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HENKIN ON NOMINALISM AND HIGHER-ORDER LOGIC

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Abstract. In this paper a proposal by Henkin of a nominalistic interpretation for second and higher-order logic is developed in detail and analysed. It was proposed as a response to Quine's claim that second and higher-order logic not only are (i) committed to the existence of sets, but also are (ii) committed to the existence of more sets than can ever be referred to in the language. Henkin's interpretation is rarely cited in the debate on semantics and ontological commitments for these logics, though it has many interesting ideas that are worth exploring. The detailed development will show that it employs an early strategy of using substitutional quantification in order to reduce ontological commitments. It will be argued that the perspective adopted for the predicate variables renders it a natural extension of Quine's nominalistic interpretation for first-order logic. However, we will argue that, with respect to Quine's nominalistic program and his notion of ontological commitment, (i) still holds and thus Henkin's interpretation is not nominalistic. Nevertheless, it will be seen that (ii) is addressed successfully and this provides further insights on the so-called "Skolem Paradox". Moreover, the interpretation is ontologically parsimonious and, in this respect, it arguably fares better than a recent proposal by Bob Hale.

Keywords: Leon Henkin • second and higher-order logic • nominalistic interpretation • substitutional quantification

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1. Introduction, definitions and overview

Since Quine's attempts to dismiss second-order logic (1947; 1948), claiming it requires the existence of abstract objects and that it is no more than "set-theory in sheep's clothing", many papers have been published on the meaning of the second-order quantifiers, their existential commitments and on the appropriate semantics for second-order logic.

Recently, some discussion on neologicism renewed the debates on this topic. In (2013) and (2019) Hale presents an interpretation for second-order logic, one motivation of which is to address Quine's claim that such logic, being no more than set theory in sheep's clothing, has "staggering existential assumptions". Hale's interpretation contemplates models whose higher-order domains contain all and only the subsets of the domain that are *definable*, both in the object language and in the metalanguage. According to him (2019, p.2649), Henkin's general semantics would not fit such a framework because it includes too much: it allows undefinable subsets to be in the higher-order domains. Nevertheless, there is a stricter interpretation proposed by Henkin himself to address Quine's critiques concerning the ontological commitments of second-order logic. Such proposal is rarely cited in these discussions on the subsequent decades. However, it contains many interesting ideas that are worth exploring. For example, in this interpretation the quantified predicate variables have no range of values, but are to be replaced by appropriate predicates of a given language. This would correspond to an interpretation having models whose higher-order domains contain all and only the relations that are definable in the object language. Thus, Hale's interpretation for second-order logic has a strong competitor in countering Quine's claims, even more so if it is considered that Henkin's interpretation was intended to be nominalistic.

Henkin's few publications on philosophical themes (1953; 1955; 1956) were on nominalism and higher-order logic. His discussion on nominalism basically dialogues with Quine and Goodman's works of the time, specially (Quine 1947) and (Goodman & Quine 1947), concerning the nominalistic reconstruction and/or understanding of concepts and formal systems for logic and mathematics. Though less explicit in (1966), in his three papers Henkin's main preoccupation is to defend the feasibility of a nominalistic interpretation for higher-order logic, the analysis of which will be the concern of the present study.

1.1. On nominalism

Nominalism is an age-old issue in philosophy and could be said to revolve around two pairs of opposing notions: that of abstract/concrete and that of infinite/finite.[1]

An abstract entity will be considered here, as usual, as something which is either causally inert or does not exist in space-time. The paradigmatic examples of abstract entities would be the objects of mathematics, as the natural numbers. A concrete entity is one that has causal efficacy and exists in space-time. The paradigmatic examples of concrete entities are the so-called physical objects.

As for the pair infinite/finite, the usual distinction between actual and potential infinity will not be made, as it does not appear in the works to be explored. Precise definitions for these terms and their feasibility from the nominalistic viewpoint will be explored later.

Nominalism will be considered here as a position which rejects the existence of abstract entities and the existence of infinitely many objects. In the sequence, we examine the reasons one would have for embracing any nominalistic project.

1.2. Nominalistic projects

One can engage in a nominalistic project concerning some discipline D and an associated theory T_D, having mainly three purposes in mind (adapted from (Burgess & Rosen 1997)):

(1) Answer the philosophical question/challenge:

  (a) Is it possible to develop D successfully, without appealing neither to abstract entities nor to infinitely many objects?

(2) Answer (a) positively, proposing a nominalistic theory T_D^n for D, and defend further that

  ⋆ the ontological commitments of T_D *are really just those of* T_D^n.

(3) Answer (a) positively, proposing a nominalistic theory T_D^n for D, and defend further that

  ⋆ the practitioners of D should abandon T_D and adopt T_D^n.

One engages in a nominalistic project with purpose (3) mainly claiming that we are finite beings and, as regards the size of the universe, the empirical evidence that we have would support better the thesis that the universe is finite, than the opposite thesis. Thus the assumption of the existence of infinitely many things in the universe cannot be properly justified. Besides this, one usually appeals to some sort of causal theory of knowledge, arguing that we can only have knowledge of things causally related to us.[2] Such views on knowledge and infinity can also be behind (1) and (2), though not necessarily.

Quine and Goodman were important proponents of nominalistic projects of the sorts specified above. Prior to their important joint work (Goodman & Quine 1947), dealing with a nominalistic interpretation of mathematics, Quine had published a paper (1947) about nominalistic interpretations of logics. In these works they probably did not have only purpose (1) in mind. The approach in (Quine 1947) could be seen as having the aim (2) (with respect to some logics). Whereas in (Goodman & Quine 1947) an explicit refusal of abstract objects and infinity (presumably due to the reasons presented above) motivates the elaboration of a nominalistic syntax, intended to enable one to regard the statements of mathematics as rule-governed but otherwise meaningless strings of symbols. In this sense, they could be seen as having the aim (3).

Henkin proposed in a series of papers (1953; 1955; 1966) nominalistic interpretations of some concepts, such as "there are more *A*s than *B*s", or "x is an ancestral of y over the relation R". Most importantly, he proposed a nominalistic interpretation of higher-order logic. In the last of these papers Henkin emphatically defends a "mathematics first" position and that foundational discussions would be important but secondary to the actual mathematical activity. Thus, one may better understand his nominalistic endeavors under the key (1) or (2).

1.3. Overview of the paper

Firstly, we expose Quine's (1947) arguments as to why the question (a), both taking D as propositional and first-order logic, can be given a positive answer, and why this does not happen with respect to second and higher-order logic. Some issues with this argument are explored and Henkin's ideas for a nominalistic interpretation of second and higher-order logic are developed in detail.

In the sequence, the proposal is assessed and it is argued that, with respect to Quine's framework for ontological commitment, Henkin's interpretation of second-order logic (and thus the extension for higher-order) ends up not being nominalistic. Hence, it does not provide a positive answer to (a) either. The interpretation would fit instead in a wider perspective, that pursues ontological parsimony whenever possible and gives priority to well-entrenched entities, over less entrenched ones.

