## RATIONALITY, THEORY ACCEPTANCE AND DECISION THEORY

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#### ABSTRACT

Following Kuhn's main thesis according to which theory revision and acceptance is always baradigm relative. I propose to outline some possible consequences of such a view First, asking the question in what sense Bayesian decision theory could serve as the appropriate (normative) theory of rationality examined from the point of view of the epistemology of theory acceptance. I ar gue that Bayesianism leads to a narrow conception of theory ac ceptance Second, regarding the different types of theory revision, i e expansion, contraction, replacement and residuals shifts, I extract from Kuhn's view a series of indications showing that theory replacement cannot be rationalized within the framework of Bayesian decision theory, not even within a more sophisticated version of that model Third, and finally, I will boint to the need for a more comprehensive model of rationality than the Bayesian expected utility maximization model, the need for a model which could better deal with the different aspects of theory replacement I will show that Kuhn's distinction between normal and revolutionary science gives us several hints for a more adequate theory of rationality in science I will also show that Kuhn is not in a position to fully articulate his main ideas and that he well be confronted with a serious problem concerning collective choice of a paradigm

### I

It seems clear that in the psycho-sociological and historical

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perspective of Thomas Kuhn's theory of science, revision of a scientific theory, and if necessary of the prevailing paradigm, is a matter of choice, choice between cognitive options which may be theories, hypotheses, principles, methodological devices, etc or whole paradigms, a choice which is always relative to cognitive (and other sorts of) objectives Nobody denies that scientific research *should* be a rational enterprise, even though Kuhn's approach opened for several of his readers the door to arationality, if not outright irrationality<sup>1</sup>

Contrary to Lakatos who saw in Kuhn's Structure of Scientific Revolution no rational cause nor reasons for a coming scientific crisis and the emergence of a new theory or a new paradigm, no way to compare theories and paradigms by common standards of rationality, because each paradigm contained its own criteria of rationality, Bayesians believe that theory choice must be rationally justified. and that it is so justified when the choice is made in accordance with the principles and rules of the bilinear model of (subjectively) expected utility maximization In the context of theory revision, the general aim for the defenders of the Bayes approach 1s to furnish a representation of epistemic states (the statics) and of state transitions (the dynamics) caused by epistemic inputs Epistemic states can be represented by probability functions defined over a language (or a set of possible worlds) A probability function provides a measure of the individual's degree of belief in the sentences or propositions As a first of several rationality constraints, these probability functions have to satisfy the probability axioms<sup>2</sup> guaranteeing coherence in a set of degrees of belief Bayesians of all sorts accept Thomas Bayes' "Dutch book" argument exploited by Frank Ramsey (1926) if one does not subscribe to the axioms of the probability calculus, a clever bookmaker will be in a position to exploit you while

you are forced to make bets in an incoherent manner, to be inconsistent between the odds acceptable on different propositions, you will lose whatever the truth-value of the propositions you believe in <sup>3</sup>

The second component of Bayesianism concerns belief kinematics governed by the principle of Bayesian conditionalization for cases where epistemic inputs cause a change of belief according to a rule describable in probabilistic terms by learning E, the agent shifts from Prod to  $Pr_{new}$ , where the difference between  $Pr_{old}(A/E)$  and  $Pr_{new}(A)$ must be greater than 0 and  $Pr_{old}$  (A/E) > 0 The principle of conditionalization states that if you start with a probability function P, and then learn something that is captured by the sentence E, you should shift to a new probability function P' defined by P'(A) = P(A/E), the conditional probability of A given E, this new probability being just the ratio P(A & E)/P(E) when P(A) > 0 There are a lot of guestions concerning the justification of the principle of intertemporal credal conditionalization as normatively necessary for the rationality of belief change, but it remains a cornerstone of Bayesian belief kinematics (see Earman, 1992)

