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WHAT IS THE AIM OF MODELS IN FORMAL EPISTEMOLOGY?

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Abstract. It is certainly well accepted that formal models play a key role in scientific job. Its use goes from natural sciences like physics and even to social sciences like economics and politics. Using mathematics allows the researcher to consider more complicated scenarios involving several variables. Some models are developed to make predictions, others to describe a phenomena, or just to improve the explanation of events in the world. But what has all this to do with philosophy? The aim of the present paper is to investigate debates on the role of formal models in a specific philosophical subject, precisely, the epistemology of rationality. Are we able to explain why models are needed in epistemological work? This answer will be addressed on the assumptions that epistemological theorizing is committed with normative statements. More specifically, epistemologists are concerned with normative questions about what rationality requires from epistemic agents. The first goal is to discuss some assumptions about the role of mathematical models in formal epistemology undertaking. And secondly, I will argue for the following two claims: (i) formal models are useful tools for predicting consequences of normative assumption about what is intuitively required by rationality; and (ii) insofar rationality theory is normative in virtue of being instrumentalist and aiming at truth, formal models are means-end tools, therefore, for rationality, mathematical models are devices for maximizing truth in doxastic states.

Keywords: Belief • idealization • formal-epistemology • models • rationality

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1. Introduction

It is certainly well accepted that formal models play a key role in scientific job. This has been true for centuries in natural sciences like physics and chemistry, and more recently for social sciences like economics and politics. In philosophy, only in the last few decades the discussion around the use of formal models as a genuine philosophical tool has received proper attention (although formal tools in philosophical activity has been used for more than a century). In the present paper I am concerned with the use of formal models in a specific philosophical domain, that is, the epistemology of rationality.

In this piece I discuss why, despite the controversies, the use of formal models in the epistemological undertaking can better off our small box of methodological tools when doing epistemology. Throughout the text, properly on sections 2, 3, and 4, I briefly show off the main line of critics and troubles to a formal modeler. And, in the final section, I intend to offer some insights that can benefit this topic. Among my theoretical contribution, I intend to establish two theses: (i) formal models are useful tools for predicting consequences of normative assumption about what is intuitively required by rationality (as we will see, the paradoxes of consistency corroborate that intuitive assumptions about quantitative and qualitative rational doxastic states lead us to an inconsistent belief set); and (ii) while a rationality theory is normative in virtue of being instrumentalist and aiming at truth, formal models are means-end tools, therefore, for rationality, mathematical models are devices for maximizing truth in doxastic states.

2. Formal models in sciences

The scientific conception of "model" is extensive. It includes computational algorithms, mathematical frameworks, pictures, and even concrete objects. Here, I will just deal with *formal* models. By "formal" I mean that model's properties are mathematically stipulated. In other words, these models are built on mathematical frameworks (abstract structures), including logic, numbers, sets, functions, vectors, etc. Usually, part of the scientist's work consists in describing a model and linking that model to worldly phenomena through theoretical principles. Using mathematics allows the researcher to consider more complicated scenarios that involve several variables. Some models are developed to make predictions, others to describe a phenomenon, or just to improve the explanation of events in the world.

Shortly, the modeling work has two main parts: the modeling framework and the interpretation of that framework. Knowing how to interpret should be a substantial competence of the modeler. First, we need to interpret how the data enter into the model, and secondly, we need to know how to interpret the conclusions of the model. These are two distinct tasks. A clear interpretation enables a well-established distinction between what is in the model and what is in the world. Regarding, models are abstract structures, they do not require that all elements of the model be in the world, unless the interpretation requires it.

To put some meat on the bones, let's start with a couple of examples in the scientific realm. One of the most common epidemiology model used to understand the spread of an infection within a population is the SEIR (Suceptible-Exposed-Infective-Recovered). Partitioning the population into four exclusive groups, this model is constructed by the following system of differential equations:

$$\frac{dS}{dt} = \mu N - \beta(I)S - \mu S$$

$$\begin{aligned}
\frac{dE}{dt} &= \beta(I)S - (\mu+\varepsilon)E \\
\frac{dI}{dt} &= \varepsilon E - (\gamma+\mu)I \\
\frac{dR}{dt} &= \gamma I - \mu R
\end{aligned}$$

These four differential equations are used to relate the model to the rates β, γ,μ, ε at which the population migrates from one group to another. To make things clearer, each differential equation on the left side can be plotted in a graph (and here we begin to understand the famous talk about "flatten the curve") and, in this way, the model can predict how bad an epidemic could be. Each equation and, consequently, each graph is determined by the parameters denoted by greek letters. β is the infection rate (for how many people an infected person can transmit the disease), γ is period of infectivity, μ is the death rate, ε is the incubation period (the time between the infection and when you might see symptoms). The total number of individuals in the population is denoted by N, and $S+E+I+R=N$.