Finally, Hale's arguments regarding the ontological commitments of second-order logic and his proposed interpretation are briefly presented and compared with Henkin's.

2. Quine's nominalistic interpretations of logic

In order to argue that a logic can or cannot be given a nominalistic interpretation, it is necessary to have a criterion for measuring its ontological commitments. In the sequence the criterion used by Quine in his considerations on logics is presented, and its application for propositional, first, second and higher-order logic is exposed.

2.1. Ontological commitment

In the thirties and forties, Quine wrote a series papers on logic and ontology (*e.g.* 1934; 1943; 1947; 1948; 1966), which ended up revitalizing the discussions on ontology. In (1943) there appeared his famous slogan "to be is to be the value of a variable", and in (1948) he famously developed these ideas as a way to approach ontological disputes. In order to assess the ontological commitments of a given theory, many versions of a criterion were proposed, less explicit in the first works and more so in the later. In (1934; 1943; 1947; 1966) the criterion could be said to be:

Definition 2.1.0.1 (Ontological commitment)

(OC) *The ontological commitments of a theory T formulated in quantificational language resides in the intended values of its variables.*

Quine attempted to give a more precise formulation of the criterion in later works.[3] Such precise renderings were meant to be understandable purely extensionally, despite the expressions "has to be" and alike appearing in them. However, it was not long before it was realized that a precise and adequate extensional criterion for ontological commitment was unlikely to be found. It seems that later in life, Quine preferred to regard the notion of ontological commitment as an intuitive one, dispensing with formal attempts to capture it.[4] Indeed, it is on these informal grounds that Quine applies his *OC* to the various logics in (1934; 1947; 1966).

2.2. Propositional and first-order logic

In order to assess the minimum ontological commitments of propositional and first-order logic, Quine (1947) makes some proposals as regards the understanding of their respective languages.[5]

In the case of propositional language, instead of treating the propositional symbols as variables having propositions as their range, they are to be understood schematically, not having a range at all. In this way, they would be merely placeholders to be replaced by sentences of some specific language. The idea is that the meaningful use of sentences of some language does not commit one to the existence of propositions.

By the same token, in the case of first-order language the predicate symbols do not need to be treated as variables, having properties or sets as their range; nor need they be treated as names for specific properties or sets. The predicate symbols are also to be understood schematically, to be replaced by actual predicates of some language. As in the above case, to meaningfully use a predicate is not necessary to assume the existence of any abstract entity as its referent.[6]

Quine (1947, p.75) claims that the logics obtained from these languages by means of the usual axioms and rules of inference are not committed to abstract entities.

2.3. Second and higher-order logics

Let us consider now Quine's analysis of the ontological commitments of logics of second-order, the same analysis applies to the higher-order case. Let $\mathbf{M}^2$ be an extension of first-order language having quantified predicate variables of any arity. There would be at least two candidates for being values of these variables: either properties (intensional) or classes (extensional). On the grounds of having clearer identity conditions, classes are chosen as the preferred values of second-order variables.

Thus, applying *OC*, Quine concludes that any second-order language (and thus the corresponding second-order logic) is committed to the existence of classes. Apart from the eventual problems issuing from such a commitment, he refers (1947, p.78) to Cantor's theorem to argue that there would be a further problem: if one assumes there is a realm of classes through which the second-order variables of $\mathbf{M}^2$ range, one would also have to accept that there are classes which cannot be referred to using $\mathbf{M}^2$. This would happen because, by Cantor's theorem (which is provable in second-order logic), there would not be enough predicates in the language for covering them all. Quine says (*ibid*.):

> Such extension of quantification theory (...) would seem a very natural way of proclaiming a realm of universals —classes— mirroring the predicates or conditions that can be written in the language. Actually, it turns out to proclaim a realm of classes *far wider* than the conditions that can be written in the language. This result is perhaps unwelcome, for surely the intuitive idea underlying the positing of a realm of universals is merely that of positing a reality behind linguistic forms. However the result follows from Cantor's proof that the classes having objects of any given kind as members cannot be paired off exhaustively with such objects individually. Cantor's proof can be carried out within the extension of quantification theory under consideration. And from his general result it follows that there must be classes, in particular classes of linguistic forms, having no linguistic forms corresponding to them.

Summing up, Quine's argument for undermining second-order logic rests on two statements:

(i) second-order logic is committed to the existence of sets and

(ii) second-order logic is committed to the existence of more sets than can ever be referred to by predicates of the language.

The justification for (i) is *OC* together with the *objectual* interpretation of second-order quantifiers, that is, the interpretation that second-order quantifiers apply to variables having ranges of values. The justification for (ii) is Cantor's theorem, together perhaps with a "universal language conception of logic", to use Van Heijenoort's (1967) terms.[7] In the next section Henkin's attack on this argument will be explored.

3. Henkin's nominalistic interpretation for higher-order logics

The claims (i) and (ii) above hinge on a particular semantics for second-order logic, so one can tackle them by offering a different semantics. This is what Henkin (1953) did, by proposing to interpret the higher-order quantifiers in a certain *substitutional* fashion. As in the interpretation proposed by Quine for the first-order case, the predicate variables are understood schematically, not having a range of values in the usual sense: they are placeholders for predicates of a given language. Then, the formulas $\forall X \phi$ and $\exists X \phi$ are taken to mean "ϕ holds when all admissible predicates are substituted for X", and "ϕ holds when some predicates are substituted for X", respectively.

This proposal is an early one adopting substitutional quantification to address issues related to ontological economy.[8] The main step of the interpretation is the construction of a language to furnish the predicates to be substituted for the predicate variables. It is shown that under this interpretation (ii) fails. As regards (i) the idea is that this construction would only use "nominalistic sets" or aggregates, together with an idealized employment of infinity. We explore below the argument questioning (ii) and in the sequence the interpretation intended to attack (i) is developed.

3.1. Cantor's theorem and general models for higher-order logics

Henkin's argument is that Cantor's theorem has its usual meaning only when second-order logic is interpreted in the *standard* semantics where, for each arity n, the respective second-order domains are taken to be the full power-set of the n-fold domain of individuals. If one rejects this semantics, as the nominalist would, Cantor's theorem cannot be used. A formula representing the theorem in the standard semantics, such as

$$(\gamma) \quad \neg\exists F \forall X \exists x \forall y (F(x,y) \longleftrightarrow X(y))$$

remains valid in the semantics to be developed below,[9] but there is something very different now as regards its meaning. In the model-theoretic version (section 4.2), the cardinality of the higher-order domains may not exceed the cardinality of the domain of individuals. This situation is likened by Henkin (1953, p.21) to Skolem's paradox and can be explained in the usual model-theoretic terminology as follows.

