diamond\neg p$

That being said, we must first ask ourselves if it is possible to know that a given proposition p is true, but no one knows it:

(KPE) $\Diamond K(p \wedge \neg Kp)$

Let us suppose by absurd that this is possible, and therefore that:

(1) $K(p \wedge \neg Kp)$ assumption

If we do the following steps:

(2) $Kp \wedge K\neg Kp$ (K-DIST) and assumption
(3) $Kp \wedge \neg Kp$ (K-FATT) and (2)

we arrive at a contradiction. This means that our initial assumption was wrong and therefore that:

(4) $\neg K(p \wedge \neg Kp)$ discharging assumption

With this established we can do the following steps:

(5) $\Box\neg K(p \wedge \neg Kp)$ (4) and (NEC)
(6) $\neg\Diamond K(p \wedge \neg Kp)$ (5) and (MOE)

We have therefore seen that starting from:

$\Diamond K(p \wedge \neg Kp)$ (KPE)

we arrive at:

$\neg\Diamond K(p \wedge \neg Kp)$ (KPE)

That is a contradiction.

The problem is to prevent such a contradiction from occurring.

However, if we start from the assumption that we can know every true proposition (principle of knowability):

(PK) $\forall q(q \rightarrow \Diamond Kq)$

the presence of a true proposition that no one knows:

(7) $(p \wedge \neg Kp)$

is sufficient to bring us back to that same contradiction:

(8) $\Diamond K(p \wedge \neg Kp)$ (PK) and (7)

It follows that if there are true propositions that no one knows, the principle of knowability can not be valid and therefore there will be true propositions that are not knowable.

For this reason, if we want the principle of knowability to apply:

(PK) $\forall q(q \rightarrow \Diamond Kq)$

we must make sure that there are no true propositions that no one knows. Which means that every time a proposition is true there must be someone who knows it:

(9) $\forall q(q \rightarrow Kq)$

In conclusion, we can write:

$\forall q(q \rightarrow \Diamond Kq) \vdash \forall q(q \rightarrow Kq)$

3. Problematic aspects of the paradox of knowability

The first problematic aspect of the paradox of knowability is that if we want the principle of knowability to apply:

$\forall q(q \rightarrow \Diamond Kq)$

then every true proposition, besides being knowable, will also be known by someone at some time:

$\forall q(q \rightarrow Kq)$

The problem is to understand how the possibility of knowing every true proposition implies that every true proposition is known at some time.

The second problematic aspect of the paradox of knowability is that if we suppose there is a true proposition unknown at some time, that is, if we suppose that:

$(p \wedge \neg Kp)$

we must renounce the principle of knowability according to which every true proposition is knowable:

$\forall q(q \rightarrow \Diamond Kq)$

The problem is to understand how an ignorance that is only contingent can lead to an unknowability that is instead necessary.

4. Paradox of knowability in literature

There are two possible reactions to the conclusions of the knowability paradox. The first is to accept them. One way to accept the conclusions of this paradox is to argue that not everything is knowable because there are propositions that no one knows.

For example, Jenkins (2006) considers it quite obvious that there are unknowable propositions if we take into consideration the particular propositions that are the object of the paradox and have the form $(p \wedge \neg Kp)$.

Among other things, it is possible to support this position without even having to give up the principle of knowability. Provided we are willing to make some restrictions. According to Tennant (1997, pp.272–6), this can be done by limiting the principle of knowability to only those propositions that, if known, do not generate contradictions. Since in this way we can exclude precisely the propositions in the form $(p \wedge \neg Kp)$, whose knowledge as we have seen leads to a contradiction.

Another way to accept the conclusions of the paradox is to argue that everything is knowable because all propositions are known by someone.

For example, Plantinga (1982) argues that what makes it possible for everything to be known by someone is an omniscient being. In these terms, the knowability paradox would provide evidence in favor of the thesis that such a being is necessary.

The second possible reaction to the conclusions of the knowability paradox is not to accept them.

One way not to accept the conclusions of this paradox is to argue that a correct interpretation of the principle of knowability would lead to other conclusions. For example, Edgington (1985) argues that the possibility of knowing a truth should not be limited to the present situation. This means that although propositions in the form $(p \wedge \neg Kp)$ cannot be known at present, as the paradox of knowability shows, they could be known in a different situation from the present one.

Another way not to accept the conclusions of the paradox is to argue that the logical steps used to reach them are not legitimate. For example, Wright (2001) argues that the paradox of knowability provides an argument in favour of intuitionist logic. For the precise reason that in intuitionist logic the absence of propositions in the form $(p \wedge \neg Kp)$ does not imply the presence of propositions in the form: $(p \rightarrow Kp)$. And this allows us to consider all the propositions as knowable without all of them being known.

5. Correct interpretation of the factuality of knowledge

In this paper, the conclusions of the knowability paradox are accepted without taking any position on the existence of true propositions that no one knows.