As expected, I will not put down all the issues here. I just want to draw attention to two aspects of the SEIR model. First, the model lays down a formal relation between premises, such that, if all the input data are accurate, the conclusions will follow. Roughly speaking: "Garbage In, Garbage Out!". Therefore, it does not make the models only a good tool for making predictions, but also a good tool for judging the accuracy of the data inputted in the model whereas the predictions turn out to be true. The second aspect is the idealized or unrealistic nature of models like that. In the construction of the model, at least three false assumptions are made. The first one is the fact that the numbers of individuals in each group (S) (E) (I) (R) are treated as continuous instead of discrete. The second, states that the population is constant, ignoring births, deaths, etc. And third, the recovery rate is equally likely among infectives (Waltman, 2013).

The next example, and the last one, of formal models in the scientific realm is a more controversial one: the model of price in microeconomic theory. In economics, and more precisely in microeconomics, price theory is basically a function of supply and demand determined by a relation between the quantity and price of a particular commodity. And all of this is made by mathematical models. Briefly, the aim of microeconomics is to study the behavior of economic agents in making individual and collective decisions. One background supposition in these models is the controversial *homo economicus* assumption. This supposed agent in microeconomics theory is a decision-theoretic agent, that is, an agent that maximizes the expected utility of his actions, according to its rational preferences (that is, complete and transitive preferences).

At the peak of the debate, Milton Friedman published a book in defense of unrealistic assumptions in economic models. For him, a model has to be judged by its capacity to make precise predictions, not by the accuracy of its premises. In his words:

> Truly important and significant hypotheses will be found to have "assumptions" that are wildly inaccurate descriptive representations of reality, and, in general, the more significant the theory, the more unrealistic the assumptions (in this sense). The reason is simple. A hypotheses is important if it "explains" much by a little, that is, if it abstracts the common and crucial elements from the mass of complex and detailed circumstances surrounding the phenomena to be explained and permits valid predictions on the base of them alone. (Friedman 1996, p.14).

What Friedman calls a "theory" is what we are calling here a "model". In that quote we can clearly see the unrealistic aspect of assumptions in the model, and all of that is just for the sake of accurate predictions. In a nutshell: a formal model is a tool for a scientist's end (at least in Friedman's sense).

The aim here is not to give a detailed account of one mathematical model in a specific field. I believe the reader is already relatively well acquainted with a lot of examples like the ones above. All I need is a parallel in science. As we will see below, the use of models in epistemology sometimes approximate and some other times differ from the above cases, and the justification for its use needs to be done when the philosophical methodology meets or strays from scientific ones. So, let's talk about philosophy.

3. What does all of this have to do with philosophy?

In science, some mathematical models are developed to make predictions, others to describe a phenomenon, or just to improve the explanation of events in the world. What about philosophy? Despite the controversies, the use of formal models in philosophical problems has been widespread over the last decades. However, before moving on, it is important to shed light on a common philosophical practice that is not precisely a modeling work, although sometimes it is treated as one.

A usual practice within contemporary philosophy, especially that of English speakers, consists in shifting from the description of a philosophical problem in words to its development in terms of symbols and equations. Sometimes, the use of formal language allows the philosopher to express complex relations and its use makes his work more precise and compact. This is the use of formal language for the abbreviation of natural language. In the $20th$ century, the use of formal language as abbreviation became standard in epistemology. In the remarkable paper "Is Justified True Belief Knowledge?", Edmund Gettier portrays the satisfaction condition for knowledge as "*S* knows that *P iff P* is true, [...]". It was probably due to the use of formal language as abbreviation that he brightly changed the history of epistemology with only three pages. Nevertheless, this is not really an example of modeling in philosophy.

It might be the case that philosophers use formal language to describe not a particular problem, but rather a formal *model*. In this sense, the use of models in philosophy goes beyond the mere use of formal language as abbreviation. As Timothy Williamson says, “The mathematical clarity of the description helps make direct study of the model easier than direct study of the phenomenon itself”. (2017, p.161). The aim here is to investigate debates on the role of formal models in a specific philosophical realm, precisely, the epistemology of rationality. I intend to show that the epistemologist’s work can be enhanced by adding formal modeling as a tool.