Let $\mathfrak{B}$ be a model $\langle B, \langle B_n \rangle_{n\geq 1}, \ldots \rangle$, where B is a domain and, for $n = 1, 2, 3, \ldots$, B_n are the second-order domains, such that $B_n \subseteq \mathscr{P}(B^n)$, where $\mathscr{P}$ is the power-set operation and B^n is the n-fold Cartesian product of B (these operations are supposed to be defined by the background set theory). Define $\mathfrak{B}'$ to be an *expansion* of $\mathfrak{B}$ whenever $\mathfrak{B}'$ can be obtained from it by adding new n-ary relations on members of B into the second-order domains B_n. A formula ϕ is *persistent* if ϕ is true in $\mathfrak{B}'$ whenever it is true in $\mathfrak{B}$, for $\mathfrak{B}'$ an expansion of $\mathfrak{B}$. The paradoxical situation mentioned above is due to the fact that the formula (γ) expressing Cantor's theorem is not persistent. If (γ) is true in a model $\mathfrak{B}$ with first-order domain B, it holds that there is no bijective mapping from B to B_1. However, it may be that a formula expressing the existence of such a mapping is true in an expansion $\mathfrak{B}'$ of $\mathfrak{B}$. What happens in models such as $\mathfrak{B}$

is that not all subsets (as defined by the background set-theory) of the B^n's are contained in the B_n's. Therefore in these models, all universally quantified second-order formulas turn out not to be persistent. The relations available in the higher-order domains are no longer fixed by the background set-theory, they are model-dependent.[10]

In the semantics presented in the next subsection, the predicates which will replace the second-order variables will be taken from a certain language **N**. As (γ) is satisfied, from an "internal" viewpoint, there will be no one-to-one pairing of individuals and predicates, while from an "external" viewpoint there will be at most denumerably many predicates and there might be such a mapping.

The exposition of this semantics given in (Henkin 1953, pp.22-23) is sketchy and is not clear what is the exact proposal for the interpretation of second-order formulas. The presentation provided below is an attempt to develop Henkin's ideas more precisely, and allow a better assessment of this approach as regards the nominalistic agenda to which he wanted to contribute.

3.2. The second-order case

Consider the following definitions for the syntax of the second-order language. The predicates involved in the definitions will be highlighted in small caps when mentioned for the first time, and may be abbreviated with acronyms. They are to be applicable to *inscriptions*, which are taken to be concrete objects. Except where it is indicated otherwise, their adequacy for the nominalist viewpoint follows from the constructions in (Goodman & Quine 1947, §5ff). In the sequence, unless otherwise indicated, the letters from i to n stand for arbitrary numerals greater or equal to 1.

- $x_1, x_2, \ldots$ and $c_1, c_2, \ldots$ are INDIVIDUAL VARIABLES and INDIVIDUAL CONSTANTS, respectively.
- $X^n_1, X^n_2, \ldots$ and $R^n_1, R^n_2, \ldots$, for $n = 1, 2, \ldots$, are RELATIONAL VARIABLES and RELATIONAL CONSTANTS OF n PLACES, respectively.

Let ϕ be a WELL FORMED FORMULA of any of the languages to be defined below and let Γ be a collection of such objects.

- Define $\mathscr{FR}(\phi)$ as the sequence $\langle \xi_1, \ldots, \xi_k \rangle$ of FREE VARIABLES IN ϕ, where each ξ_i, $1 \leq i \leq k$, is either an individual or relation variable.
- Where ξ_j is an individual or relation variable, and ξ_i is any term of the same category, define $(\phi)^{\xi_i}_{\xi_j}$ as the SUBSTITUTION OF ξ_i FOR THE FREE OCCURRENCES OF ξ_j in ϕ. If ξ_j is free in ϕ and ξ_i is a variable, then ξ_i remains free in $(\phi)^{\xi_i}_{\xi_j}$ and thus the variables in ϕ may be renamed accordingly.

- A collection Γ of well formed formulas IS WITNESSED whenever:
  - if $\exists x_i \phi$ belongs to Γ, then $(\phi)^{c_k}_{x_i}$ belongs to Γ, for some individual constant c_k;
  - if $\exists X^n_i \phi$ belongs to Γ, then $(\phi)^{R^n_k}_{X^n_i}$ belongs to Γ, for some relational constant R^n_k.

Now we define a sequence of languages.

(**L**) Let **L** be the pure first-order language, that is, first-order language only with schematic predicate symbols and without any individual constants (which is already supposed to have a nominalistic interpretation).

($\mathbf{M}^2$) Let $\mathbf{M}^2$ be the pure second-order language, defined as follows:

- The predicate "to be a TERM OF $\mathbf{M}^2$" ($\text{TERM}^{\mathbf{M}^2}$) is defined as:
  - if x_i is an individual variable, then x_i is a $\text{TERM}^{\mathbf{M}^2}$;
  - "$\text{TERM}^{\mathbf{M}^2}$" does not apply to anything else.
- The predicate "to be a WELL FORMED FORMULA OF $\mathbf{M}^2$" ($\text{WFF}^{\mathbf{M}^2}$) can be defined as follows:[11]
  - if X^n_j is a n-ary relational variable and $t_1, \ldots, t_n$ are $\text{TERM}^{\mathbf{M}^2}$, then $X^n_j t_1 \ldots\ t_n$ is a $\text{WFF}^{\mathbf{M}^2}$;
  - if X^n_j is a relational variable and ϕ is a $\text{WFF}^{\mathbf{M}^2}$, then $\forall X^n_j \phi$ is also a $\text{WFF}^{\mathbf{M}^2}$;
  - as usual for other operators and nothing else is a $\text{WFF}^{\mathbf{M}^2}$.

($\mathbf{M}^{2'}$) Let $\mathbf{M}^{2'}$ be the following extension of the language $\mathbf{M}^2$:

- The predicate "to be a TERM OF $\mathbf{M}^{2'}$" ($\text{TERM}^{\mathbf{M}^{2'}}$) holds for all objects for which $\text{TERM}^{\mathbf{M}^2}$ holds and, additionally:
  - if c_i is an individual constant, then c_i is a $\text{TERM}^{\mathbf{M}^{2'}}$;
- The predicate "to be a WELL FORMED FORMULA OF $\mathbf{M}^{2'}$" ($\text{WFF}^{\mathbf{M}^{2'}}$) holds for all objects for which the corresponding predicate of $\mathbf{M}^2$ holds, additionally:
  - if R^n_i is a n-ary relational constant and $t_1, \ldots, t_n$ are $\text{TERM}^{\mathbf{M}^{2'}}$, then $R^n_i t_1 \ldots\ t_n$ is a $\text{WFF}^{\mathbf{M}^{2'}}$.
- The predicate "to be a CLOSED WELL FORMED FORMULA OF $\mathbf{M}^{2'}$" ($\text{CWFF}^{\mathbf{M}^{2'}}$) is defined as expected.

(**N**) Let the language **N** be an extension of **L** whose vocabulary includes new individual constants u_1, u_2, u_3, etc. Moreover, for each ϕ which is WFF$^{\mathbf{M}^{2'}}$ such that $\mathscr{FR}(\phi)=\langle x_{i_1},\ldots,x_{i_n}\rangle$, the vocabulary of **N** includes the n-ary first-order relation constant F^n_ϕ. The predicates TERM$^{\mathbf{N}}$ and WFF$^{\mathbf{N}}$ are defined as expected.

Finally, we define some formal systems with respect to the above languages.

