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MUCH ADO ABOUT THE MANY

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Abstract. English distinguishes between singular quantifiers like 'a donkey' and plural quantifiers like 'some donkeys'. Pluralists hold that plural quantifiers range in an unusual, irreducibly plural way over common objects, namely individuals from first-order domains and not over set-like objects. The favoured framework of pluralism is plural first-order logic, PFO, an interpreted first-order language that is capable of expressing plural quantification. Pluralists argue for their position by claiming that the standard formal theory based on PFO is both ontologically neutral and really logic. These properties are supposed to yield many important applications concerning second-order logic and set theory that alternative theories supposedly cannot deliver. I will show that there are serious reasons for rejecting at least the claim of ontological innocence. Doubt about innocence arises on account of the fact that, when properly spelled out, the PFO-semantics for plural quantifiers is committed to set-like objects. The correctness of my worries presupposes the principle that for every plurality there is a coextensive set. Pluralists might reply that this principle leads straight to paradox. However, as I will argue, the true culprit of the paradox is the assumption that every definite condition determines a plurality. The course of the discussion will proceed in three steps: First I outline the debate between singularists and pluralists, then I define the syntax and semantics of PFO and finally I present my argument for the claim that PFO's semantics is committed to set-like objects.

Keywords: Plural quantification • plural logic • ontological commitment

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1. Introduction

English contains two kinds of quantifiers. There are singular quantifiers like 'a donkey' and plural quantifiers like 'some donkeys'. It is a controversial question how plural quantification is best understood. According to *singularism* plural quantifiers range in the ordinary way, known from Tarski semantics, over unusual, set-like objects like sets, classes, mereological sums etc. *Pluralism*, on the other hand, holds that plural quantifiers range in an irreducibly plural way, namely objects from first-order domains.

These different views on the nature of plural quantification come with different logical frameworks for regimenting natural language plurals. Singularists opt for second- or higher-order logic with one of its standard set theoretic semantics. So, it seems that, according to the singularist viewpoint, sentences involving plural quantifiers carry ontological commitments to set-like objects. To illustrate, consider the well-known Geach-Kaplan sentence

(1) Some critics admire only one another.

It can be proved that the sentence's satisfaction conditions cannot be expressed by any sentence of first-order logic, at least, if we restrict ourselves to first-order sentences containing at most (first-order translations of) the predicates occurring in (1).[1] The satisfaction conditions of (1) are of course expressible in monadic second order logic via the second order sentence (2). So, singularists can happily take (2) to be a proper formalization of (1).

(2) $\exists S(\exists u Su \land \forall u(Su \to Cu) \land \forall u \forall v(Su \land Auv \to Sv \land u \neq v)$

Pluralists, on the other hand, use plural first order logic (PFO) to regiment plural quantification. PFO is an interpreted first-order language that supposedly is capable of expressing plural quantification without reducing it to singular quantification. Pluralists argue for their position by claiming that the standard formal theory based on PFO is both ontologically neutral and really logic. These properties are supposed to yield many important applications concerning second-order logic and set theory that alternative theories supposedly cannot deliver. I will show that there are serious reasons for rejecting at least the claim of ontological innocence. Doubt about innocence arises on account of the fact that, when properly spelled out, the PFO-semantics for plural quantifiers is committed to set-like objects.

I will proceed as follows. In the next section the syntax and semantics of PFO will be given as well as as the standard axiomatic theory for plural quantification. This will set the stage for establishing my main thesis in the third section: Contrary to what pluralists claim, PFO's semantics is committed to set-like objects.

2. Plural Logic and its Justification

For some non-empty index set I we assume families of singular variables, plural variables , singular constants and plural constants, each indexed by I (for simplicity we may assume I to be countably infinite). The *terms* of PFO are given by the following Backus-Naur form (BNF), where $i \in I$.

$$t \ ::= \ x_i \mid xx_i \mid c_i \mid cc_i$$

Here the x_i, xx_i denote singular and plural variables and the c_i, cc_i singular and plural individual constants, respectively. Let a *plural* term, T, be a plural variable or plural constant and a *singular* term be a singular variable or a singular constant.