The herein case studies will be typical models in formal epistemology and, more precisely, the epistemology of rationality. Firstly, epistemic rationality is about rational belief. There are two main notions of belief: Full and Partial Belief (also known as belief *simpliciter* and credences, respectively). The former is a qualitative notion, whereas the latter is a quantitative one. Epistemology has traditionally considered belief (*simpliciter*) as an indispensable constituent for knowledge in addition to justification and truth. On the other hand, credence plays a key role in other scientific domains, like probability and decision theory foundations. The credence encroachment in epistemology fostered the emergence of what we know today as Bayesian Epistemology, i.e., epistemology developed with credences and probability theory. Both of them got substantial development in their formal aspects over the years. Before moving on, let’s take a brief look at each of them.

Starting with credence, its formal development was simultaneous with the emergence of the subjective probability conception (see Ramsey 1926; and Savage 1972). Understood as synonymous, credence and subjective probability have got their canonical normative framework based on the axioms of probability theory. Roughly speaking, for an epistemic agent to be rational, the credence doxastic attitude should obey probability theory. Basically, probability theory is grounded in three axioms, plus a definition of conditional probability:[1]

(Non-negativity): $P(A) > 0$

(Normality): $P(W) = 1$

(Finite Additivity): If $P(A \cap B) = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

(Conditional Probability): $P(A|B) = \frac{P(A \cap B)}{P(B)}$ (with $P(B) \neq 0$)

These formal conditions are the starting point of bayesian epistemology.

On the belief side, it is a little more difficult to postulate a canonical normative framework. I intend to follow here some postulates that are compatible with two of the most famous model for belief: AGM theory (see Alchourrón, et al, 1985) and

Epistemic Logic (see Hintikka, 1962). I am assuming that, to be rational, an epistemic agent should have a belief set that complies with the following postulates:

$Bel(W)$;

$\neg Bel(\emptyset)$;

If $Bel(A)$ and $A \subseteq B$, then $Bel(B)$;

If $Bel(A)$ and $Bel(B)$, then $Bel(A \cap B)$.

Roughly, this is basically the logical closure requirement for a belief set.[2]

For present purpose, we will regard these logical/mathematical postulates as describing a model for the rationality of the aforementioned doxastic states. Instead of immersing in these models and their use in epistemology, I am focusing on the methodological aspect usually implicit in their use. It is not hard to see that the above examples do not follow the same pattern of scientific models, as shown previously. Rather, epistemic models are not tools to confirm empirical hypotheses, or their premises, from well-realized predictions. In general, they also do not suit to predict how real human beings behave and, in some cases, they contradict some human behavior patterns. And both of them take epistemic agents as idealized, something like an epistemic superhero. So, are we able to explain why formal models are relevant in philosophical work?

By the way, it is well known that these models do not represent how humans behave. But someone can contend that these kinds of epistemic models are strictly normative, and that means that they set rules that human agents "should" follow. However, these are really hard requirements for epistemic agents. After all, an outcome of the aforementioned epistemic models is the requirement that epistemic agents be logically omniscient, that is, they need to believe with maximal confidence in every logical truth and believe in every proposition entailed by his present belief set. Before starting to analyze the normative status of formal models in epistemology, I would now like to make the notion of "idealization" present in this discussion clearer.

4. What are idealized models?

If you have read a formal epistemology piece, you probably have already faced the notion of "idealized agents". This is a common assumption in almost all formal work on epistemology. For some epistemologists, this kind of assumption sounds like a freak, whereas others take it as so obvious that it does not even deserve to be explicitly mentioned in the text. But what is really assumed when an epistemologist considers an idealized agent in his theory? This is the question I will take up in what follows.

According to Michael Titelbaum (Forthcoming), one kind of idealization simplifies for the sake of mathematical calculus — the same as the examples in the scientific domain from the last topic. An ideal agent, on the other hand, is ideal in the sense of doing what non-ideal agents should do, or by being better than non-ideal agents in a normative dimension. Such relevant distinction also arises in David Christensen (2003):

> [. . .] such a model may idealize normatively: it may seek to represent ideally rational beliefs — beliefs that exhibit a kind of perfection of which actual humans are incapable. But it may also idealize in the way in which countless purely descriptive models idealize: it may assign a number to a quantity whose application to real instances is not completely precise. (Christensen 2003, p.145)

With this point in play, I will inspect this relevant distinction taking into account the methodological aspects generally assumed in philosophical practice and their contrast to science.