($\mathscr{M}^2$) Let $\mathscr{M}^2$ be the logic formed by the language $\mathbf{M}^2$ and the usual AXIOM SCHEMAS and/or RULES OF INFERENCE.[12]

($\mathscr{M}^{2'}_\omega$) Let $\psi_1, \psi_2, \psi_3, \ldots$ be an ENUMERATION OF FORMULAS THAT ARE CWFF$^{\mathbf{M}^{2'}}$. By the usual method, taking the formulas ψ_n as axioms, one obtains successive collections of formulas $\mathscr{M}^{2'}_n$ that are CONSISTENT WITH RESPECT TO $\mathscr{M}^2$ and witnessed.[13] Each $\mathscr{M}^{2'}_n$, for $n = 1, 2, 3, \ldots$, is thus a finite collection and the corresponding predicates "TO BE AN AXIOM OF $\mathscr{M}^{2'}_n$" can be defined for each n. Define now $\mathscr{M}^{2'}_\omega$ as the system having as axioms all formulas which are axioms of $\mathscr{M}^{2'}_n$, for $n = 1, 2, 3, \ldots$. Such system is MAXIMALLY CONSISTENT.[14]

The collection of formulas $\mathscr{M}^{2'}_\omega$ must be infinite and this is naturally a delicate issue of this interpretation. We will analyse later Henkin's argument to the effect that the corresponding predicate "TO BE AN AXIOM OF $\mathscr{M}^{2'}_\omega$" could still qualify as nominalistic.

The interpretations for **N** and the higher-order languages are given in the sequence. Their quantifiers are interpreted substitutionally, however, the quantifiers of the meta-theory are still meant to be objectual, thus, *OC* still applies to them.

The interpretation for the language N

N is interpreted by $\mathfrak{I}$ as follows:

- The terms and formulas from **L** by hypothesis already have a nominalistic interpretation;
- For terms t_i, set $\mathfrak{I}(t_i) = c_i$, where c_i an individual constant;
- Let $F^n_\phi t_{i_1} \ldots t_{i_n}$ be a WFF$^{\mathbf{N}}$, such that $\mathscr{FR}(\phi) = \langle x_{j_1}, \ldots, x_{j_n}\rangle$. Then we have that $F^n_\phi t_{i_1} \ldots t_{i_n}$ is true at $\mathfrak{I}$ iff $\phi^{\mathfrak{I}(t_{i_1})\ldots\mathfrak{I}(t_{i_n})}_{x_{j_1}\ \ldots\ x_{j_n}}$ is an axiom of $\mathscr{M}^{2'}_\omega$, where $\phi^{\mathfrak{I}(t_{i_1})\ldots\mathfrak{I}(t_{i_n})}_{x_{j_1}\ \ldots\ x_{j_n}}$ means the SIMULTANEOUS SUBSTITUTION of $\mathfrak{I}(t_{i_1}), \ldots, \mathfrak{I}(t_{i_n})$ for the free occurrences of $x_{j_1}, \ldots, x_{j_n}$ in ϕ, respectively;
- $\forall x\phi$ is true at $\mathfrak{I}$ iff $\phi^{u_i}_x$ is true at $\mathfrak{I}$ for every individual constant u_i.
- The clauses for truth-functional connectives are the expected ones.

The interpretation for the language $\mathbf{M}^2$

Let $F^n_{\phi_1}, F^n_{\phi_2}, \ldots$ be an ENUMERATION OF THE N-ARY PREDICATES OF **N**, for $n = 1, 2$, etc. Now the interpretation $\mathfrak{I}'$ of the language $\mathbf{M}^2$ will be defined relative to the interpretation $\mathfrak{I}$ of the language **N**.

- If x_i is an individual variable, set $\mathfrak{I}'(x_i) = u_i$
- If X^n_j is the j^{th} n-ary relational variable, set $\mathfrak{I}'(X^n_j) = F^n_{\phi_j}$, where $F^n_{\phi_j}$ is the j^{th} n-ary predicate of **N**.
- Let $X^n_j x_{i_1} \ldots x_{i_n}$ be a WFF$^{\mathbf{M}^2}$. Given the above two itens, define:
  - $X^n_j x_{i_1} \ldots x_{i_n}$ is true at $\mathfrak{I}'$ iff $F^n_{\phi_j} u_{i_1} \ldots u_{i_n}$ is true at $\mathfrak{I}$.
- If $\forall \xi \psi$ is a WFF$^{\mathbf{M}^2}$, for ξ an individual or n-ary relational variable, then:
  - $\forall \xi \psi$ is true at $\mathfrak{I}'$ iff ψ is TRUE AT EVERY INTERPRETATION $\mathfrak{I}'^*$ DIFFERING FROM $\mathfrak{I}'$ AT MOST IN THE VALUE ATTRIBUTED TO ξ.
- The clauses for truth-functional connectives are defined as usual.

In order to avoid quantification over interpretations in the case of formulas of the form $\forall \xi \psi$, one can add the individual constants u_k and predicates F^n_ϕ to the language $\mathbf{M}^2$.[15] This is another issue with the intended nominalistic interpretation for $\mathbf{M}^2$, we will come back to it later.

3.3. The higher-order case

The interpretation for second-order language $\mathbf{M}^2$ given above can be lifted for higher-order languages. Henkin (1953, p.24) remarks that in a sentence of the sort $G(F^k)$, where the predicate F^k appears in argument position, it is not necessary to read the predicate as being a name for a corresponding class or property, *pace* Quine. One can hold that $G(F^k)$ means that the predicate F^k itself has G. The same happens for quantified predicate variables occurring in argument position, as in $\forall X^k G(X^k)$.

Henkin does not develop the details for the higher-order case in (1953), but refers the reader to his papers (1949; 1950). The approach of (1950) is rather different in that it depends on choice functions available in the object language. In order to avoid them, he points out (1950, p.89–90) that one would need to follow the route of (1949). One way to do so is to construct a higher-order model, whose higher-order domains contain all and only definable relations of each respective order (see section

4.2 below for details). Another way is to "flatten" the higher-order predicates into individuals of various sorts. In this case we would interpret a second-order predicate G^α as an individual G^α of the sort α, and the first-order predicates F^β would also be interpreted as individuals F^β of the sort β, and $c_1^\gamma, \ldots, c_k^\gamma$ individuals of the sort γ. Thus, the relation each of G^α and F^β has with its arguments is interpreted through special first-order membership relations $\mathscr{E}^n$, one for each sort. E.g. $F^\beta c_1^\gamma \ldots c_k^\gamma$ is true iff $\mathscr{E}^i(\mathrm{F}^\beta, c_1^\gamma, \ldots, c_k^\gamma)$ is true; similarly, $G^\alpha F^\beta$ is true iff $\mathscr{E}^j(\mathrm{G}^\alpha, \mathrm{F}^\beta)$ is true. The same would be done for predicates of higher-order having various types. This is the usual and perspicuous approach proposed by Manzano (1996) for the general semantics for higher-order logic.