If scientific inquiry is controlled by considerations analogous to those that are relevant in practical matters, the decision maker must not only have i) a finite list of cognitive options (theories, etc.) from which he can choose  $(O_1, , O_m)$ , a finite list of states of nature  $(S_1, , S_n)$  with probability assignments relative to the total evidence  $\mathbf{p}(S_j, \mathbf{e})$ such that the total available evidence entails that only one of these can be true, each option being consistent with the total evidence ii) The decision maker must also be able to determine and describe in all relevant aspects relative to his epistemic goals and *desiderata* the outcomes or results  $(r_{ij})$ which are realized when a certain state of nature  $(S_j)$  is the case iii) The rational decision maker must be in a position to make a utility assignment (unique up to the choice of a 0 point and unit) to each specified result,  $\mathbf{u}(\mathbf{r}_{u})$ , in the light of the objectives, epistemic, practical, moral and aesthetic goals he may combine in the numerical measure of this value or utility These utility assignments have to be made in accordance with the structural principles governing the combination of preferences (desirabilities or intensities of desires concerning outcomes seen in the light of epistemic goals) with degrees of belief (probabilities for the states of nature) These coherence principles of preferences (and probabilities) concern preference orderings, such as the principle of comparability (weak preference orderings have to be reflexive, complete and transitive), continuity, monotony, substitution and reduction<sup>4</sup> iv) According to the Bayes rule, a rational decision maker has to choose the cognitive option which maximizes the expected utility (relative to the available evidence)  $maxEU(O,e) = max^{n} = 1$  $\mathbf{p}(S_n, \mathbf{e}) \mathbf{u}(\mathbf{r}_n)$ 

The outlined Bayesian programme offers a probabilistic epistemology where theory choice does not really consist in accepting a theory as true but merely in probabilifying different versions of it over time Within the framework given by Bayes, scientists my decide to devote time, treasure and trouble to one research programme rather than another But this is a decision of institutional investment which does not really amount to the adoption of an epistemic attitude regarding a theory (except when the probability of its truth approaches 1 which nearly never happens) In this case, one has to focus more on the utility component than on the probability component

Kuhn's conception of theory choice is remarkable for its omission of any reference to the probability of theories or degrees of belief and confirmation of hypotheses among which one has to chose <sup>5</sup> But when the choice has to be made given a system of utility assignments, Bayes' programme itself gives no instructions as to the way utility assignments have to be made in the first place The advantage of Kuhn's paradigm approach is to focus on this aspect of the decision problem (epistemic preferences, cognitive values, epistemic entrenchment, etc.) in order to broaden the Bayesian perspective which is clearly too narrow to cover the whole complex of theory choice This appears clearly from the writings of Thomas Kuhn since his seminal *Structure of Scientific Revolution* 

## Π

Traditionally, four types of epistemic changes and theory revisions were distinguished in matters of theory-choice These types have the following characteristics

**Expansion.** a change that happens when "a shift is made from  $K_1$  to  $K_2$  containing  $K_1$  obtained by adding a sentence e to  $K_1$  and forming a deductive closure" (Levi, 1983, 25)

**Contraction** a change obtained when a shift is made from  $K_1$  to  $K_2$  when some beliefs are retracted from  $K_1$  and no new belief is added, the contracted state of belief being closed under logical consequences (Levi, 1983, 25, Gardenfors, 1988, 60)

**Replacement:** a sentence representing the epistemic input to a consistent  $K_1$  (containing h) causes a shift to a consistent  $K_2$  (containing  $\neg h$ ), the input sentence leads to a contradiction of the beliefs already in  $K_1$  Replacement takes place when a scientist is "converted from a commitment to one theory to a commitment to another conflicting with the first" (Levi<sup>6</sup> (1983), 26)

**Residual shift** shifts that are not classifiable by the other three categories (as mentioned by Levi for completeness)

According to Gardenfors and Levi, even though Kuhn does not explicitly say so, epistemic changes which occur in "normal science", are revisions that may be characterised as forms of expansions and contractions Both authors tried to explicate scientific progress in the context of "normal science" in a Bayesian framework showing that the only expansion method is Bayesian conditionalization understood as expansion of belief (see Gardenfors (1988), sections 52 and 57) The epistemic desideratum for expansion is to add to a given corpus new error-free information, which means supplying a function that is an information-determining probability, further criteria for the evaluation of the expansion strategies may also be necessary When these criteria and epistemic goals for expansion strategies are given (see Levi (1983), chapter 2), the Bayesian decision rule of the maximum expected utility can be applied in the context of "normal" expansion It may be possible that the epistemic goals conflict, necessitating a trade-off between, for instance, risk of committing an error and gain of informational value This trade-off depends naturally on the scientist's evaluations of the informational value of potential answers, on his risk-prone or risk-aversive attitude, and on a series of practical aims a scientist is after in a given context