4.1. Idealization as simplifying

Idealization is often connected with simplicity. That simplicity can be understood from two distinct aims: as a mathematical simplification, or as a simplification of the phenomena. Certainly, sometimes these two aims take place simultaneously. Starting with the first, idealization by mathematical convenience is more common than you might expect. Coming back to our examples from the first section, scientific models often simplify for mathematical convenience. And this also happens within our epistemic models. Now, the following parallel can be drawn: inasmuch as SEIR epidemic models assume that the number of people can be represented by real numbers (and we know that the number of people in a population is always discrete), then it makes perfect sense that bayesians assume that credences can also be represented by a continuous line of real numbers. Now, this comparison immediately raises the question: is this a valid parallel?

The requirement that degrees of belief be represented by real-valued functions (probability functions) seems to be a kind of idealization by mathematical convenience: there is not anything more or less rational in representing epistemic agents with continuous, instead of discrete, functions, or with cardinal instead of ordinal doxastic states. In this sense, some aspects of a model framework are built just for the sake of mathematical tractability. A good case is the existence of alternative theories to degrees of belief, other than the bayesian mainstream model of subjective probability. Some philosophers (see, e.g., Shafer 1976; Spohn 2009), with daring logical/mathematical constructions, have searched for alternative ways to represent the same credence phenomena, and small differences in understanding the relevant phenomena led them to completely different frameworks. Despite the substantial debate about the normative assumption of each of these theories (which is not my focus here), the distinct mathematical framework behind each one has an instrumental purpose whose intention is to better apply it in a suitable conception of degrees of belief (credences).[3]

Going to the second simplification type in models, we have the phenomenon simplification. In some cases, we can simplify by "being close enough". This happens when a topic is too complex to handle directly. The example of *homo economicus* in Microeconomic seems to fit well here, and, I believe, it also fits with most kinds of philosophical theories of rationality. According to Sven Hanson,

> The reason why we idealize-simplify is that philosophical subject-matter is typically so complex that an attempt to cover all aspects will entangle the model to the point of making it useless. (Hanson 2008, p.16).

In such a case, a theory that tries to address all aspects of the relevant phenomenon would turn out unattainable in our actual theoretical context. For that reason, the modeler detaches an important variable from the theory and investigates that isolated point. One consequence is that the outcome might be a model that looks very different from the studied facts. Following Hanson, such deviation should always be judged relatively to the purpose of the model and how it is used: if any pertinent feature was "lost" in the idealization, we should consider how much we will lose in simplicity if such a feature is included. In his own words, "[p]hilosophical or scientific model-making is always a trade-off between simplicity and faithfulness to the original" (Hanson 2000, p.164).

Sometimes, the simplification employment in philosophical problems is very ungrateful. In a clever way, Audrey Yap asserts that "either epistemic logic fails to be epistemic or it fails to be logic" (Yap 2014, p.03). For some, the quest for an epistemic logic is an unattainable goal by its own nature. For others, such not-really-epistemic deviations in epistemic logic are just a side effect from the enterprise of being a logic. I believe that most of this quarrel can be solved by a better understanding of the formal models' role on the distinction about what is in the world and what is in the model, and about the kinds of idealization underlying such theory.

4.2. Idealization as Cognitive Perfection

As previously discussed, a fraction of bayesian formalism can be regarded as a type of idealization as simplification. Nevertheless, the requirement that agents should comply with probability axioms seems to bear a normative component: agents that disrespect them in their reasoning process might be suffering some kind of loss. Such "loss" has been inquired a lot by bayesian literature already, being a finance loss (from a Dutch Book argument), or being an epistemic loss (from an Accuracy Dominace Argument).[4] Hence, an epistemic agent that complies with probability axioms is ideal in the sense of doing what non-ideal agents "should" do. And the relevant kind of idealization here is an instance of cognitive perfection.

Idealization as cognitive perfection is a type of idealization that we do not commonly see in empirical models of science.[5] Dealing with normative notions is recognized as a properly philosophical undertaking. In this sense, it is a common place to suppose normative concepts in an epistemic work, and here is where our job moves away from the scientific one. About the distinction between normative and descriptive inquiry, Gilbert Harman says:

> Actually, normative and descriptive theories of reasoning are intimately related. [...] it is hard to come up with convincing normative principles except by considering how people actually reason, which is the province of a descriptive theory. On the other hand it seems that any descriptive theory must involve a certain amount of idealization, and idealizations is always normative to some extent (Harman 1986, p.07).