However none of these approaches fits well with the procedure employed for the second-order case and with Henkin's own descriptions for the higher-order case.[16] To follow them, we would have to keep the higher-order predicates "unflattened" and, at each order, the formulas having predicate variables would be interpreted in terms of predicates available in a lower order. This route is developed in detail below for third-order language $\mathbf{M}^3$ where we have second-order predicate constants of any type. To obtain those constants we use open well-formed formulas of $\mathbf{M}^2$. The same method should be employed to obtain an interpretation for fourth-order $\mathbf{M}^4$, whose second-order variables would be replaceable by such second-order predicate constants of $\mathbf{M}^3$, and so on.

In order to obtain appropriate second-order predicates from open-formulas of $\mathbf{M}^2$, let us briefly define some predicates for type symbols $\mathscr{T}^2$ up to second-order:

- $\langle 0, 1, \ldots, 1\rangle$ is $\mathscr{T}^1$, for any sequence of n symbols "1", $n = 1, 2, 3, \ldots$,
- if the members of the sequence $\tau_1, \ldots, \tau_n$ are either equal to "1" or are $\mathscr{T}^1$, and there is at least one which is $\mathscr{T}^1$, then $\langle 0, \tau_1, \ldots, \tau_n\rangle$ is $\mathscr{T}^2$.

An individual variable x has type equal to "1", an n-ary predicate variable X^n ($n = 1, 2, 3, \ldots$) has type equal to "$\langle 0, 1, \ldots, 1\rangle$", with n occurrences of "1". Recall that we are referring indistinctly to variables, relational or individual, using the meta-variables ξ_1, ξ_2, etc. We define an attribution of types to the *open* well formed formulas ϕ of $\mathbf{M}^2$ as follows:

- if $\mathscr{F}\mathscr{R}(\phi) = \langle \xi_1, \ldots, \xi_k\rangle$, with variables having types $\tau_1, \ldots, \tau_k$, respectively, then the type of ϕ is $\langle 0, \tau_1, \ldots, \tau_k\rangle$.

For a WFF$^{\mathbf{M}^2}$ ϕ having a type τ that is $\mathscr{T}^2$, associate a predicate constant G_ϕ^τ with the same type. Thus we have that:

- $G_{\phi_1}^{\tau_1}, G_{\phi_2}^{\tau_2}, G_{\phi_3}^{\tau_3}, \ldots$ are SECOND-ORDER PREDICATE CONSTANTS (RCON2).[17]

All well-formed formulas of $\mathbf{M}^2$ are well-formed formulas of $\mathbf{M}^3$. The latter have in addition formulas involving the new second-order predicate constants:

- if G^{τ}_{ϕ} is RCON2 with $\tau = \langle 0, \tau_1, \ldots, \tau_k \rangle$, and the variables $\xi_1, \ldots, \xi_k$, relational or individual, have the types $\tau_1, \ldots, \tau_k$, respectively, then

$$G^{\tau}_{\phi}\xi_1 \ldots \xi_k \text{ is a WELL-FORMED FORMULA OF } \mathbf{M}^3 \text{ (WFF}^{\mathbf{M}^3}\text{)}.$$

The interpretation of $\mathbf{M}^3$

Let $\mathfrak{I}'$ be the interpretation of $\mathbf{M}^2$ above, we define an extension $\mathfrak{I}''$ to interpret the second-order predicate constants of $\mathbf{M}^3$. Let G^{τ}_{ϕ} be a RCON2, $\tau = \langle 0, \tau_1, \ldots, \tau_k \rangle$, where $\mathscr{F}\mathscr{R}(\phi) = \langle \xi_{i_1}, \ldots, \xi_{i_k} \rangle$. Let the variables $\xi_{j_1}, \ldots, \xi_{j_k}$ have the types $\tau_1, \ldots, \tau_k$, respectively, then

- $G^{\tau}_{\phi}\xi_{j_1} \ldots \xi_{j_k}$ is true in $\mathfrak{I}''$ iff $\phi^{\xi_{j_1} \ldots \xi_{j_k}}_{\xi_{i_1} \ldots \xi_{i_k}}$ is true in $\mathfrak{I}''$.

4. General assessment

Henkin's proposal is then a reasonably smooth continuation from the nominalistic interpretation given by Quine for first-order logic: predicate variables bound by quantifiers would not range over anything, but be replaceable by actual predicates of a given language.

As remarked before, the above interpretation presupposes that the majority of the predicates in small caps have been given an adequate nominalistic account, notably "to be a first-order formula",[18] and "to be a true first-order formula". The nominalistic adequacy of "to be a higher-order formula" and the related predicates are guaranteed given that the first-order counterparts are so. Now, the predicates whose nominalistic adequacy need to be examined are:

1. to be an axiom of $\mathscr{M}^{2'}_{n}$, for $n = 1, 2, 3, \ldots$,
2. to be an axiom of $\mathscr{M}^{2'}_{\omega}$,
3. to be a true higher-order formula.

In this section we analyse whether they can be given a satisfactory nominalistic account.

4.1. On abstract entities and infinity

The issue over abstract entities is entangled with the one about infinity in the interpretation of the section 3. In it, all references to collections are meant to be *nominalistic*, for example, the collection of free-variables of a given formula ϕ is not meant to be the *set* of free-variables of ϕ, but the *aggregate* of such inscriptions. The delicate point of the interpretation is that infinitely many inscriptions are needed to construct $\mathscr{M}_{\omega}^{2'}$, so it is an infinite aggregate. Henkin argues (more on this below) that the assumption of the availability of infinitely many concrete objects is an idealization, thus $\mathscr{M}_{\omega}^{2'}$ would be an ideal object. Nevertheless, it is difficult to sustain that an ideal infinite aggregate is still a concrete object, as it does not seem to satisfy none of the defining features of concrete objects exposed on section 1.1.

Moreover, the construction of $\mathscr{M}_{n}^{2'}$, for $n = 1, 2, 3, \ldots$, is given inductively and it is not possible to apply induction over some collection of formula-inscriptions, if the expression "collection of formula-inscriptions" refers to nothing but an *aggregate* of formula-inscriptions. The concrete bunch itself cannot serve as the object on which to apply induction: the concrete bunch determines different collections but in the induction at issue we are interested in just one. The different collections the bunch determines cannot themselves be called concrete, it is more appropriate to treat them as being abstract, that is, just sets. Thus, it seems that one cannot avoid the usage of abstract entities in this task.

A further issue is that, already in (Quine 1953), the attempt to base the case for nominalism purely on the rejection of abstract objects was no longer considered feasible and a better dividing line was proposed. Quine says (1953, p.129):

> From a mathematical point of view, indeed, the important opposition of doctrines here is precisely the opposition between unwillingness and willingness to posit, out of hand, an infinite universe. This is a clearer division than that between nominalists and others as ordinarily conceived, for the latter division depends on a none too clear distinction between what qualifies as particular and what counts as universal.