PFO has the following predicates. It has logical predicates $=$ and $\prec$, the former standing for identity and the latter for the inclusion relation *is one of*. Furthermore for every $n \in \mathbb{N}$ there are families $\{R_i^n\}_{i \in I}$ of n-place non-logical predicates (indices are mostly suppressed in what follows).

The following BNF determines the set of PFO-formulas.

$$\varphi \ ::= \ R^n \tau_1 \ldots \tau_n \mid t \prec T \mid \neg\psi \mid (\psi \wedge \chi) \mid \exists\alpha_i\psi$$

Here, τ_i are singular or plural terms, t is a singular, T a plural term and α_i is a singular or plural variable.[2] Universal quantifiers and the other connectives are defined in the standard way. That's all the syntactic machinery we will need, so let's stop here and turn to semantics.

The Geach-Kaplan sentence (1) receives the following translation into PFO:

(3) $\exists xx(\forall u(u \prec xx \rightarrow Cu) \wedge \forall u \forall v(u \prec xx \wedge Auv \rightarrow v \prec xx \wedge u \neq v)$

Whether this formula captures the intuitive truth-conditions depends of course on the way the semantics of PFO is formulated. So what is the semantics?

The desiderata for a semantics are clear: Pluralists want to frame the semantics in a metalanguage that does not carry any commitments to set-like things; so the metalanguage should not be enriched by set theory. At the same time the metalanguage should be as precise as can be.

To achieve these aims, some pluralists like Boolos and Rayo opt for a translation function from PFO-formulas into some target language (Boolos 1998b; Rayo 2002). While Boolos uses the language of second-order set theory, Rayo uses some fragment of English, suitably enriched with indices to mark binding relations. The two clauses of Rayo's translation function, θ, that concern plural terms go as follows:

- $\theta(x_i \prec xx_j) =$ it$_i$ is one of them$_j$
- $\theta(\exists xx_j\varphi) =$ there are some things$_j$ such that $\theta(\varphi)$

But this proposal is at least wanting. Translation functions from some source language to some target language of course are a viable tool for interpreting the source language. But whether this interpretation satisfies some desired properties in the end depends on whether the interpretation of the target language itself has these properties. So, to show that their translation functions indeed induce a semantics of PFO that fulfills the desiderata of not involving set-theory and being precise, Boolos and Rayo have to provide a semantics for their respective target languages having the properties desired by the Pluralist. Until this has been done this proposal cannot be fully evaluated.

A better proposal starts from the following idea Oliver & Smiley (2006; 2016). Set-theoretic semantics can be imitated pluralistically: Sets used for the semantics of singular logics can be simulated by pluralities in a precise sort of way. Plural quantification is then used as a primitive notion in the metalanguage.

Sets of 1-tuples are naturally imitated by pluralities of their members. However, here a problem arises: Plural quantification does not yield a natural interpretation of n-place relations (sets of n-tuples), where $n > 1$. The semantic problem is solved as soon as we have a pairing function on the relevant domain; that is, a function π such that $\pi(u, v) = \pi(u', v')$ iff $u = u'$ and $v = v'$. For then quantification over 2-place relations can be imitated by plural quantification over ordered pairs. So, pluralities of pairs go proxy for 2-place relations. By iteration of the pairing function we can represent n-tuples and thus n-place relations. So what's the function?

If we want to avoid stipulating the existence of a pairing function as a primitive mathematical object we can use formal mereology and plural quantification to simulate talk of ordered pairs. One variant of this approach is Burgess's method of double images (Burgess et al. 1991). The intuitive idea is this. Suppose infinite reality contains two non-overlapping microcosms, proper parts containing within itself images of everything there is. Then the pair (Possum, Bruce) is represented by the mereological fusion of Possum's image in the first microcosm and Bruce's image in the second microcosm. This can be made precise in third-order logic interpreted in a theory combining mereology and plural quantification.