Note that this kind of idealization is not an exclusivity of formal epistemology, it also happens in traditional epistemology and in others philosophical fields. Even in a clique of traditional epistemologists, an intrinsic connection between the ideal agent assumption and the normativity of epistemology is often assumed.

The conflict between how we should begin theorizing, whether normatively or descriptively, is on the background of most works on the epistemology of rationality: there is always a theoretical trade-off between considering how people actually reason and considering how an ideal agent should reason based on "normative principles". About this kind of trade-off, Harman says:

> How then are we to begin to figure out what these principles of revision are? There seem to be two possible approaches. We can begin by considering how people actually *do* reason, by trying to figure out what principles they *actually follow*. Or we can begin with our "intuitions" as *critics* of reasoning (Harman 1986, p.09).

In this sense, "normative principles" would be a certain kind of "philosophical intuition" (although I am not able to specify exactly what kind it is). Following Harman's quote above, I will call such conflict (about how we should begin theorizing) the Harman's Dilemma. This will be central on the next topic, and I shall return to that distinction soon.

Back to the notion of idealization, it is often hard to decide whether an idealization is made intending mathematical convenience, intending a simplification of the phenomenon, or to show what an ideal agent should do. A paradigmatic case is the logical omniscience in standard epistemic logic. The fact that agents of epistemic logic clearly have capacities that are not possessed by human beings has led some to contend against epistemic logic as a theory about actual knowers. One possible answer is to argue that logical omniscience is a side effect of the formal modeling (and its idealization) in epistemic logic. But it is not clear what kind of idealization underlies the requirement of deductive closure (the main responsible for the problem of logical omniscience), whether it is a normative one with capacities of unlimited cognitive agents or a simplification for the sake of the theoretical undertaking.

In the remainder of this paper, I will argue that the use of a methodological approach that employs formal models to some epistemic problems is a point of no return. However, doing epistemic modeling in a suitable way requires the exact understanding of its role as a model. To do that, we need to consider the above distinction about different kinds of idealization: different motivations should be taken into account when we wish to criticize a model. In this sense, a model should not be criticized as inadequate simply because it invokes idealizations or false assumptions. And finally, I want to draw attention to the normative assumption on a specific theory of rationality and the characteristics of idealized agents in its model.

5. Why do epistemic modeling?

In this section, I will support two main reasons for the epistemic modeling enterprise. The first is motivated by the key element that distinguishes epistemic modeling and scientific modeling: its normative assumption. My purpose is to reinforce that some epistemic problems involving normative assumptions, especially about epistemic rationality, cannot be properly understood without some kind of formal modeling. And, to finish, the second reason for doing epistemic modeling is motivated by a specific theory of rationality: the instrumental theory of rationality. I shall introduce a couple of reasons for taking rationality as a means-end tool, and explain how epistemic modeling can help us with the quest for a theory of rationality. On the whole, I believe that the following considerations can represent progress in the current literature about this subject.

5.1. Formal Models for Normative Theories

Faced with the objections that real agents fail to comply with the requirements of most epistemic models, some formal epistemologists argue that model idealization is a tool for normative theories, and it is more about how agents "should" behave than about how agents actually behave. According to Titelbaum (Forthcoming) and Colyvan (2013), epistemic theories of rational belief are normative in the sense that ideal agents are "epistemically better" than real ones. So, Bayesian and Logical models (as presented in section 3) are models managing "normative facts". According to Colyvan (2012, p.1340), as part of normative theories, formal models must be taken as a prescription on how we should reason, organize our beliefs, and so on.

One possible way to understand what "normative facts" are is regarding them, quoting Harman (1986, p.09) again, as "intuitions" arising from us as "*critics* of reasoning". In the present section, close to Titelbaum (Forthcoming) and Colyvan (2013), I will accept that epistemic formal models are models for some kind of normative theory about rationality. And assuming that theories of rationality are normative, their models must include some kind of "normative intuition" (as critics of reasoning) as input. At this time, epistemic modeling is deeply similar to the scientific one; it looks like scientific modeling but with normative facts. However, different from the scientific manner of drawing conclusions from the inputted data, epistemic modeling is also able to judge the suitability of such inputs, and this can be done by paying attention to the consequences which result from what is inputted in the model. As Wheeler (2012, p.233) asserts, "discovering robust features of a problem can reshape your intuitions. In precisely this way formal epistemology can be used to train philosophical intuitions". Summarizing, formal models can calibrate our intuitions about epistemic rationality.