It is noticeable that in (Goodman & Quine 1947) and (Quine 1947; 1953), no distinction is made between potential and actual infinity, so it seems that both would have to be rejected indistinctly by the nominalist. However, it is not clear how the concept of infinity (indistinctly taken) is to be understood and opposed to the concept of finiteness. If the (former?) nominalist has been convinced that the distinction abstract/concrete is not usable and is ready to employ second-order language under the full interpretation, he could define precisely the pair finite/infinite using Dedekind's notion: a domain G is infinite (finite) if there is (there is not) an injective and non-surjective mapping from G to itself. Now if he thinks there are cases where the distinction abstract/concrete is clear and would rather avoid using abstract entities whenever he can, he perhaps would try to use Dedekind's notions under the interpretation of section 3. As Henkin (1953, p.26) points out, the nominalist will have trouble if he tries this route. Due to the fact that such an interpretation is relative to a given language, the notions of finitenesss and infinity end up themselves being relative.

There are some interesting asymmetries between the notions of infinity and finiteness which are worth exploring further in this context. The first one concerns how "easy" it is for the nominalist to obtain a precise definition for each concept. In the case of infinity, the nominalist is able to provide a formal definition already in his accepted system of first-order logic, as there is a first-order sentence ϕ that is true only if the domain of interpretation is infinite, that is, ϕ has only infinite models.[19] Thus, using a reasonable notion of definability, the concept of infinity can be formally defined in first-order logic.

Now the concept of finiteness is not first-order definable, even with respect to the broader notion of definability employed in the above paragraph. Moreover, the nominalist cannot appeal to second-order logic under the interpretation of section 3 to properly define it. Let δ be a formulation of Dedekind finiteness in the second-order language $\mathbf{M}^2$. Under the proposal of section 3, the meaning of δ is dependent on the interpretation of the reference language **N**, which means that δ may be true with respect to some interpretation of **N**, and false in another. From the "viewpoint" of $\mathbf{M}^2$, the domain of interpretation would be deemed finite in the first case and, in the second, infinite.

Connected with the above remarks, the second asymmetry is that the $\mathbf{M}^2$-sentence $\neg\delta$ (defining infinity) is persistent, as opposed to δ (recall definition of section 3). This means that the notion of infinity has a stable character that finiteness lacks: if a domain is deemed infinite by an interpretation $\mathfrak{I}$ ($\neg\delta$ is true at $\mathfrak{I}$), and thus **N** has the relevant injective and non-surjective function, one is certain this will remain so upon "expansions"[20] of $\mathfrak{I}$. On the other hand, if δ is true at $\mathfrak{I}$, the domain thus being deemed finite, then there is no guarantee that it will not be deemed otherwise upon "expansions" of $\mathfrak{I}$.

Despite the resistance the concept of finiteness has for being precisely captured by the nominalist, he could still claim that the intuitive notion of finiteness is clear enough. He could take it to be a primitive notion, define infinity from it and build his case against infinity. Trying to conciliate the usage of infinity with the nominalistic demands, Henkin argues that in order for his interpretation to be constructed, one does not need to assume that the universe really contains infinitely many inscriptions, but only *make believe* or *pretend* that this is so. This pretense is not to be considered a hypothesis about reality, but as an *idealization* and would be thus ontologically innocuous (Henkin 1953, p.27). The idea is that such pretense would be analogous to those involved in the construction of models in science in general, the main purpose of which being to simplify analyses and calculations. For example, consider some cases where the domain to be investigated is finite but immensely complex, as the behaviour of water molecules in a hull or air molecules under an airplane wing. Here it is much simpler to disregard the object of study as composed of individual atoms interacting with one another, and consider the whole as a continuous (thus infinite) fluid. For most applications, the conclusions one draws for infinitely many objects also hold for a very large number of them, a sort of "upside down induction".

Let us grant Henkin's argument on the use of pretense for the case of $\mathscr{M}^{2'}_{\omega}$ and that there is no commitment to sets in the definition of the systems $\mathscr{M}^{2'}_{n}$. Even then, an important issue remains. As it was remarked, the quantifiers in the meta-theory are still understood objectually, and the interpretations for the higher-order languages $\mathbf{M}^2$, $\mathbf{M}^3$, etc., must appeal to quantification over interpretations. This is not necessary in interpretations for extensions of these languages with individual constants u_1, u_2, etc. and with predicates F^n_{ϕ}, etc. In the extended languages, the interpretation clauses for the quantifiers only require quantification over individual constants and predicates. As regards, $\mathbf{M}^2$, $\mathbf{M}^3$, etc., it it is not clear how to avoid quantification over interpretations without using the *ad-hoc* device indicated in note 15. Therefore, unless quantification over interpretations is itself explained substitutionally or by some other method, the proposal of section 3 would still be committed to abstract entities.

Coming back to Henkin's use of pretense, notice that it is different from the uses of *fictions* in Quine (1947) and later in Field (1980). In these works the use of fictions is nominalistically acceptable, but only if it is inessential and no more than a way of speaking. Indeed, Field (*ibid*) spends a great deal of effort in the attempt to justify the acceptance of fictions in this manner. On his turn, Henkin allows for uses of pretense which are not dispensable, despite being only fictions.[21] By requiring a non dispensable use of pretense and adopting Quine's framework for ontological commitment, reference to fictions would not be ontologically innocuous and would still have to be counted as ontological commitments.

Therefore, the commitments of the interpretation of section 3 are:

- in the object language, at most denumerably many concrete objects;
- in the meta-language, denumerably many concrete objects and denumerably many sets of concrete objects.

Though ontologically parsimonious, the interpretation still does not fit in the nominalistic perspective as it was characterized in section 1.1. Thus, predicates 1–3 of the beginning of this section have not been given a nominalistic understanding, and the question (a) of section 1.2 did not receive a positive answer for second and higher-order logic.

In order to construct his interpretation, Henkin needs to invoke a non dispensable pretense. However the allowed use of such form of pretense does not mean that anything goes, otherwise the whole project of providing the interpretation for higher-order logic would be trivialized. What then is the difference between (1) pretending that there are denumerably many concrete entities and sets of these, and (2) pretending that there are infinitely many sets of anything whatsoever, sets of these and so on? Henkin's justification for the choice of (1) (1953, p.28) would probably go on the following lines: compared with (2), pretense (1) involves a more reasonable assumption about the quantity of entities, and the entities involved in the pretense are more entrenched. Both pretenses have epistemic costs that have to be weighted against the smoothness and simplicity they offer. In this case, the epistemic cost/benefit for (1) would be better. However, there remains a possible issue with the interpretation of section 3: the quantifiers for the first-order language **L** are (presumably) treated objectually, and in the extensions, $\mathbf{N}, \mathbf{M}^2, \mathbf{M}^3$, etc, they are treated substitutionally. This topic is explored in the sequence.

4.2. Substitutional and objectual interpretations for the quantifiers

Nothing was said about the interpretation of the first-order language **L** apart from the assumption that it is already nominalistic. On the first-order level, substitutional quantification is known to be disliked on the grounds that it may dodge ontological issues rather than solving them. Not to mention some serious issues that first-order language presents with substitutional quantifiers, as shown by Shapiro (1991). So it would be perhaps more harmonious if one could provide an objectual interpretation for higher-order quantifiers that is proper higher-order[22] and also ontologically parsimonious. This is straightforward: the substitutional interpretation of section 3 is in a certain sense equivalent to an objectual interpretation, having as the higher-order domains all and only the definable[23] subsets of the respective Cartesian products. In the sequence such an interpretation is briefly exposed.