To ensure a well-defined definition of the semantics predicate interpretations must be definite: Let an n-place relation (plurality of n-tuples) on a plurality D be *definite*, if for any individuals $a_1, \ldots, a_n$ from D, the relation either holds or does not hold of $(a_1, \ldots, a_n)$. Let a *model* be a pair $M = (D, I)$ such that D is a plurality and I a function (plurality of tuples) such that

(i) $I(c_i)$ is some individual from D.

(ii) $I(cc_i)$ are some individuals from D.

(iii) $I(R^n)$ is a definite n-place relation on D.

Let an *assignment* g in model $M = (D, I)$ be a function (plurality of tuples) defined on singular and plural variables such that $g(x_i)$ is some individual from D; $g(xx_i)$ are some individuals from D.

Let the *denotation* of a term τ with respect to (w.r.t.) a model $M = (D, I)$ and an assignment g in M be defined as follows:

(i) $\tau^{M,g} = I(\tau)$, if τ is a singular or plural constant.

(ii) $\tau^{M,g} = g(\tau)$, if τ is a singular or plural variable.

Satisfaction of a PFO-formula φ w.r.t. a model $M = (D, I)$ and an assignment g in M is defined as follows. (Here, $g \sim^{\alpha_j} g'$ means that g' differs from g at most in the value of the variable α_j.)

(i) $M, g \models R^n\tau_1, \ldots, \tau_n \Leftrightarrow I(R^n)$ applies to $(\tau_1^{M,g}, \ldots, \tau_n^{M,g})$.

(ii) $M, g \models \neg\psi \Leftrightarrow M, g \not\models \psi$.

(iii) $M, g \models (\psi \wedge \chi) \Leftrightarrow M, g \models \psi, \chi$

(iv) $M, g \models \exists x_j \psi \Leftrightarrow M, g' \models \psi$, for some assignment g' in M with $g \sim^{x_j} g'$.

(v) $M, g \models \exists xx_j \psi \Leftrightarrow M, g' \models \psi$, for some assignment g' in M with $g \sim^{xx_j} g'$.

Let a plurality of formulas Σ be *satisfied* by model M and assignment g in M ($M, g \models \Sigma$), if $M, g \models \psi$, for all ψ from Σ. A formula φ *follows from* a plurality of formulas Σ ($\Sigma \models \varphi$) iff $M, g \models \varphi$, for all models M and assignments g in M with $M, g \models \Sigma$. φ is *valid* ($\models \varphi$), if $M, g \models \varphi$, for all models M and assignments g in M.

In the context of PFO various axiomatic theories of first-order plural quantification can be formulated. The standard one, $\mathbb{PFO}$, has the following axioms (Linnebo 2003). The first is a comprehension schema stating that the φ exist, if some individual satisfies φ. The second guarantees that all pluralities are non-empty. Finally, the third axiom says that coextensive pluralities satisfy exactly the same PFO-formulas.

(C) Let φ be a PFO-formula that contains x free but contains no occurrence of xx. Then the following formula is an axiom.

$$\exists x\varphi(x) \rightarrow \exists xx\forall x(x \prec xx \leftrightarrow \varphi(x))$$

(NE) $\forall xx\exists x(x \prec xx)$.

(EX) For any formula φ the following is an axiom:

$$\forall xx\forall yy(\forall z(z \prec xx \leftrightarrow z \prec yy) \rightarrow (\varphi(xx) \leftrightarrow \varphi(yy)))$$

PFO has more expressive strength than singular first-order logic: In first-order logic there is no set of sentences that characterize the standard model of arithmetic $\mathcal{N}$ up to isomorphism. So, the induction axiom of Peano arithmetic cannot be expressed in first-order logic. In PFO, however, the induction axiom can be expressed as follows:

$$\forall xx(0 \prec xx \wedge \forall x(x \prec xx \rightarrow Sx \prec xx) \rightarrow \forall x(x \prec xx))$$

Adding this sentence to the remaining Peano postulates produces a theory extending $\mathbb{PFO}$ that has up to isomorphism only $\mathcal{N}$ as its model Oliver & Smiley (2006).