To develop this point, I wanna take the famous Lottery Paradox as a case study. The Lottery Paradox (Kyburg, 1961) shows that our initial intuitions about what is rational can conflict with each other. Think about the following case. Let's begin by supposing you bought a lottery ticket. You know that the lottery is fair and exactly one ticket will win. You have got a very low confidence that your ticket will be the winner (it depends on the lottery size). So, it seems reasonable to believe that your ticket will not be the winner. But, your ticket does not have anything special among the others, and you are also entitled to believe the same thing for all the other tickets. Therefore, by the end, you believe that each individual ticket will lose. However, you also believe that at least one ticket will win. By the end, the agent has an inconsistent belief set.

That was an informal presentation of the lottery paradox. We can see that there is no need of a remarkable formal capacity to understand its paradoxical aspect. However, noting the existence of a paradox does not seem enough for us: we want to understand what is grounding the paradox and, if it is possible, offer a solution (nobody likes to be trapped by paradoxes!). Now, let's look at a more formal presentation of the lottery paradox.

Suppose the existence of an intuitive relationship between rational belief and rational credences built by a threshold model: believe in A is rational if and only if the subjective probability in A is greater than some threshold s, and s is a real number between 0.5 and 1. This is known as the Lockean Thesis (see Foley 1992). Suppose the threshold value is $s = 0.9$. Thus, to believe in a given proposition A, it credence must be above 0.9, i.e., $P(A) > 0.9$. Now, think about a fair small lottery with 100 tickets. So, it is rational to assign credence of 0.01 to the propositions that "ticket i will be the winner" (given $i = 1, \ldots, 100$, for all tickets). So, for each individual ticket, the credence in the proposition "it is not the case that ticket i will be the winner" is 0.99, and $0.99 > s$; that is, $P(\neg A_i) = 0.99$. Therefore, if $P(\neg A_i) > s$, so $Bel(\neg A_i)$. But we know that exactly one ticket will win, which means that $P(A_1 \cup \ldots \cup A_{100}) = 1$, and $1 > s$. So, we believe that one ticket will win: $Bel(A_1 \cup \ldots \cup A_{100})$. And given that we are justified in $Bel(\neg A_i)$, for all i, a conjunction rule enables the belief in this following set of not-winner-tickets propositions: $Bel(\neg A_1 \cap \ldots \cap \neg A_{100})$. Nevertheless, according to De Morgan's Law: $(A_1 \cup \ldots \cup A_{100}) = \neg(\neg A_1 \cap \ldots \cap \neg A_{100})$. As expected, an inconsistent belief set!

Why did I show the same paradox twice? Well, in the former example we notice that it is possible to observe informally that there is something wrong with our beliefs in that case. On the other hand, it is impossible to understand what is really going on without using some of the formal theories of rational belief. The lottery paradox is not a simple effect of language misuse. The paradox shows us that there is a deep problem with our best theories about rational belief. I am supposing that what is wrong in lottery cases is what Carl Hempel (1962) called "The Nonconjunctiveness of Statistical Systematization": the probability of a conjunctive set of propositions falls inasmuch as we add more propositions in the conjunction. And this problem cannot be simply answered informally, its formal structure is part of the paradox's own nature. Hence, trying to give an answer to the lottery paradox requires a deep study on the bayesian and logical model of rational belief, as seen previously.

The problem of only looking at the Lottery Paradox informally is that we would hardly be able to isolate relevant factors and realize what is really happening. Epistemic modeling enables us to split the problem in three distinct parts: the model of rational belief, the model of rational credence, and the threshold model. In this sense, epistemic models are close to the scientific ones, but with normative facts about what is required by rationality. In a few words, formal models can help us predict consequences and shed light on normative theories of rational belief. About this, Colyvan asserts,

> In case of the normative theories, the fruits might be thought to include: meshing with intuitions about what to do in given situations, shedding light on other situations where intuitions fail, and perhaps facilitating elegant representation theorems (Colyvan 2013, p.1347).

Now, we can ask ourselves: given the incompatibility between the models of rational belief, which of them must be rejected? Should any be rejected? The answer to that question depends on what we should take as a judgment criterion for each model. In other words: What makes a theory of rationality a good one? And here we need to come back to the Harman's Dilemma, from the last section: doing theory of rationality requires managing the conflict between how people actually behave and our philosophical intuitions about what a rational requirement is. Insofar as rational theory, as assumed, has a normative component, at some moment intuitions must come into the model. In the lottery case, we may question, just to cite two examples, what is the normative status of the consistence requirement (i.e., your beliefs should not contradict each other), or we may question the normative status of conjunctiveness requirement (i.e., all of your beliefs can be laid in a single big conjunction). Given their intuitive appeal, only a philosopher could question something like that. But what are these normative intuitions and how can we justify them? I cannot fully answer this hard question here, but I hope to, in the following pages, be able to address some interesting matters about normative intuitions and the modeling practice in epistemology.