On the other hand, contraction also may be described and reconstructed as a contraction of probability functions (with the help of specific postulates formulated by Gardenfors (1988), section 5 7) in such a way that contraction can be regarded as "backward" conditionalization. The epistemic aims and *desiderata* of contraction being the minimization of unnecessary loss of informational value (which seems counterproductive in the light of the general epistemic objective of scientific research which is to augment pertinent information)

In spite of all the relative success of the Bayesian

programme, Kuhn<sup>7</sup> thinks that for "revolutionary science" when theory replacement (and much more) takes place. "debate over theory-choice cannot be cast in a form that resembles logical or mathematical choice" (1970, 199) More pessimistically, he maintains that in debates "over theorychoice there can be no recourse to good reasons instead theory must be chosen for reasons that are ultimately personal and subjective, some sort of mystical apperception is responsible for the decision actually reached" (199) It is the contention of Kuhn that theory replacement cannot be rationalized within the framework of Bayesian decision theory His main reason is to show that the kind of change in "revolutionary science" involves a paradigm shift, which is a change in value commitments not as being a matter of degrees of belief in the first place, but as a matter of epistemic preferences, cognitive values and utilities The probability strategy and the way of comparing degrees of belief relative to an identical and comparable background no longer works in this context because "revolutionary science" is marked by the replacement of the very same framework, it is marked by a paradigm shift in Kuhn's terms In this case, there no longer exists a common background against which both alternatives (of theory-choice) could be epistemically evaluated "Sir Karl takes it for granted that proponents of competing theories do share a neutral language adequate to the comparison of such observation reports I am about to argue that they do not If I am right. says Kuhn, then 'truth' may, like 'proof' be a term with only intra-theoretic application" (1970b, 265-66) Kuhn has several reasons to reject Bayesianism, the strongest one being the following

In a period of theory change when theory replacement and paradigm shift take place, there is no way to conditionalize on the information that a new and heretofore unarticulated theory  $T_2$  has been introduced This would be nonsensical, because conditionalization can only take place when prior to the time of replacement there was a well defined probability for this information, i.e. for the new theory  $T_2$  But this possibility is clearly ruled out, except when one is willing to attribute an arbitrary probability value or initial probabilities for as yet unborn theories This is not, as Kuhn showed convincingly by his detailed historical investigations, what really happened during scientific revolutions The transfer of allegiance from paradigm to paradigm does not obey Bayesian decision algorithms (Kuhn (1970b), 260), the important factor in paradigm shift is persuasive argumentation, not logical proof by conditionalization (ibidem, 261) The meaning of this statement is to say that theory choice is a shift in value commitments These commitments are normally changed, as a matter of sociological fact, by persuasion because this change does not depend on degrees of belief, but on a change of preferences which constitute only a prelude to the possibility of proof The statement does not mean that paradigm shift is an intuitive or even mystical affair Kuhn totally rejects in the end On the contrary, there must be good reasons for any theory choice These reasons concern value commitments, the neglected part in the Bayesian approach It is well understood that normally scientific research is supposed to be value free relatively to noncognitive values, value freedom being itself an ideal requesting impartiality (stipulating that acceptance of a theory should depend solely on cognitive values), autonomy (scientific research programmes should be pursued with regard to the implementation of the cognitive objectives of a specific scientific community) and neutrality (scientific research should be independent of any particular noncognitive value perspective 8