To justify their position pluralists hold that $\mathbb{PFO}$ has properties that allow important applications to second-order logic and set theory, applications singularism allegedly cannot deliver. Most prominent among these properties is the following

(O) Ontological innocence: $\mathbb{PFO}$ is not ontologically committed to set-like objects.

However, as will be shown in the next section, this kind of justification does not work, since $\mathbb{PFO}$ is not ontologically innocent.

3. Ontological guilt

There are serious reasons for doubting that $\mathbb{PFO}$ is ontologically innocent. Here is a first reason: The comprehension axioms of $\mathbb{PFO}$ contain existentially bound plural variables. We should adopt the logical conception of objecthood, which entails that the existential first-order quantifier is existentially loaded. More precisely the logical conception says the following: If a true sentence S contains a subexpression Q that logically functions as an existential first-order quantifier, then there exist those objects that Q must be taken to range over for S to be true. So, $\mathbb{PFO}$ is ontologically committed to pluralities (Linnebo 2003).

The pluralist can reply in at least two ways.

1. Why accept the logical conception? To be correct the argument needs a convincing defense of the logical conception which its proponents have not provided so far.
2. The logical conception yields only a ’formal’ notion of objecthood: Nothing is said about the nature of the objects. So, even if (b) is true, what the argument shows at most is that $\mathbb{PFO}$ is committed to *thin* pluralities (collections that are nothing over and above their members). So the pluralist can accept the argument, but at the same time hold that all his pluralities are thin. So, talk of pluralities is just shorthand for talk of their members.

The first reply is a fair challenge.[3] However, it’s not clear either why one should accept alternative conceptions of objecthood. One of the most formally delevoped alternatives is the Meinongian one, where existence is treated as a primitive predicate. Why should we accept *that*?

The second reply works for simple plural predications like ’There are some cheerios in my bowl’. But it is highly counterintuitive when it comes to more complicated plural constructions, where cross-reference occurs. To see this consider the pluralist version of the induction axiom translated into English:

(4) Whenever there are some natural numbers$_i$ such that 0 is one of them$_i$ and for every natural number n that is one of them$_i$, $n+1$ is one of them$_i$, then every natural number is one of them$_i$.

It is very intuitive to assume that utterances of (4) make claims about pluralities over and above their members. It is very natural to regard the occurrences of 'them' to refer to objects of this sort (Parsons 1990).

Another reason for doubt concerns the way the satisfaction of PFO-formulas is determined. There are some preliminaries to the argument. The first is logical, the second embodies two assumptions concerning semantic values.

The satisfaction of a PFO-formula φ w.r.t. M, g is determined only by the interpretation of φ's constants and predicates under M and by the values of φ's finitely many free variables under g. Formally, this means that the PFO semantics has the coincidence property.[4]

Theorem 1. *Let $M = (D, I)$ be a model and g, h assignments in M. Then we have:*

(a) $g \restriction FV(\tau) = h \restriction FV(\tau) \Rightarrow \tau^{M,g} = \tau^{M,h}$, *for every term* τ.

(b) $g \restriction FV(\varphi) = h \restriction FV(\varphi) \Rightarrow (M, g \models \varphi \Leftrightarrow M, h \models \varphi)$, *for every formula* φ.

Proof. (a) is immediate. The atomic, Boolean and singular quantifier cases of (b) are straightforward. The plural quantifier case goes as follows. $M, g \models \exists xx\psi \Leftrightarrow M, g' \models \psi$, for some $g' \sim^{xx} g \Leftrightarrow M, h' \models \psi$, for some $h' \sim^{xx} h$ (induction hypothesis, for $h'(xx) = g'(xx)) \Leftrightarrow M, h \models \exists xx\psi$ $\square$

As a corollary we get the following. Let $FV(\varphi) \subseteq \{v_1, \ldots, v_n\}$, where every v_i is plural or singular. Instead of $M, g \models \varphi$ we may write $M \models \varphi[a_1, \ldots, a_n]$, where $g(v_i) = a_i$. In particular, the plural quantifier clause of the semantics can now be presented as follows:

(v') $M \models \exists xx_i\varphi[a_1, \ldots, a_n] \Leftrightarrow M \models \varphi[a_1, \ldots, a_n, (b_j)_{j \prec J}]$, where $(b_j)_{j \prec J}$ are the values of an indexed family (a plurality of pairs each consisting of an index j and a uniquely determined object b_j).