In many cases, we are not able to distinguish if the normative requirement in a theory of rationality is an effect of philosophical intuitions or if the respective intuition is an effect of the model. For example, we can ask a couple of questions: What justifies the non-contradiction principle in doxastic logic? Is it the model that justifies the requirement or the requirement that justifies the model? Is it a philosophical intuition that justifies classical logic or the logic that justifies the intuition? Of course, these are very complex and fascinating questions that encompass epistemology, philosophy of logic, science, and math. And all of this relates to the question of what makes a theory of rationality a good one.

Trying to make things clearer, I will show an interesting example in Bayesian model of rational credence. I am calling "The nonconjunctiveness requirement" the probabilistic rule that says that the probability of two or more events is never strictly greater than the probability of only one of them. The fact that real agents seldom comply with this requirement gives rise to the famous "Conjunction Fallacy Experiment", or also known as the "Linda Problem". In a time-honored work (Tversky and Kahneman 1983), the psychologists Amos Tversky and Daniel Kahneman tested the people's intuitions on probabilistic judgments. What they found was that the participants of the experiment deviate from the nonconjunctiveness requirement if the less probable scenario is more "representative" than the more probable and less "representative" one. What they concluded was that the participants follow some kind of "heuristic" in their reasoning, instead of some probabilistic rule. And this is not necessarily a bad thing. The reasoning guided by the "heuristic" procedure is cognitively easier than some reasoning guided by some abstract bayesian rule. But what does all of this mean?

The last paragraph suits well as an example of how much the Harman's Dilemma is significant to our methodological decision in doing theory of rationality. Occasionally, facts about how people reason do not fit well with our normative theories of rationality. But again: what justifies our theory of rationality? Well, for bayesianism, the aforementioned (section 4.2) and well established arguments for probabilism is grounded both on the pragmatic advantage from Dutch Book Arguments and the epistemic advantage from Accuracy Dominance Arguments. Hence, the nonconjunctiveness requirement is not a rule grounded only in our "intuition as critics of reasoning", rather, it is also grounded in our bests arguments about how a bayesian agent maximizes truth (or wellness, in the practical instance). And this is not the case only for the nonconjunctiveness requirement, but for all probabilistic rules. What this means is that bayesian requirements for rationality need not be grounded mainly on an ideal philosophical intuition resulting from an exceptional mind, but rather a possible way to maximize a pre-established goal.[6]

5.2. Formal Models as Means-End Tools

The final topic of this paper is an attempt to close some open questions from the last section and to convince the reader of the benefits of doing epistemic modeling. Rather than being a simple model for normative theories of rationality, we can regard bayesian and logical models as instrumental tools. The following analogy can be useful. Insofar as a model in Decision Theory can provide the agent with an action path that maximizes expected utility in a complex context where the agent is unable to "perceive" (or calculate) the best option, an epistemic model can provide for the agent (or for someone watching him) the best path for truth maximization in complex circumstances. This means that formal models of theories of rationality suit well with the instrumental notion of rationality, that is, taking rationality as a tool for means-end calculus.

The idea of judging a rational requirement by its capacity to promote truth maximization is not something new. Proponents of Epistemic Utility Argument (see Joyce 1998; Easwaran 2016; Easwaran and Fitelson 2015) contend that the rationality of an epistemic state is settled by a function that measures the epistemic utility (or disutility) of having that epistemic state in a specific world. By the way, rational requirements in epistemic utility theory are requirements of standard utility theory with the goal of maximizing the epistemic value. And the most common assumed epistemic goodness is truth, or also, "accuracy.[7] As mentioned in section 4.2, Dutch Book and Accuracy Dominance arguments are commonly used to justify probabilistic rules for bayesianism, and both are based on norms of utility theory, mainly on the decision-theoretic notion of dominance.[8]

What is provided by epistemic utility theory makes rationality theory something more tractable. As a result, the role of idealized agents in modeling becomes something better oriented. The lottery paradox is a good example. Some of the newest and well develop solutions to the paradox result from super complex models with cognitive perfect agents.[9] But this does not mean that they are not good solutions to the paradox only because real epistemic agents are not able to carry out the requirements within the models. Rather, in this case, these models can precisely explain what is wrong with our epistemic suppositions. Idealized agents are just theoretical devices to improve our understanding about truth maximization in belief sets, not a sample of a human being to be followed. Therefore, the role of epistemic modeling in philosophical problems about lotteries is to improve our understanding of the relationship between rational belief and credence concerning truth maximization, not to teach us how to bet in a real lottery.