Let $\mathfrak{B}$ be a model of the sort defined in the beginning of the section 3. We need some more quick definitions: let ψ be an $\mathbf{M}^2$-formula with $\mathscr{FR}(\psi) = \langle x_{i_1}, \ldots, x_{i_n} \rangle$. By $\mathfrak{B} \Vdash \psi[b_1, \ldots, b_n]$ it is meant that ψ is true at $\mathfrak{B}$ when its variables $x_{i_1}, \ldots, x_{i_n}$ are assigned the elements $b_1, \ldots, b_n$ from B, respectively. An n-ary first-order relation **R** on $\mathfrak{B}$ is $\mathbf{M}^2$-*definable* if and only if there is a formula ψ with $\mathscr{FR}(\psi) = \langle x_{i_1}, \ldots, x_{i_n} \rangle$ such that $\mathbf{R} = \{\langle b_1, \ldots, b_n \rangle \in B^n \mid \mathfrak{B} \Vdash \psi[b_1, \ldots, b_n]\}$.

Define $\mathfrak{B}$ to be an *op*-model (ontologically parsimonious model) for $\mathbf{M}^2$ if and only if all its higher-order domains contain all and only the $\mathbf{M}^2$-definable relations. It is possible to construct an *op*-model $\mathfrak{B}$ for $\mathbf{M}^2$ using the formal system $\mathscr{M}^{2'}_{\omega}$ as follows. Define ICON to be the set of individual constants $c_1, c_2, \ldots$. Take ψ to be an open formula of $\mathbf{M}^{2'}$ with $\mathscr{FR}(\psi) = \langle x_{i_1}, \ldots, x_{i_n} \rangle$, and let $\mathbf{F}^n_{\psi}$ be the set $\{\langle c_{j_1}, \ldots, c_{j_n} \rangle \in$

ICON^n | $\psi^{c_{j_1}\ldots c_{j_n}}_{x_{i_1}\ldots x_{i_n}} \in \mathscr{M}^{2'}_{\omega}\}$. Then, define the second-order domains B_n to be $\{\mathbf{F}^n_\psi \mid \psi$ is an open formula of $\mathbf{M}^{2'}$ with $\mathscr{F}\mathscr{R}(\psi) = \langle x_{i_1}, \ldots, x_{i_n}\rangle\}$ and define $\mathfrak{B} = \langle \text{ICON}, \langle B_n\rangle_{n\geq 1}\rangle$.

By construction every relation $\mathbf{R}$ contained in the higher-order domains of $\mathfrak{B}$ is $\mathbf{M}^2$-definable. Suppose an n-ary relation $\mathbf{R}^n_j$ on the elements of $\mathfrak{B}$ is $\mathbf{M}^2$-definable by δ. It is easy to see that $\mathfrak{B}$ satisfies the comprehension axiom $\exists X^n \forall x_1 \ldots x_n(X^n x_1 \ldots x_n \leftrightarrow \delta)$, then $\mathfrak{B}$ satisfies $\forall x_1 \ldots x_n(X^n x_1 \ldots x_n \leftrightarrow \delta)$ for some $\mathbf{R}^n_l \in B_n$, which must then be identical to $\mathbf{R}^n_j$. Therefore, $\mathfrak{B}$ is an *op*-model.

Construing the second-order quantifiers of $\mathbf{M}^2$ objectually and using *op*-models, the ranges of second-order variables are limited to sets and relations that can be defined in $\mathbf{M}^2$. It is clear that an $\mathbf{M}^2$-sentence ϕ is true at $\mathfrak{I}'$ of section 3 iff it is true at $\mathfrak{B}$. An *op*-model can be constructed for languages of higher-order in a similar fashion. So if the substitutional character of the interpretation of section 3 is disliked, this objectual one can be adopted at the cost of allowing in the object language some commitments of the meta-language.

5. Concluding remarks

Quine claimed that second-order logic is ontologically committed (i) to sets and (ii) to more sets than could possibly be referred to in the language.

Henkin's idea was to give an interpretation for higher-order logic which answered both (i) and (ii). This was to be done by extending the nominalistic syntax proposed by Quine and Goodman and changing the understanding of second and higher-order quantifiers. In order to define the interpretation, one would only make reference to finite collections of concrete objects, to nominalistically acceptable predicates and to an idealized infinite collection of concrete objects. In this way the resulting interpretation would be nominalistic and (ii) would be false under it, as it depends on the standard semantics for second-order logic.

The alternative interpretation for the second and higher-order quantifiers fits well with Quine's own considerations on relation symbols in the first-order case **L**. The interpretation proposed for these quantifiers is substitutional: the bound predicate variables are read as mere placeholders to be replaced by predicates of a given language. Then, the idea is to extend **L** with predicates F^n_ϕ which will be substituted for the predicate variables of the second-order language $\mathbf{M}^2$. Such predicates F^n_ϕ are on their turn interpreted with respect to a certain formal system $\mathscr{M}^{2'}_{\omega}$. The higher-order case is simply an extension of the above approach.

Under this interpretation (ii) is indeed false, though (i) remains true. However, the commitment does not appear in the object language through the use of relation variables, as Quine would have it, but on the meta-language: in order to give the interpretation for the second and higher-order quantifiers in $\mathbf{M}^2$, $\mathbf{M}^3$, etc, one quantifies (in the objectual sense) over interpretations. Moreover, the predicates "to be an axiom of $\mathscr{M}_n^{2'}$" and "to be an axiom of $\mathscr{M}_\omega^{2'}$" cannot be given a nominlistic rendering in Quine's framework, for they irreducibly require sets and denumerably many objects for their definition.

Therefore, Henkin's interpretation is not nominalistic, though it pursues ontological parsimony and prefers well entrenched entities. In this sense, the interpretation successfully addresses those claims concerning the alleged excessive ontological commitments of second and higher-order logic. As these claims are also countered by Bob Hale in a recent proposal of a semantics for second-order logic, it is of interest to compare both approaches, even if briefly.

5.1. A brief comparison with Bob Hale's semantics for second-order logic

Hale proposes (2019) an interpretation for second-order language which aims to counter Quine's claim (1970, p.66) that "Set theory's staggering existential assumptions are cunningly hidden in the tacit shift from schematic predicate letter to quantifiable set variable".[24]

As regards *OC* and quantification into predicate position, Hale claims that there is no *new* commitment introduced, but merely the generalization of a previous one. The use of first-order predicates would already commit one to the existence of properties. However, no reasons are given for this, and Hale does not attack Quine's argument (section 2.2 above) that a predicate does not need to refer to anything in order to be meaningful.