The argument furthermore presupposes two principles concerning semantic values. The first is widely used in semantics of natural languages, but difficult to make precise.

Principle 1 (Semantic Admissibility)**.** *If there is a bijective operation π (which need not be a set) from members of a collection F onto members of a collection G and π is associated with a procedure for 'transforming' members of F into members of G, then F is a collection of admissible semantic values for some syntactic category C iff G is.*

The idea is that the semantic role played by some object can equally be played by another corresponding to the first and being constructible from it. Typically 'constructible' means that the bijection is expressible as a so-called combinator in extensions of the (untyped) λ-calculus. Here a combinator is a λ-term containing neither any free variables nor any non-logical constants. We'll work in the language of simple type theory that extends the simply typed λ-calculus (Church 1940, Hindley 1997). As base types we choose the objects e and t, where e stands for the type of individuals and t for the type of truth-values.

Here is an example. In type-theoretic semantics of natural language there are two collections of admissible semantic values for proper names: Individuals (type e) and principal filters of individuals (type $((e,t),t)$. The bijective transformation procedure from the domain of type e onto the domain of type $((e,t),t)$ is given by the followoing combinator.

$$\lambda x_e.\lambda Y_{(e,t)}.Y(x).$$

The second principle extends the ontological commitments of a theory to the admissible semantic values of the variables of the theory's language.

Principle 2 (Extended Commitment)**.** *A theory T is committed to Fs, if F is a collection of admissible semantic values for the variables of T's language, where the variables may have any finite order.*

The intuition is that a theory is committed to entities the variables of the theory's language range over as well as to objects that are uniquely constructible from such entities and so are suited to do the same semantic work.

Now, we are ready to state the argument for the claim that $\mathbb{PFO}$ is committed to set-like objects. Recall the quantifier clause for PFO as presented above.

(v') $M \models \exists xx_i\varphi[a_1,\dots,a_n] \Leftrightarrow M \models \varphi[a_1,\dots,a_n,(b_j)_{j\prec J}]$.

Let $g(v_i) = a_i$, where $FV(\exists xx_i\varphi) \subseteq \{v_1,\dots,v_n\}$. Obviously, the values of the family $(b_j)_{j\prec J}$ can be bijected onto the plurality of these values. It's not straightforward to find a suitable λ-term. If we admit a type j for the index collection J, we could let (j,e) be the type of lists of length $|J|$ of individuals ($|J|$ denoting the cardinality of J) and use the term $\lambda x_{(j,e)}.\lambda yy_e.\forall z_e(z \prec yy \leftrightarrow \exists y_j x(y) = z)$. This combinator only denotes a function from lists of individuals to sets of pluralities. But according to the extensionality axiom of $\mathbb{PFO}$, these sets are singletons, so no harm is done.

So, according to the principle of semantic admissibility, pluralities belong to the admissible semantic values of plural variables of PFO. So, according to extended innocence, $\mathbb{PFO}$ is committed to pluralities. This concludes the first step of the argument.

The first step invites the pluralist reply that these pluralities are only thin, nothing over and above their members, so that the pluralist's theory is innocent after all. But this easy way out is blocked by the second step. The second step shows that $\mathbb{PFO}$ is committed to sets. It crucially involves an answer to the question: Under what conditions do some things form a set?

More precisely: Let some things xx form a set y, if u is one of the xx just in case $u \in y$. In PFO: $\forall u(u \prec xx \leftrightarrow u \in y) =: F(xx, y)$. So, to rephrase the above question: Let some things xx be given. Under what conditions is there a set y such that $F(xx, y)$? My answer is: Under any conditions. Let's give this generous principle a name (Linnebo 2010).