It turns out that when someone does not comply with one rational requirement, this does not necessarily imply that he has broken an intuitive rule, neither that his behavior has not followed some descriptive pattern: maybe he might just have not maximized truth in his belief set. An here we can go back to Harman's Dilemma and add a further option to the package of theoretical choices: understanding epistemic rationality can be done by considering how people actually behave, what is intuitively required by rationality, and also by looking at what models advise. Inasmuch as formal models fit an instrumental conception of rationality, the advantage is that we do not need to assign so much power to philosophical intuitions about normative concepts in our quest for a theory of rationality.

One of the fruitful consequences of this view is the existence of a clear distinction between what is strictly normative and what is rational. Although rationality theory must deal with normative elements at some moment, this does not imply that a theory of rationality gives us reasons about what to do "all things considered". What human beings should do with rational doxastic states, that is, the question "Why be rational?", is not an issue for a theory of rationality, but rather a question for a philosophical field concerning "normative reasons".[10] In this sense, to assume that bayesianism is a proper model of credence rationality does not imply that epistemic agents "ought to" reason according to probabilistic axioms or to assign maximal confidence in all logical truths. To sum up, whether or not agents ought to maximize truth in their doxastic system is not an issue for rationality on its own. Although I acknowledge that this is not a "solution" to problems like the logical omniscience one, I believe that a better understanding of the relationship between logic, models, normativity and rationality can lower its destructive power.

I hope it is now clear why epistemic modeling can improve the quest for a theory of rationality. If the answer to the previous question "Why should we comply with bayesian requirements?" was only "Because they are in agreement with our intuitive rational principles", now, the answer can be "because they also maximize truth in epistemic states". Hence, we do not need to rest all the theory on normative intuitions about rational requirements and, I believe, that it is enough to justify epistemic modeling. Why, then, limit our options?

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Notes

[1]Through the paper, I will assume that the objects of doxastic attitudes are propositions. And propositions must be understood as a set of possible worlds. So, we have $P(A)$ denoting the probability of some proposition $A \subseteq W$, where W is the universal set that contains all possible worlds. And the same goes for belief, with $Bel(A)$ denoting the belief in some proposition $A \subseteq W$.

[2]Either probabilistic or qualitative belief requirements are considered herein only in its synchronic aspect. You can check some remarks on the diachronic topic in Genin and Huber (2021).

[3]A wide debate about these alternative conceptions of degrees of belief can be found in Huber and Schmidt-Petri (2009) and Genin and Huber (2021).

[4]If the reader is not acquainted with the literature of arguments for probabilism, see Pettigrew (2019), and Vineberg (2016).

[5]The reader now might think about the "homo economicus assumption", as we saw earlier in section 2. But even in that scenario, if we assume Friedman's notion of modeling in microeconomic theory, the "homo economicus assumption" has a pure instrumental purpose, as a means to facilitate predictability. In this sense, I believe that a frontier between scientific and philosophical undertaken can be drawn by the way which field deals with that kind of normative assumption in its model: while empirical science uses normative assumption as an instrument, philosophy is the main responsible for taking that assumption as the primary aim of the investigation. But I acknowledge the existence of subjects that we are not able to fit totally by that distinction, as, for example, some interpretations of Game Theory. A remarkable discussion about Game Theory and the "Normative vs. Descriptive" conception can be found in Binmore (1997).

[6]It is relevant to note that the both Dutch Book Arguments and Accuracy Dominance Arguments have assumption that are not accepted by everyone. The reader can check Pettigrew (2019) and Vineberg (2016) for these controversies. I acknowledge that the "acceptance" of these theses is conditional for my argument.

[7]While truth is taken as a qualitative notion, accuracy is a quantitative one: a belief can be more or less accurate according to its proximity to truth.

[8]The principle of Dominance warrants that, if A and B are two distinct attitudes, A is dominated by B if and only if, for any possible world in W, the utility of B is greater than the utility of A.

[9]See, for example, Leitgeb (2017) and Lin and Kelly (2012).

[10]On the distinction between rationality and normativity, see Broome (2013) and Kolodny (2005).

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