According to Gardenfors (1988, sections 46 and 4 7), the value commitments relate essentially to what he calls the epistemic entrenchment of some components of a theory "A change in paradigm typically involves a radical change in the ordering of epistemic entrenchment, and, vice versa, a substantial change of the degrees of epistemic entrenchment of the theses in a scientific field is a strong indication of what Kuhn calls a 'scientific revolution'" (ibidem, 88) It is important to notice that the degree of entrenchment with the formal properties of transitivity, conjunctiveness (connectivity), etc is not fixed by how probable a belief is judged to be but rather by how important the belief is to inquiry and deliberation Within the paradigm approach to the idea of epistemic entrenchment, what is included in the paradigm (according to Lakatos, what is included in the "core") is the most entrenched part, which is given up only in paradigm shift A paradigm shift, described by Kuhn, involves, then, a radical shift in the ordering of epistemic entrenchment The ordering of this entrenchment depends typically on the epistemic values which must be seen as essentially context dependant, where the context includes, as Kuhn has shown, much more than the degrees of belief of the scientific decision maker In relation to paradigm shift in chemistry, for instance, Kuhn noted that according to the phlogiston theory, qualitative facts such as colour and taste of substances were more important than quantitative information such as measured weights, after Lavoisier and Dalton these quantitative properties were considered as being more fundamental For what reasons? The reasons have to do with the list of epistemic desiderata Kuhn cites accuracy, simplicity, fruitfulness (explanatory and predictive power), consistency and scope These items function as cognitive values which determine paradigm choice They are the common values

members of the scientific community are supposed to share However, these common values do not guarantee, Kuhn reminds us, that each individual member of the community makes the same decision in the "same" circumstances there is "no neutral algorithm for theory-choice, no systematic procedure which, properly applied, must lead each individual in the group to the same decision" ("Postscript", 206) Individuals may apply these criteria differently to the same problem, and this diversity is maintained for very good reasons As long as the most deliberate and best considered judgments in science may be wrong - which is the mark of faillibilism, "it is vitally important that different individuals decide in different ways" (1970b, 241) Furthermore, the criteria are imprecise, and individuals may differ about their applications in concrete cases In many circumstances they may clearly prove to conflict with one another, "accuracy may, for example, dictate the choice of one theory, scope the choice of its competitor" (1977, 322) For this and other reasons, simple decision theory is not useful in such contexts These considerations indicate that theorychoice must be made in accordance with a more elaborate model of decision making, a model which incorporates more details in its utility component

# III

It is my contention that Kuhn has conclusively argued that theory-choice needs a more comprehensive model of rationality than the one recommended by the Bayesian approach He repeatedly explained that existing theories of rational decision are "not quite right, and that we must readjust or change them to explain why science works as it does" (1970b, 264) They are not only descriptively inadequate but also defective from the normative point of view Kuhn did not only ask a (descriptive) question concerning the criteria which scientists are *observed* to follow when evaluating theories, but at the same time he asked a normative question to know in what sense these criteria are also *rational* bases for their judgment (1983, 563) I would like to point out in this final part two types of contributions one might expect from the paradigm approach of Kuhn The first, a positive one, concerns the way to articulate in more detail the utility or value component of the expected utility maximization model The second, a negative remark, concerns a serious difficulty which arises from the Kuhnian conception of the procedure for theory-choice (with paradigm change)

a) As Hempel said some years ago, an account of scientific inquiry as a rational pursuit, "will have to specify certain goals of scientific inquiry as well as some methodological principles observed in their pursuit, finally, it will have to exhibit the instrumental rationality of the principles in relation to the goals" (1979, 58) In Kuhn's view, a paradigm incorporates these elements A paradigm is conceived as a constellation of group commitments not only concerning shared beliefs in heuristics and symbolic generalizations, but also, and most importantly, epistemic (and other) value commitments Shifts are made as a function of prioritises among cognitive values and relative importance of epistemic criteria discussed extensively by McMullin (1996) and expanded significantly by Lacey (1997)

In several of his papers following the "Postscript", Kuhn gives us his list of goals of scientific inquiry, the list of epistemic *desiderata* or values which serve as the criteria for evaluation of the outcomes of the several cognitive options to judge the adequacy of a theory (including its methodological principles) Kuhn knows that he does not yet possess a complete and eventually well articulated list of these goals and criteria Such an enterprise supposes that criteria for choice can be unambiguously stated As concerns this problem, Kuhn is justifiably pessimistic little progress has been made for this matter Sometimes he declares that a full articulation within a univocal decision algorithm is "not a quite attainable ideal" (1970b, 326) His list of cognitive values contains the following items accuracy, intra- and inter-theoretic consistency, broad scope, simplicity and fruitfulness These shared values of a scientific community are said to be effective, but their effectiveness does not depend on their being sufficiently articulated to dictate the choice of each individual who subscribes to them As Kuhn's case studies clearly illustrated, these criteria admit variation from individual to individual, variation in interpretation, application and relative weighting