Principle 3 (Collapse). *For every plurality xx there is a set y such that $F(xx, y)$.*

Collapse and the extensionality axiom of $\mathbb{PFO}$ jointly entail that there's an injection from pluralities to sets. Trivially, the converse holds as well. So, we have a bijection from pluralities onto sets. This bijection can be represented by the combinator $\lambda xx.\lambda y.\forall z(z \prec xx \leftrightarrow z \in y)$. Again this yields an operation from pluralities to singletons of sets. Since there's no set of all singletons this bijective operation is not a set.

Thus, since by the first step of the argument pluralities are among the admissible semantic values of PFO-plural variables, by semantic admissibility we have that sets are equally among the admissible semantic values of plural variables. Thus, according to extended commitment $\mathbb{PFO}$ is committed to sets.

But the pluralist may reply that the collapse principle opens the door to a version of Russell's paradox and so should be rejected. The reasoning goes as follows. Consider again the plural comprehension axiom scheme.

$$\exists x\varphi(x) \to \exists xx\forall x(x \prec xx \leftrightarrow \varphi(x))$$

and let $\varphi(x) = x \notin x$. Since there are non-self-membered sets, we arrive at the Russell plurality rr of all and only the sets that are not elements of themselves:

$$\forall x(x \prec rr \leftrightarrow x \notin x)$$

By collapse rr forms a Russell set:

$$\exists r\forall x(x \in r \leftrightarrow x \prec rr)$$

So we have the following characterization of the Russell set r.

$$\forall x(x \in r \leftrightarrow x \notin x)$$

By instantiating the quantifier ∀ with respect to r we can conclude

$$r \in r \leftrightarrow r \notin r$$

But the conclusion that this argument shows collapse to be a non-starter is much too quick. For there are compelling reasons for believing that collapse is true. After all, a set is completely determined by its elements. So every plurality yields a precise characterization of a set, namely the set of all and only the members of the plurality.

So the pluralist's argument against collapse proves to be a genuine paradox, not only for the friends of collapse, but for everyone: We are confronted with several plausible assumptions which jointly lead to contradiction. Dealing with this paradox appropriately requires examining each of these assumptions rigourously and not rushing to rejecting one of them (Linnebo 2010).

Besides collapse the assumptions leading to the paradox are the following:

1. Naive plural comprehension (NPC): For every satisfied definite condition C there is the plurality of all and only the objects satisfying C. (NPC is embodied in the plural comprehension axiom of $\mathbb{PFO}$.)
2. Absolute generality concerning sets (AG): It is possible to quantify over absolutely every set.

The true culprit of the paradox is NPC. Collapse and AG are innocent and should be maintained, since their denial leads to highly problematic consequences. Denying collapse leads to the limitation-of-size view of sets, which arbitrarily draws cardinal boundaries for sethood, where there are none. If collapse were false there should be a smallest cardinal number κ such that every class that is strictly smaller in size than κ is a set, but every class that is greater than or equal to κ fails to be a set. However, identifying such a κ and justifying its status as the dividing line between sets and non-sets in a non-arbitrary way is a task that is far from being accomplished so far.[5]

Denying AG is at best fully at odds with the way mathematicians use their own mathematical language, ascribing to them a systematic violation of the restrictions their language supposedly has to obey. These alleged violations often occur with highly important theorems. Hartogs' theorem is a key fact concerning well ordered sets and is crucial for establishing Zermelo's general fixed point theorem. Hartogs' theorem says that there is a definite operation χ on the universe of sets that associates with every set x a well ordered set $\chi(x)$ such that there is no injection from $\chi(x)$ to x. Obviously, Hartogs' theorem quantifies over absolutely *every* set and set theorists are happy to acknowledge that.