All that needs to be done is to correctly interpret the premise of the factivity of knowledge:

(K-FATT) $Kp \to p$

Which, being valid regardless of the content of the proposition p, is in fact placing a constraint on the knowledge operator K. And when we attribute characteristics to the knowledge operator K, we are also changing its meaning. Just like in formal axiomatic systems, where the symbols used require one interpretation rather than another depending on the axioms used to introduce them.

To affirm, as the principle we are considering here does, that the knowledge of the proposition p also determines its truth, has a very precise consequence. Namely that this knowledge must be assumed to be certain. Because only if a proposition is known with certainty, that knowledge can be sufficient on its own to characterize the proposition p as true.

This new interpretation of the knowledge operator K obviously also changes the meaning of the principle of knowability:

(PK) $\forall q(q \to \Diamond Kq)$

Which should no longer be interpreted as the possibility of knowing each proposition, but of knowing it with certainty. And it is precisely this new meaning that makes the previous conclusions of the paradox of knowability harmless, as we are about to see.

6. Solution of the second problem of the paradox of knowability

We know that the premise (K-FATT) causes the following expression:

$Kp \wedge K\neg Kp$

to become:

$Kp \wedge \neg Kp$

producing a contradiction.

This means that if we consider the above premise valid, we must consider the term $K\neg Kp$ not compatible with Kp.

However, if we interpret the knowledge operator K as certain knowledge, this contradiction allows us to accept the conclusion of the paradox in a harmless way. In fact, in that case, the expression $K\neg Kp$ must be interpreted as knowing with certainty that no one knows the proposition p with certainty. But only if the proposition p cannot be known with certainty (denial of the principle of knowability), can we be certain that no one knows it with certainty. We can then write:

(10) $K\neg Kp \rightarrow \neg\Diamond Kp$

7. Solution of the first problem of the paradox of knowability

According to the paradox of knowability, a world in which every proposition can be known (PK):

$\forall q(q \rightarrow \Diamond Kq)$

is also a world where everything is known (PK):

$\forall q(q \rightarrow Kq)$

However, if we interpret the operator of knowledge K as certain knowledge, even this conclusion becomes harmless.

For the simple reason that in a world where everything can be known with certainty there can be no room for something that no one knows with certainty. In fact, the only way to know with certainty that there is something that no one knows with certainty is that this something is part of what no one can know with certainty: as shown to us by (10). But we are hypothesizing precisely a world in which this cannot happen.

8. Introductions of synthetic and analytic truths

Once we understand that the principle of the factivity of knowledge (K-FATT) must be referred to certain knowledge, the problem arises of how to introduce an operator which refers to a different type of knowledge. For example, to a knowledge like the scientific one, which we remember is never certain knowledge.

This problem is overcome by introducing the following operator of synthetic knowledge K_s:

(11) $K_s p \leftrightarrow K\neg Kp$

according to which to know synthetically the proposition p implies to know with certainty that no one knows it with certainty.

Thanks to (10) we will be able to write:

(12) $K_s p \rightarrow \neg \Diamond K p$

that is, if we know something synthetically, we cannot have a certain knowledge of it. The consequence of this is to have a synthetic knowledge that is completely equivalent to ordinary beliefs, and also to make every possible distinction between them depend on the justifications that have led us to consider them to be true. Justifications that in the case of scientific knowledge will be given by scientific practice, in the case of religious knowledge by faith, and so on.

Similarly we can define the following operator of analytic knowledge K_a:

(13) $K_a p \leftrightarrow KKp$

according to which to know analytically the proposition p implies to know with certainty that someone knows it with certainty.

Thanks to (K-FATT) we can write:

(14) $KKp \rightarrow Kp$

Moreover, if it is true that we are knowing a proposition p with certainty, then we are also knowing with certainty that we are knowing it with certainty. And this allows us to write:

(15) $Kp \rightarrow KKp$

and therefore:

(16) $K_a p \leftrightarrow Kp$

In this way we discover that knowing the proposition p analytically and knowing it with certainty is the same thing.

We can finally note that if we introduce the following operator of inevitable knowledge K_i:

(17) $K_i p \leftrightarrow K_a p \vee K_s p$

the corresponding version of the principle of knowability:

(PKI) $\forall p(p \rightarrow \Diamond K_i p)$

does not fall into the paradoxes of starting. In fact the following expression:

(18) $(p \wedge \neg K_i p)$

is always false because the following expression is always false:

(19) $\neg K_i p$

To realize this, we just need to do the following steps from (19) and verify that they lead to a contradiction:

(20)	$\neg[K_a p \vee K_s p]$	(17)
(21)	$\neg[(KKp) \vee (K\neg Kp)]$	(20), (11) and (13)
(22)	$\neg[Kp \vee \neg Kp]$	(21) and (K-FATT)
(23)	$\neg Kp \wedge Kp$	(22) and De Morgan $\vee$

9. Conclusion

We have seen that by interpreting the knowledge operator K as certain knowledge, or even as analytical knowledge, the paradox of knowability ceases to be a true paradox. Allowing us to say that if we want to know everything with certainty there must not be propositions whose truth is synthetic: as shown to us by (12).

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