Accuracy can be interpreted in the broad perspective of empirical adequacy, say as range of quantification and development of measuring techniques, exactness and number of successful predictions, best approximation to truth, high degree of confirmation, corroboration or verisimilitude, self-evidence, etc., some of which Kuhn explicitly excluded Consistency can be understood in terms of selfconsistency of a theory, closure under logical consequences and logical omniscience, conceptual coherence (homogeneity), nomological entrenchment, idealized equilibrium of sets of sentences, compatibility with other theories, and many other meanings Optimality of scope could mean that a theory's consequences should extend beyond observations, laws or subtheories it was initially designed to explain But what counts as optimal may vary from time to time, from individual to individual Simplicity generally understood as the law of least effort may cover actual computational labour requested to make predictions, complexity of the mathematical apparatus of a theory, of procedures to extrapolate from data, for example of curve-fitting, simplicity1 when the criterion is used to select a hypothesis for further test, simplicity2 as the final arbitration criterion when decisive evidence is lacking Simplicity is independent of truth because the question is not to know if nature itself is simple. The last criterion in the list is *fruitfulness* which allows a large range of interpretations. Contrary to Popper who insists on fruitfulness in problem raising,<sup>9</sup> Kuhn insists on efficiency in problem solving, on the bearings of theories on subsequent results, on the many signs of progressiveness of a research programme, beside overall usefulness in practical technological control of natural processus

Not only does Kuhn admit variation in interpretation of the epistemic values, but also different judgments of these values in concrete situations of application where values can be weighted differently by different individuals Even though they look ambiguous in application and may then be an insufficient basis for a shared algorithm of choice, they nevertheless specify what scientists must consider in reaching a decision, what they may consider relevant, what they are required to justify as the basis for choices they make (see Kuhn (1977), 331) A further problem arises from the fact that the specified cognitive values repeatedly prove to conflict with one another when you cannot satisfy both at the same time with the same theory Then one has to tell how to make trade offs, how to combine them in a single multi-criterial decision algorithm

The multiple variations within the value component of a hypothetical decision procedure for theory-choice and paradigm change show that it is not only difficult to articulate in due detail the different criteria and cognitive values, but that it is even more difficult to elaborate on such a basis a unique decision algorithm forcing each individual to make the same decision in an identical or similar context Kuhn comes to a sceptical conclusion concerning such an enterprise the identified variations are too heavily context dependant However, multi-criterial decision analysis<sup>10</sup> should recommend itself as a general framework for the formulation of principles of combination, principles of domination and principles of trade-off between the different criteria

b) The scepticism nourished against a shared decision algorithm did not prevent Kuhn from maintaining that in paradigm change and theory-choice, "it is the community of specialists rather than its individual members that make the effective decision" ("Postscript", 200) "As in political revolutions, so in paradigm choice - there is no standard higher than the assent of the relevant community " (1970, 94) Even more specifically, Kuhn asserts that it is a "group licensed way of seeing" and solving a problem ("Postscript", 189) Rational decision for acceptance of scientific theories appears to be not individual choice, but collective or social choice This kind of choice must guarantee, against individual and subjective variations, the obrectivity of the whole enterprise What is at stake is the following Given the preferences of the individual members of a scientific community concerning the interpretation, mode of application and weighting of the common values and epistemic criteria, the problem is to fix a procedure according to which the community is in a position to specify what is collectively best or best overall for science The problem for Kuhn 1s now to give a clear account of how the group decision is to be effected Earman (1992, 199) finds "this idea puzzling", and he does not find in Structure of Scientific Revolution any hints as to the decision procedure which would end up in a rational consensus when individual members of the scientific community have divergent degrees of belief obtained from different value commitments It is a fact that we cannot learn from Kuhn's writings what normative rationality constraints have to be imposed on a hypothetical collective decision procedure