I'll quickly and very roughly elaborate on the failure of NPC.[6] But this can only be the beginning of a fair trial, which I have not given to the denial of collapse or of

AG so far. Now, under what circumstances does a definite condition C fail to define a plurality? This is the case, if it is not determinate for every object whether it is one of the Cs. Under what conditions is it not determinate for every object whether it is one of the Cs? My answer is: The determinacy of the Cs follows from the conjunction of (i) and (ii).

(i) For every object it is determinate whether C does or does not hold of it.

(ii) The plurality of C-candidates exists and is determinate. That is, for every object it is determinate whether or not it is one of the possible C-satisfiers or not.

If (i) holds, that is if C is definite, but the Cs are not determinate, (ii) must be false. If (ii) were true, we could go through every member of the plurality of C-candidates and check whether it satisfies C or not. In this way we would arrive at the plurality of Cs.

Finally the question arises: When is there no definite plurality of C-candidates? The answer lies in the iterative conception of sets, which is the right notion of sethood. This notion can be spelled out quite technically, but its core is a simple rule: Whenever some things are 'constructed' their set can be formed. Consider the definite condition of being self-identical. Assuming there were a definite plurality of candidates for being self-identical, the core rule of the iterative conception would demand that the set of those candidates exists. But since the condition of being-self identical is definite, we would arrive at the set of all self-identical objects and so at the set of all sets. But there is no such set.[7]

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Notes

[1]The standard model-theoretic proof is due to David Kaplan and is summarized in Boolos (1998a).

[2]My syntactic clause for $\prec$ is more restrictive than the corresponding clause of Oliver and Smiley's mid-plural logic. There the inclusion predicate $\prec$ may be flanked by plural terms on both sides. See Oliver & Smiley (2016). Nothing of what follows depends on the particular syntactic treatment of the inclusion predicate I have chosen. This more restrictive syntax for the inclusion predicate is of course not new. See Lennebo (2003).

[3]The first reply is a fair challenge, because the logical conception is confronted with various problems that its proponents have not been able to address in a satisfactory way so far. For instance, note that there are declarative sentences of English which contain subexpressions that are commonly taken to function as first-order quantifiers and yet which intuitively do not carry any ontological commitment to objects the relevant subexpressions appear to quantify over. For example, consider the sentence 'First Perseus worshipped some gods$_i$ but then he found out that there are no gods and so he stopped worshipping them$_i$ altogether'. (Subscripts indicate anaphoric binding) Intuitively this sentence carries no ontological commitment to gods, yet it contains the phrase 'some gods' which is standardly treated as a first-order existential quantifier. To avoid these counterexamples friends of the logical conception use a very relaxed notion of paraphrase function that is supposed to send the problematic sentence to a sentence of some suitable first-order language. This first-order target sentence is supposed to be truth conditionally equivalent to the problematic sentence and its intuitive ontological commitments are meant to coincide with those predicted by the logical conception. However, in many cases the paraphrase functions used are not defined at all or not defined by recursion on the syntax of the source language. So in many cases it is unclear how the paraphrase mechanism is to be compatible with a systematic, i.e. compositional approach to natural language semantics.

[4]In the following FV denotes a function from terms and formulas to sets of variables, that assigns to each term or formula the set of variables that occur free in that term or variable. FV is defined recursively in the standard way. As usual, if $f : x \to y$ is a function from a set x to a set y and $u \subseteq x$, then $f \restriction u := f \cap (u \times ran(f))$ is the restriction of f to u, where $ran(f)$ is the range of f.

[5]One may worry that collapse is not compatible with an adequate semantic treatment of those collective predicates like 'work together' which do not have a fixed arity. The semantic values of such predicates, the objection goes, are neither sets of n-tuples nor pluralities of n-tuples, for any fixed n. Instead, the objection concludes, predicates like 'work together' should be interpreted via real pluralities. Now, I accept the claim that collective predicates with no fixed arity should not be assigned sets or pluralities of n-tuples of individuals as