The interpretation proposed is model-theoretic, but Hale imposes the condition that the second-order domains should not include the full power-set of the respective n-fold domain of individuals, but only *definable* subsets. He argues that to restrict the notion of definability to the object language is needlessly crippling, and saw no good reason not to allow definability in the meta-language (*ibid*., p.2650).[25] However, Hale did not define precisely his meta-language and the adequate notion of definability for it. The only detail we are given about the meta-language is that it would be "an extension of the object language to include a certain amount of set-theoretic vocabulary" (p. 2651). Without this information we cannot properly evaluate the ontological commitments of Hale's interpretation. By contrast, the ontological commitments of the interpretation of section 3 are clear: in the object language, at most denumerably many concrete objects; in the meta-language, denumerably many concrete objects and sets of them. If the objectual interpretation of section 4.2 is chosen, then the commitment to the existence of sets of concrete objects also occurs in the object language.

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Notes

[1]The pair "universal/particular" is ubiquitous in the traditional debate, but many of the arguments that are relevant to the present discussion and which involve this pair are motivated by considerations over abstract entities.

[2]It is usual to employ this characterization of knowledge and argue that it would be impossible to obtain knowledge about abstract objects. Concerning this, an important paper is (Benacerraf, 1973). Other examples can be found in (Burgess & Rosen 1997).

[3]*E.g.* Quine (1948, p.32) gives the following version: "we are convicted of a particular ontological presupposition if, and only if, the alleged presupposition has to be reckoned among the entities over which our variables range in order to render one of our affirmations true."

[4]On this, see (Chateaubriand, 2003).

[5]For such arguments to be coherent, it will be supposed that Quine and Goodman's (1947) proposal for a nominalistic syntax is correct.

[6]The underlying idea is that one would be able to attribute meaning to a predicate by giving conditions of use.

[7]In such a conception of logic the domain of interpretation would be everything there is.

[8]During the 1960-80s, many works were published on this issue, mainly concerning first-order languages. A good example is Marcus (1978), where in the final remarks (p.361) there is even a suggestion for the development of a thoroughly substitutional approach up to the higher-order.

[9]For soundness results, refer to Manzano (1996).

[10]The *General* or *Henkin* models are those satisfying a minimal condition: all definable first-order relations are in the corresponding higher-order domains. Incidentally, one can prove that if $\mathfrak{B} \Vdash \exists F \forall X \exists x \forall y (F(x, y) \leftrightarrow X(y))$, then a certain definable first-order relation is not in the corresponding domain of $\mathfrak{B}$, thus $\mathfrak{B}$ would not be a Henkin model.

[11]Note we only give the usual recursive definition for this predicate. The proper nominalistic definition should not use recursion but follow the route established in (Goodman & Quine 1947, p.116).

[12]An example can be found in (Manzano 1996).

[13]The consistency of a collection of formulas in second-order logic is reduced to the consistency of a related collection of propositional formulas (Henkin 1953, p.22). The witness condition is assured by the following:

- At the nth extension $\mathscr{M}_n^{2'}$, if nth formula has the form $\exists X_i^j \phi$, then to obtain $\mathscr{M}_{n+1}^{2'}$ the axiom $\exists X_i^j \phi \rightarrow (\phi)_{X_i^j}^{R_k^j}$ is added to $\mathscr{M}_n^{2'}$, where R_k^j is the first j-ary relation constant not appearing in ϕ nor in $\mathscr{M}_n^{2'}$. A similar process goes for closed formulas of the form $\exists x_i \phi$, where the axiom $\exists x_i \phi \rightarrow (\phi)_{x_i}^{c_k}$ is added.

[14]A formal system is MAXIMALLY CONSISTENT if for each closed formula ϕ of its language, either ϕ, or $\neg\phi$ is an axiom of the system.

[15]An alternative is to extend the interpretation $\mathfrak{I}$ of **N** with definitions for the predicate variables and second-order quantifiers, obtaining $\mathfrak{I}^e$, so that it is possible to define the truth of second-order quantified formulas, *i.e.* $\forall X_j^n \psi$ would be true at $\mathfrak{I}^e$ iff $\psi_{X_j^n}^{F_\phi^n}$ were true at $\mathfrak{I}^e$ for every predicate F_ϕ^n; analogously for first-order quantified formulas. However, as the language $\mathbf{M}^2$ does not include the language **N** this proposal seems unreasonable and *ad hoc*.

[16]He says

> (...) we consider how to interpret formulas in which these higher-order variables are quantified. Again, we shall consider a formula $(\phi)A$ to be true just in case every formula A' is true which results from A by the substitution for the variable ϕ of an admissible predicate. (1953, p.24))

[17]These predicates can de obtained easily if we have the lambda operator available in $\mathbf{M}^2$: for variables $\xi_1, \ldots, \xi_n$, relational or individual, with types $\tau_1, \ldots, \tau_n$, and ϕ a well-formed formula of $\mathbf{M}^2$, we would have the second-order predicate $\lambda\xi_1 \ldots \xi_n.\phi$, with type $\tau = \langle 0, \tau_1, \ldots, \tau_n \rangle$.

[18]Although Henkin in (1953) and (1955) did not mention Quine and Goodman's program for providing a nominalistic syntax, in (1966, p.192, fn. 10) some important issues are raised against such a program. Also, Weir (2019, sec. 5) claims that the notion of well formed formula is ill defined in Quine and Goodman's system.

[19]Under a common notion of definability, for a sentence to define the collection of infinite models, it would have to capture them directly so to speak, and not through extensions of them. This means that if ψ defines the collection of infinite models, and a model contains only an infinite domain and nothing else (that is, a model on the empty vocabulary) it should, according to this notion of definability, be included in the collection of models of ψ. Now one could say that this notion of definability is too strict. A metaphor fits nicely here: sometimes to detect an infinite structure one needs to add some reagent to cope with the visual limitations of the system at hand. For first-order logic, this would mean adding a binary relation symbol. The first-order sentence $\forall x \neg Rxx \wedge \forall xyz((Rxy \wedge Ryz) \rightarrow Rxz) \wedge \forall x \exists y Rxy$ has only infinite models, but these models need to contain a binary relation. This laxer notion of definability is known as *projective definability*.

[20]This is a correlate of the notion defined in the beginning of section 3. Here, by "expansion of $\mathfrak{I}$", it is understood an increase of available predicates from the reference language **N** to be substituted for predicate variables.

[21]In this sense, his approach would be similar to the one developed by Leng (2010). I owe this point to an anonymous reviewer.

[22]In contrast with the first-order many-sorted one presented in subsection 3.3.

[23]With respect to the object language.

[24] This claim is made after a remark about the relation between the comprehension axiom in set-theory: $\exists x \forall y (y \in x \leftrightarrow F(y))$, and its second-order version: $\exists G \forall y (G(y) \leftrightarrow F(y))$. The latter would be trivially inferred from the validity $\forall y (F(y) \leftrightarrow F(y))$.

[25]It is claimed that if the subsets definable in the meta-language are allowed to be in their respective second-order domains, the categoricity result for second-order arithmetic still goes through and, for the same reason, the usual methods for proving compactness, completeness and Löwenheim-Skolem fail. The argument purporting to show why categoricity would still hold is based on the claim that the relevant subset of the domain of individuals, needed in the categoricity proof, would be included in Hale's models, as it would be definable in the meta-language (p.2667).

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