In a period of revolutionary science and paradigm change, there is a real clash between the supporters of a new paradigm and the rest of the scientific community, there is a clear lack of intersubjective agreement Kuhn must then show how intersubjective agreement or consensus is restored when individual scientists start from different value commitments He must show what can count as a rational process of convergence He must indicate how rational agreement is to be reached on interpretation, application and relative weighting of common values for theory-choice in the first hand Do all the members have to agree when the group "decides" on a theory or paradigm? Is it a mater of a majority vote? Or do we simply have to exclude from the group of specialists those who disagree? In this case, we are back to the problem of circularity a scientific community is defined in terms of a shared paradigm. and a paradigm is what members of the scientific community have in common, circularity Kuhn invited philosophers do avoid This is not the only problem when paradigm shift is to be reconstructed as a rational enterprise We have to ask Kuhn where the "good reasons [are] for being persuaded", and what reasons are "ultimately decisive for the group" (1970a, 199) The main problem, acutely formulated by Earman (1992) is that the collective cannot decide, "it cannot rationally decide to agree if the individuals disagree" This is certainly true in matters of science probably different from politics

A further problem would be to indicate the relation between cognitive values and epistemic entrenchment, that is to show how cognitive values determine degrees of entrenchment of beliefs which influence the epistemic commitment function (a function that determines, as a rule, how an epistemic state would change as a result of various inputs), and to show how this function is related to conditionalization

Let us conclude I have indicated in what sense Bayesianism offers a narrow account of rational decision making in science, and that this account is inadequate for the case of "revolutionary science" Kuhn, in focusing on the cognitive value component of decision theory, gives several hints for a more elaborate model, but the paradigm approach is also confronted with difficult problems in sofar as it gives no idea of the rationality constraints for rational agreement and rational consensus in paradigm change

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# Notes

<sup>1</sup> In Lakatos' eyes, theory choice is a matter of "mob psychology" "Thus, in Kunh's view scientific revolution in irrational, a matter for mob psychology" (Lakatos (1970), 178)

- Ax 1  $0^2 \mathbf{p}(\mathbf{A})^2 = 1$  for all sentences A in L (nonnegativity),
- Ax 2 p(T) = 1 for the probability of truth (normalization),

Ax 3 For all sentences A, B logically distinct

 $(\neg(A \& B)), p(A \cup B) = p(A) + p(B) (additivity)$ 

<sup>3</sup> It is, however, not certain that the "Dutch Book" argument is a knock down argument (see Schick, 1986)

<sup>4</sup> Continuity with  $x_a$  the best and  $x_2$  the worst option in X, there exist for each alternative x in X a probability **p** such that a rational person is indifferent between x (for sure) and the lottery  $[\mathbf{p}x_a, (1-\mathbf{p})x_2]$ 

Monotony with two probability distributions between the options x and y, a rational person prefers the one which is better and has a higher probability

Substitution if a rational person is indifferent between probability distributions  $x^*$  and x from X,  $x^*$  and x can then be substituted in any context without modifying the preferences

Reduction or complexity a rational person is indifferent between a composed lottery (whose results are themselves lotteries) and the simple lottery obtained by multiplication of the probabilities in accordance with the rules of the probability calculus <sup>5</sup> This was rightly observed by Earman (1992, ch 8 3)

<sup>6</sup> Replacement may be treated by Gardenfors (1988) under the heading of "revision" While Levi (1983, 26) makes reference to Kunh's "revolutionary science", Gardenfors does not

<sup>7</sup> The main references for this position are Kuhn (1970a), "Postscript", Kuhn (1970b, 1977 and 1983)

<sup>8</sup> An excellent discussion of theses matters can be found in Lacey (1997)

<sup>9</sup> "Science never pursues the illusionary aim of making its answers final, or even probable Its advance is, rather, towards the infinite yet attainable aim of ever discovering new, deeper and more general problems, and of subjecting its ever tentative answers to ever renewed and ever more rigorous tests" (Popper (1959), 281) How can fruitfulness in problem rising be a *desideratum* of scientific inquiry? How could a problem solved ever raise more deeper problems? It is quite clear that when conclusions advocated as solutions to a problem, are in conflict with previous views or data, there is no problem solved and no virtue!

<sup>10</sup> See especially Keeny and Raiffa (1993) analysing decisions with multiple, competing objectives illustrated by applications of multiattribute analysis addressing value trade-offs, the structuring of objectives and the measurement of their achievements