their proper semantic interpretation. But that is fully compatible with the truth of collapse. This can be shown as follows. Assume that we identify pluralities with those non-empty sets which contain at least two elements, individuals with singleton sets and the inclusion relation with the subset relation. Then collapse is true, since every plurality is identical to a set. However, it is trivial to formulate an adequate semantics for predicates like 'work together' that formalizes the intuitive plural ontology mentioned above. Thus giving 'work together' a proper semantic treatment does not rule out collapse. To show this I will outline a semantics with the required properties for the variable-free fragment of PFO without identity. Models are structures $(\mathscr{P}(D), \subseteq, f)$, where D is a non-empty set, $\mathscr{P}(D)$ denotes its power set, the subset relation interprets the inclusion predicate $\prec$ and f is an interpretation function such that: $f(c) \in \mathsf{Ind} := \{\{d\} : d \in D\}$, for every singular constant c, $f(cc) \in \mathscr{P}(D) \setminus \mathsf{Ind} \cup \{\emptyset\}$, for every plural constant cc and $f(R) \subseteq \mathscr{P}(D)^n$, for every n-place non-logical predicate R. Truth in a model is defined in the standard way. The predicate 'work together' can be translated as a 1-place predicate, whose interpretation in a model $(\mathscr{P}(D), \subseteq, f)$ is a set of subsets of D. This semantics allows to represent the predicate's lack of a fixed arity by treating it as applying to sets of individuals, whose cardinalities are only bounded by the cardinality of the set D underlying the model domain, i.e. $\mathscr{P}(D)$. Of course the semantics outlined is a simplified set-theoretic version of G. Link's lattice-theoretic plural semantics. See Link (1998).

[6]The following sketchy elaboration owes very much to the ideas put forth in Yablo (2006). Cf. also the recent work of Ø. Linnebo on modal set theory, which seems to sympathize with a lot of these ideas. See Linnebo (2018). It may be conjectured that many of these ideas can be formalized in a framework combining plural logic and modal set theory.

[7]In (Oliver & Smiley 2016) Oliver and Smiley present an axiomatization of their full plural logic which replaces the plural comprehension axiom schema of $\mathbb{PFO}$ with an axiom schema characterizing their exhaustive description operator. This is a variable-binding operator creating plural terms from formulas according to the syntactic clause that $[x : \varphi]$ is a term, where φ is any formula of full plural logic and x a singular variable (I insert square brackets for better readability). Intuitively, the term $[x : Rx]$ denotes the things that individually satisfy the formula Rx. The axiom schema characterizing the exhaustive description operator is meant to capture this intuition. In PFO-notation the axiom schema reads: $\forall y(y \prec [x : \varphi(x)] \leftrightarrow \varphi(y))$, where φ is a formula of full plural logic and $\varphi(y)$ contains y free where $\varphi(x)$ contains x free. Oliver and Smiley see many virtues in this axiom schema in comparison with the comprehension schema: the former avoids the 'clumsy existential antecedent' of comprehension and allegedly allows to characterize pluralities instead of merely making an existential claim about them. The first point may be conceded, but it amounts to questions of simplicity of formulation and as such does not bear that much substantial weight. On the other hand, the second point is highly questionable. First, surely the comprehension axioms do not merely make existential claims, since they make no existential claims at all. What each one of them says is that the plurality of all and only the objects satisfying φ exists, *if* some objects satisfy φ. Every comprehension axiom is a conditional whose antecedent and consequent make existential claims. But saying that some things exist, if something exists is not in itself making an existential claim. Secondly, on the assumption that something satisfies a formula, the comprehension axiom for that formula characterizes the plurality whose existence is guaranteed by the axiom and there being satisfiers of the formula. It is a plurality containing exactly those objects that verify the formula. So its not true, as Oliver and Smiley

suggest, that the comprehension axioms do not characterize pluralities. They do so, if the relevant formulas are satisfied by something. However, the question whether to stick with the comprehension schema or opt for the axiom schema governing the exhaustive description operator does not affect the worries I raise in the main text concerning the comprehension axioms. For presupposing the other axioms of full plural logic, comprehension is derivable from the axiom schema governing the exhaustive description operator, as Oliver and Smiley themselves demonstrate in (Oliver & Smiley 2016).

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